

UiO : **Department of Physics**
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Bandlimited fractional differential operators

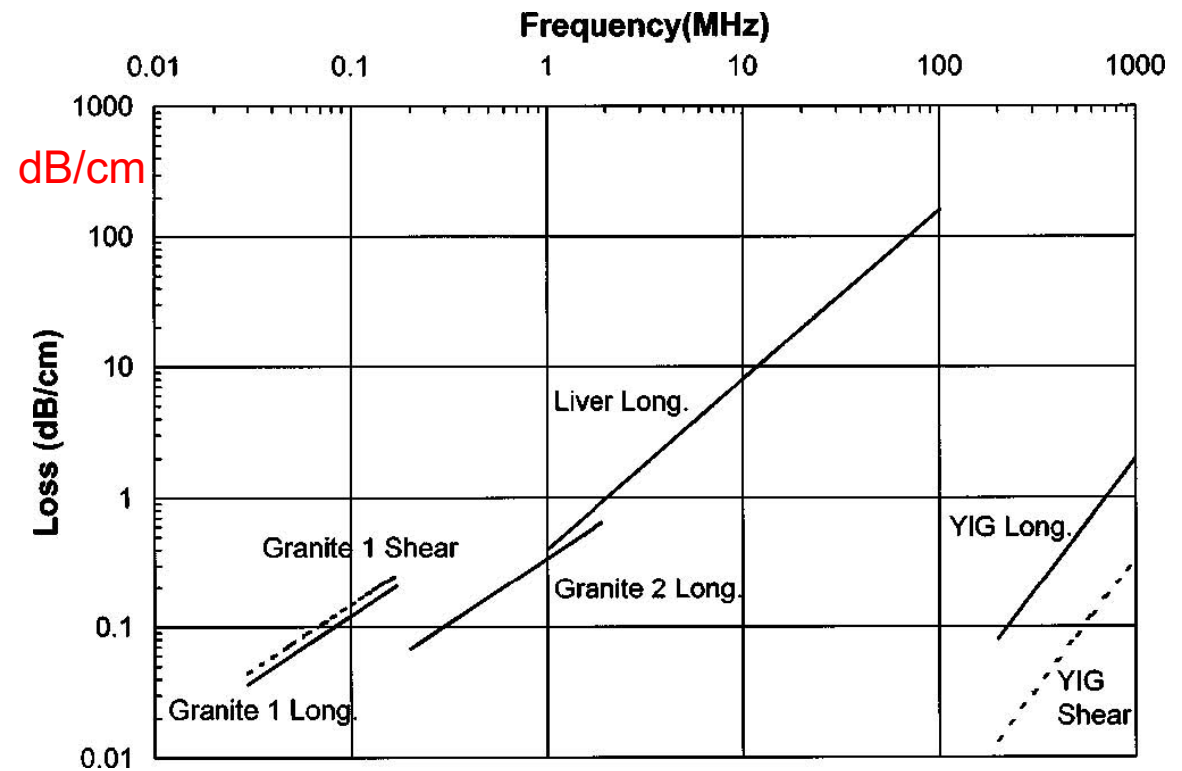
Sverre Holm



- Many phenomena can be described by power laws in the frequency domain which are different from f^2 . Often attenuation in medical ultrasound, shear wave properties in elastography imaging, and sub-bottom wave propagation in underwater acoustics are better described by something which nearly follows f^1 .

Limited range power laws: loss ω^y

- Medicine, geophysics, ...
- Longitudinal, pressure:
 - Granite: $y \approx 1$
 - Liver: $y \approx 1.3$
- Shear:
 - YIG: $y = 2$
(Yttrium indium garnet)
 - Granite: $y \approx 1$
- Szabo and Wu, “A model for longitudinal and shear wave propagation in viscoelastic media”, JASA (2000).



Data for shear and longitudinal wave loss which show power-law dependence over four decades of frequency.

Bandlimited fractional differential operators

- The inverse Fourier transform of a frequency power-law corresponds to a non-integer or fractional derivatives. However, the measurements are always band-limited, i.e. for medical ultrasound one only has measurements from say 1 to 20 MHz. The Fourier property, however, is valid for all frequencies from 0 to infinity.
- Here we want to develop new operators that correspond better to measurements of the frequency response of real band limited propagation media.

Derivative of arbitrary order

- Derivative of order α :

$$\frac{d^\alpha f(t)}{dt^\alpha} =_a D_t^\alpha f(t)$$

- $\alpha < 0 \Leftrightarrow$ integration
 - a and t : limits in defining integral
- Fourier transform (neglecting initial cond's):

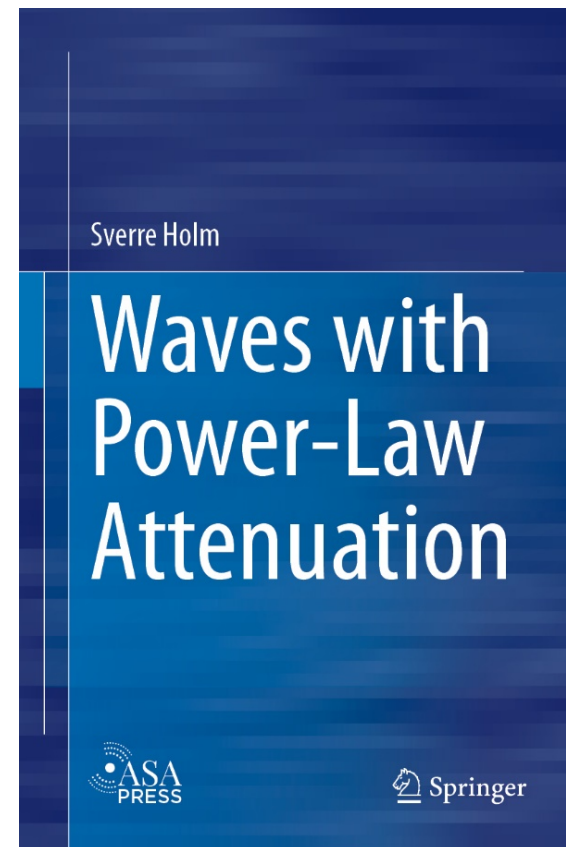
$$FT \left(\frac{d^\alpha f(t)}{dt^\alpha} \right) = (i\omega)^\alpha F(\omega)$$

References

- S. Holm, [Waves with Power-Law Attenuation](#), 2019, Springer, chs. 1, 5.1-5.3, and 9.4.
- E. Torgersen, [Niels Henrik Abel får enda en fjær i hatten](#), Titan, februar 2020.

Anomalous wave propagation

- Interpretation of frequency variation in shear speed
- Interpretation of frequency variation in shear attenuation





<https://titan.uio.no/naturvitenskap/2020/niels-henrik-bel-far-enda-en-fjaer-i-atten>

Niels Henrik Abel laget formler som i dag brukes til å beregne aksjekurser og bølger som bryter. Foto: Stein Henningsen, Colourbox og Severin Worm-Petersen

Niels Henrik Abel får enda en fjær i hatten

Aksjekurser, ultralydbølger i kroppen og turbulens blir lettere å beregne når forskerne benytter seg av det som kalles ikke-heltallige deriverte. Nå viser det seg at Niels Henrik Abel hadde gjort mer på dette feltet enn det som tidligere var antatt.

30.10.2020

Av Eivind Torgersen

26. februar, 2020