SUBJECTIVE LOGIC AND BAYESIAN BELIEF REASONING

Tutorial at FUSION 2022

4 Juli 2022

Audun Jøsang

UiO : University of Oslo

About me

- Prof. Audun Jøsang, UiO
  Research interests
  - Information Security
  - Reasoning under uncertainty
- Bio
  - Telecommunications Engineer, 1988
  - MSc Information Security, London 1993
  - PhD Information Security, NTNU 1998
  - Associate Prof. QUT, Australia, 2000
  - Prof. UiO, Norway, 2008

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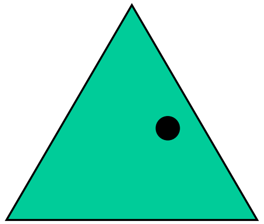
Tutorial overview

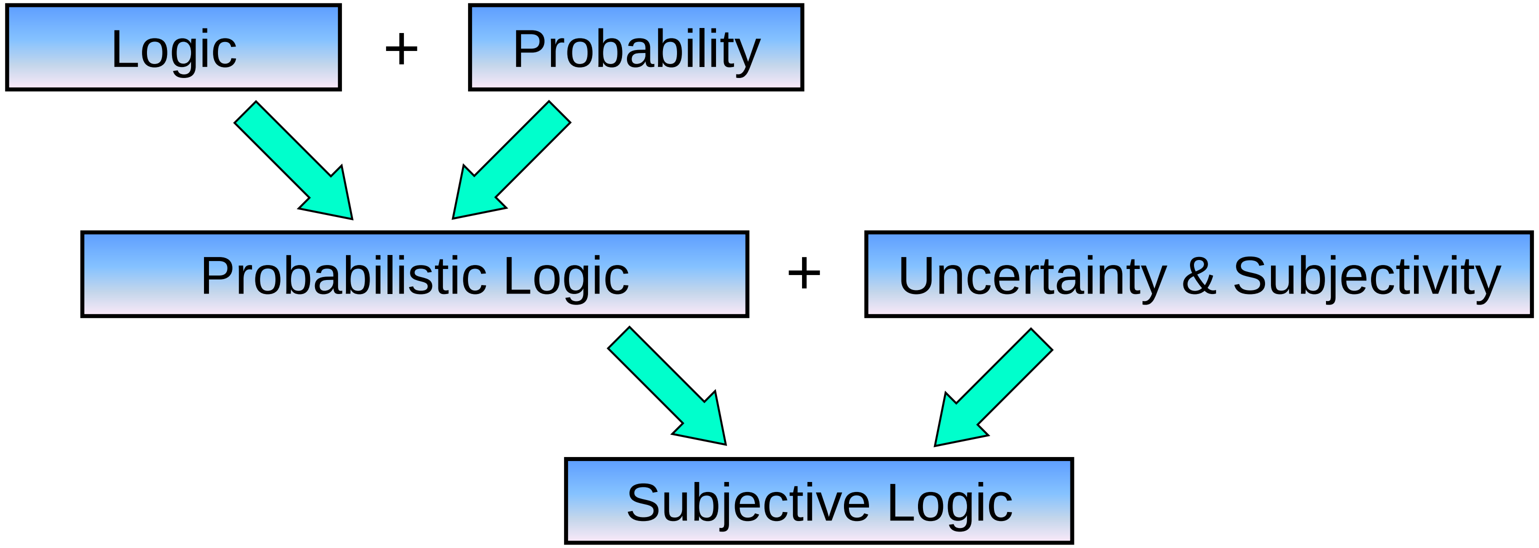


1. Representations of subjective opinions

2. Operators of subjective logic

3. Bayesian belief reasoning:
   – Trust fusion and transitivity
   – Trust networks
   – Bayesian reasoning
   – Subjective networks

The General Idea of Subjective Logic



Example Correspondences

Binary Logic	Generalization → Probabilistic logic
AND: $x \wedge y$	Product: $p(x \wedge y) = p(x)p(y)$
OR: $x \vee y$	Coproduct: $p(x \vee y) = 1 - (1 - p(x))(1 - p(y))$
MP: $\{x \rightarrow y, x\} \Rightarrow y$	Deduction: $p(y) = p(y \mid x)p(x) + p(y \mid \bar{x})p(\bar{x})$
Contraposition CP: $x \rightarrow y \Leftrightarrow \bar{y} \rightarrow \bar{x}$	Bayes' theorem $p(x \mid y) = \dfrac{p(y \mid x)a(x)}{p(y \mid x)a(x) + p(y \mid \bar{x})a(\bar{x})}$ $p(x \mid \bar{y}) = \dfrac{p(\bar{y} \mid x)a(x)}{p(\bar{y} \mid x)a(x) + p(\bar{y} \mid \bar{x})a(\bar{x})}$
MT: $\{x \rightarrow y, \bar{y}\} \Rightarrow \bar{x}$	Abduction: $p(x) = p(x \mid y)p(y) + p(x \mid \bar{y})p(\bar{y})$

a: base rate

Aleatoric and Epistemic Uncertainty

Aleatoric uncertainty

- Aleatoric uncertainty results from knowledge that is conflicting or balanced
- Low aleatoric uncertainty when probability is close to P=0 or P=1
- High aleatoric uncertainty when P= ½
- *E.g.: Probability of heads when flipping coin is* P= ½ *, and hence high aleatoric uncertainty, but dynamics of situation are known, hence low epistemic uncertainty.*

"alea" = "dice" in Latin

Epistemic uncertainty

- Epistemic uncertainty results from lack of knowledge
- Low epistemic uncertainty when circumstances and dynamics of the situation are known
- High epistemic uncertainty when circumstances and dynamics of the situation are unknown
- *E.g.: Probability that Oswald was the assassin of US president Kennedy in 1963 is* P= ½ *, but lacking knowledge, and hence high epistemic uncertainty.*

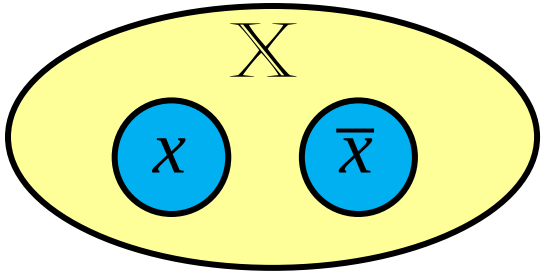
"epistemology" = study of knowledge and understanding

Domains, variables and opinions

Binary domain $\mathbb{X} = \{x, \overline{x}\}$

Binary variable $X = x$

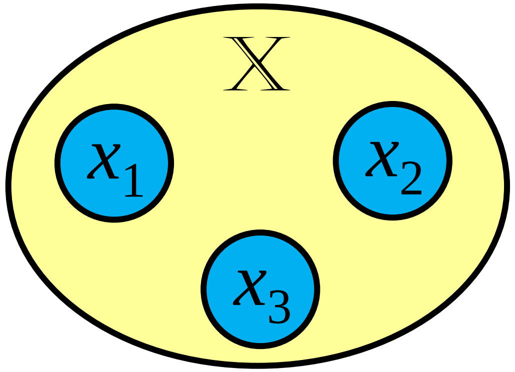
Binomial opinion



3-ary domain $\mathbb{X}$

Random variable $X \in \mathbb{X}$

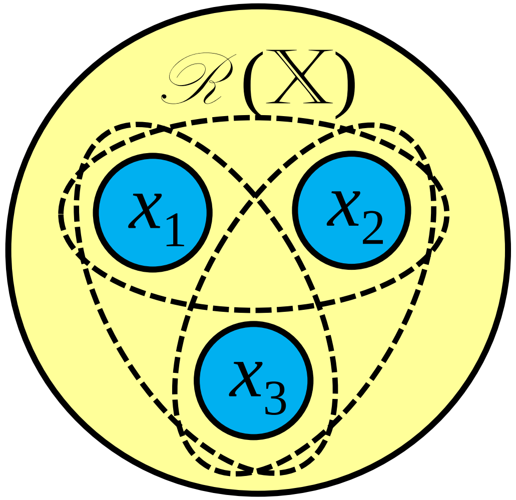
Multinomial opinion



Hyperdomain $\mathscr{R}(\mathbb{X})$

Hypervariable $X \in \mathscr{R}(\mathbb{X})$

Hypernomial opinion



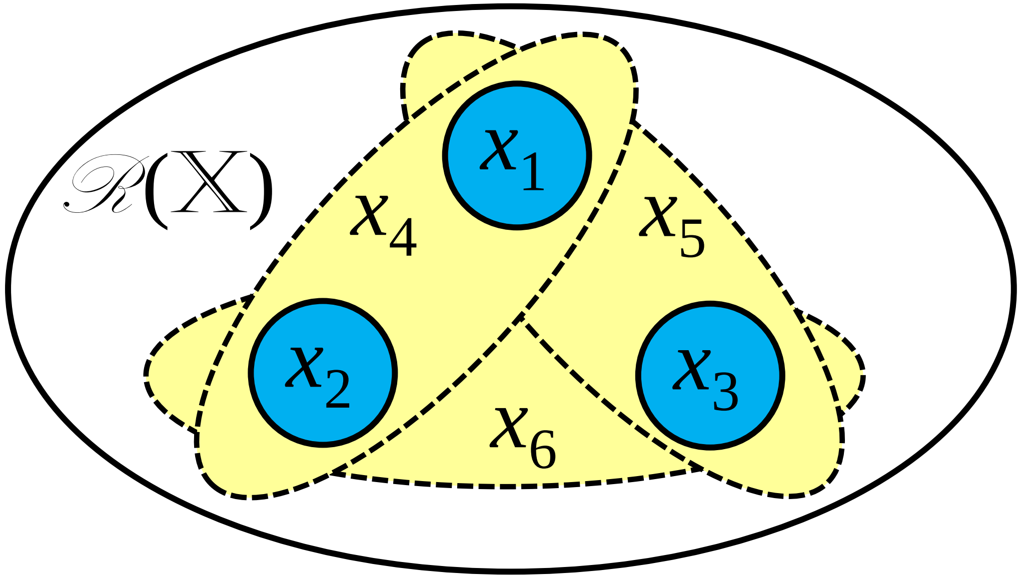
Hyperdomains

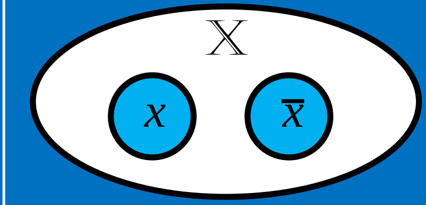
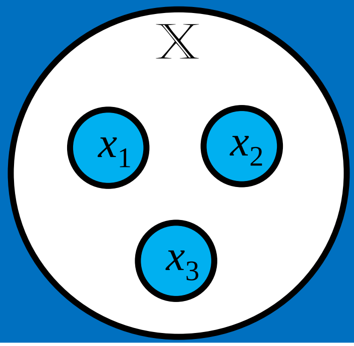
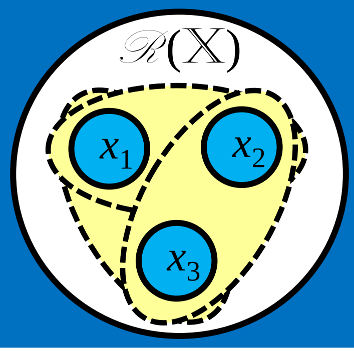
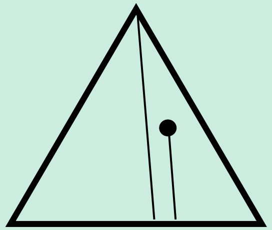
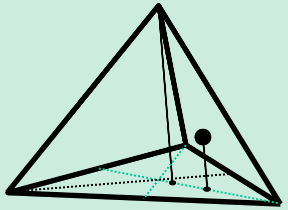
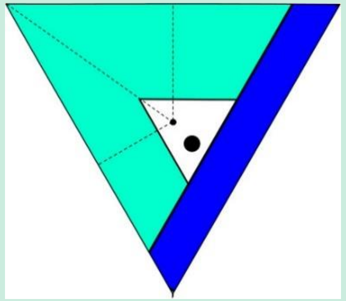
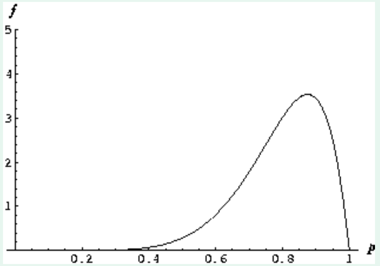
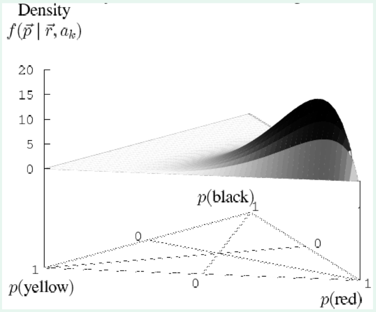
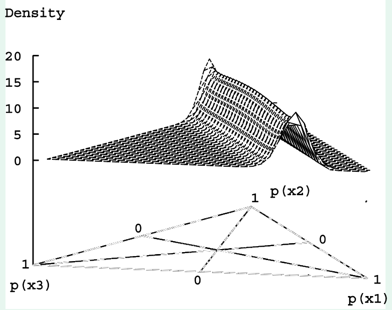
- A domain $\mathbb{X}$ is a state space of distinct state values
- Powerset $\mathscr{P}(\mathbb{X}) = 2^{\mathbb{X}}$, set of subsets, including $\{\mathbb{X},\emptyset\}$
- Reduced powerset $\mathscr{R}(\mathbb{X}) = \mathscr{P}(\mathbb{X}) \setminus \{\mathbb{X}, \emptyset\}$
- Hyperdomain $\mathscr{R}(\mathbb{X}) = \{ x_1, x_2, x_3, x_4, x_5, x_6 \}$
- $\mathscr{C}(\mathbb{X})$ called Composite set
- $\mathscr{C}(\mathbb{X}) = \{ x_4, x_5, x_6 \}$
- Cardinalities:

  $|\mathbb{X}| = 3$ in this example

  $|\mathscr{P}(\mathbb{X})| = 2^{|\mathbb{X}|} = 8$ in this example

  $|\mathscr{R}(\mathbb{X})| = 2^{|\mathbb{X}|} - 2 = 6$ in this example



	Binomial Opinion	Multinomial Opinion	Hypernomial Opinion
	Binary domain $\mathbb{X}$ Binary variable $X = x$	n-ary domain $\mathbb{X}$ Random variable $X \in \mathbb{X}$	hyperdomain $\mathscr{R}(\mathbb{X})$ Hypervariable $X \in \mathscr{R}(\mathbb{X})$
Domain			
Geometric opinion represen-tation	 triangle	 tetrahedron	 belief mosaic
PDF represen-tation	Beta PDF over x  FIG 1: Beta function after 7 positive and 1 negative results	Dirichlet PDF over $\mathbb{X}$ 	Hyper Dirichlet over $\mathbb{X}$ 

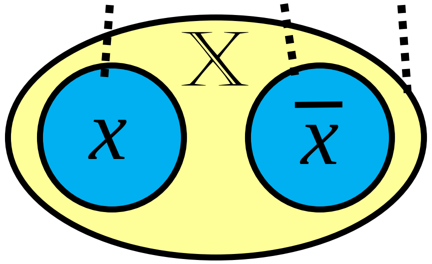
Binomial subjective opinions

- Belief mass and base rate on binary domain
  - $b_x^A = b(x)$ is source A's belief in x
  - $d_x^A = b(\bar{x})$ is source A's disbelief in x
  - $u_x^A = b(X)$ is source A's epistemic uncertainty about x
  - a_x^A is the base rate of x

Binomial opinion $\boxed{\omega_x^A = (b_x^A, d_x^A, u_x^A, a_x^A)}$

base rate of x

Binary domain



$$b_x^A + d_x^A + u_x^A = 1$$

Base rates (also called Priors)

- In probability theory and statistics, a **base rate** refers to **category probability** unconditioned on evidence.
- “**Prior probability**” is the same as “**base rate**”.
- For example, if it were the case that 0.01% of persons in a population have tuberculosis, then the base rate of tuberculosis is 0.01%.
- Given a positive or negative result of a medical test, the posterior probability can be calculated by taking into account the base rate.

Barycentric representation of binomial opinions

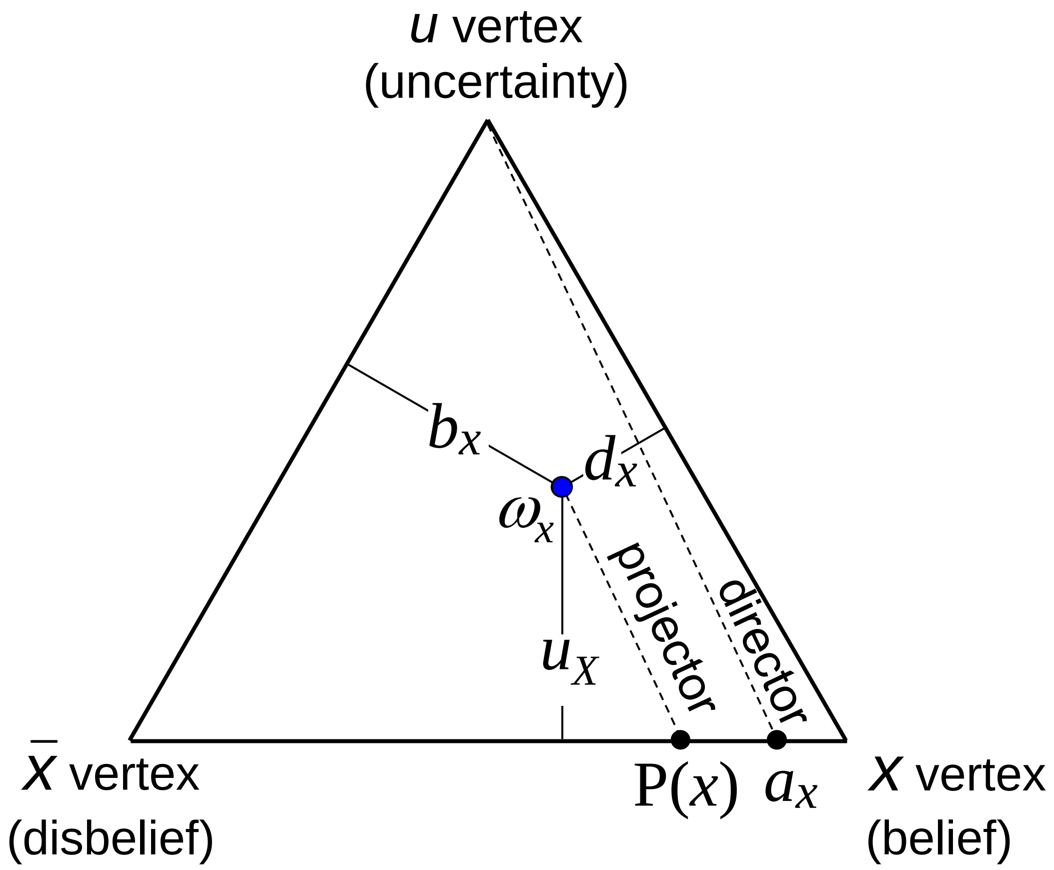
- Ordered quadruple:

$$\omega_x = (b_x, d_x, u_x, a_x)$$

  - b_x : belief
  - d_x : disbelief
  - u_x : epistemic uncertainty (lack of evidence)
  - a_x : base rate
- Point defined by additivity:

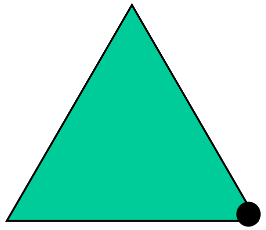
$$b_x + d_x + u_x = 1$$

- Projected probability: $\mathrm{P}(x) = b_x + a_x \cdot u_x$

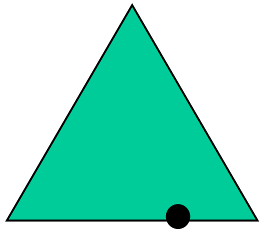


Example ω_x =(0.4, 0.2, 0.4, 0.9), P(x) = 0.76

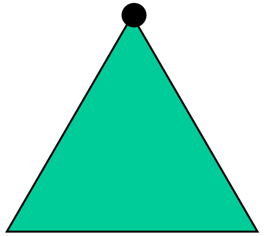
Opinion types



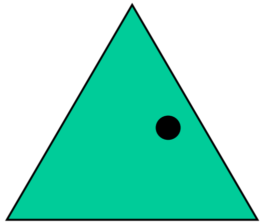
Absolute opinion: $b_x=1$. Equivalent to TRUE.
Low aleatoric and epistemic uncertainty.



Dogmatic opinion: $u_x=0$. Equivalent to probabilities.
Low epistemic uncertainty.



Vacuous opinion: $u_x=1$. Equivalent to UNDEFINED.
High epistemic uncertainty.



General uncertain opinion: $u_x \neq 0$.

Beta PDF representation

$$\text{Beta}\ (p(x);\ \alpha, \beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} p(x)^{\alpha-1}(1-p(x))^{\beta-1}$$

$$\alpha = r + W a$$

$$\beta = s + W(1\text{-}a)$$

r: # observations of x

s: # observations of $\bar{x}$

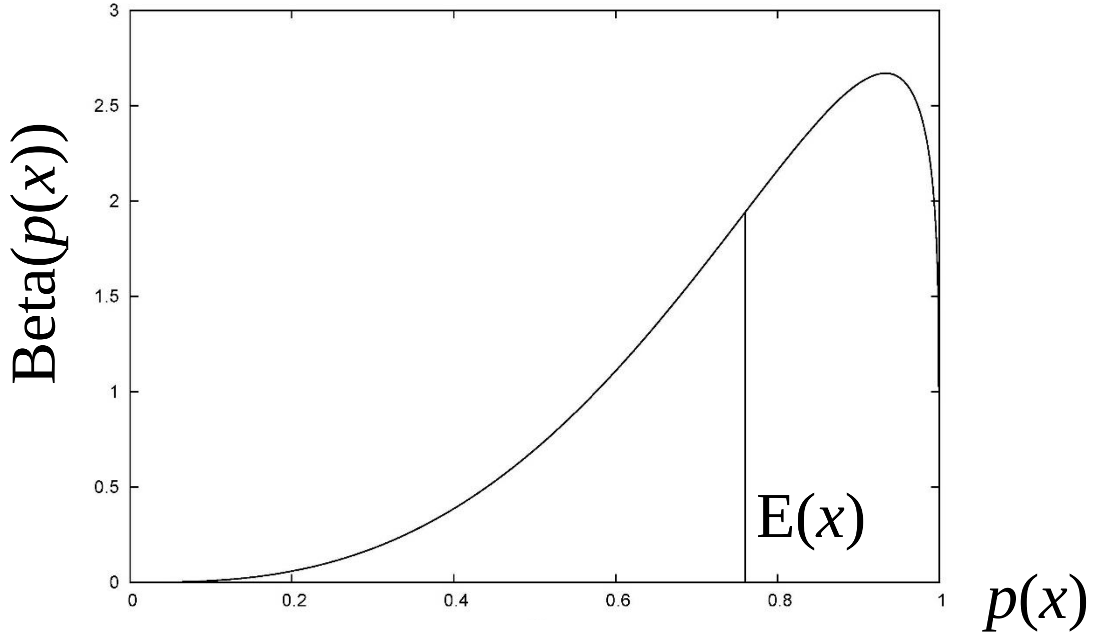
a: base rate of x

$W = 2$: non-informative prior weight

$\mathrm{E}(x)$: Expected probability

$\mathrm{E}(x) = \mathrm{P}(x)$

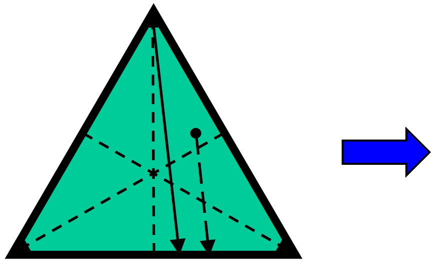
Beta Probability Density Function



Example: $r = 2$, s = 1, $a = 0.9$, $\mathrm{E}(x) = 0.76$

Binomial Opinion ↔ Beta PDF

- (r,s,a) represents Beta PDF evidence parameters.
- (b,d,u,a) represents binomial opinion.
- $\mathrm{P}(x) = \mathrm{E}(x)$
- Op → Beta: $\begin{cases} r = Wb/u \\ s = Wd/u \\ b+d+u = 1 \end{cases}$



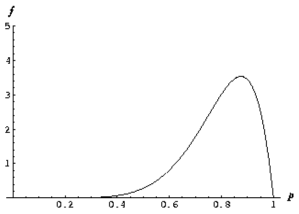


FIG 1: Beta function after 7 positive and 1 negative results

- Beta → Op: $\begin{cases} b = \dfrac{r}{r+s+W} \\ d = \dfrac{s}{r+s+W} \\ u = \dfrac{W}{r+s+W} \end{cases}$

$W = 2$

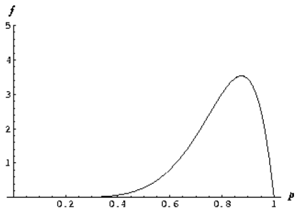
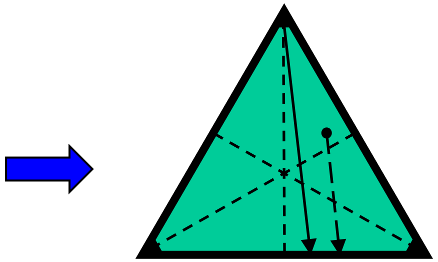
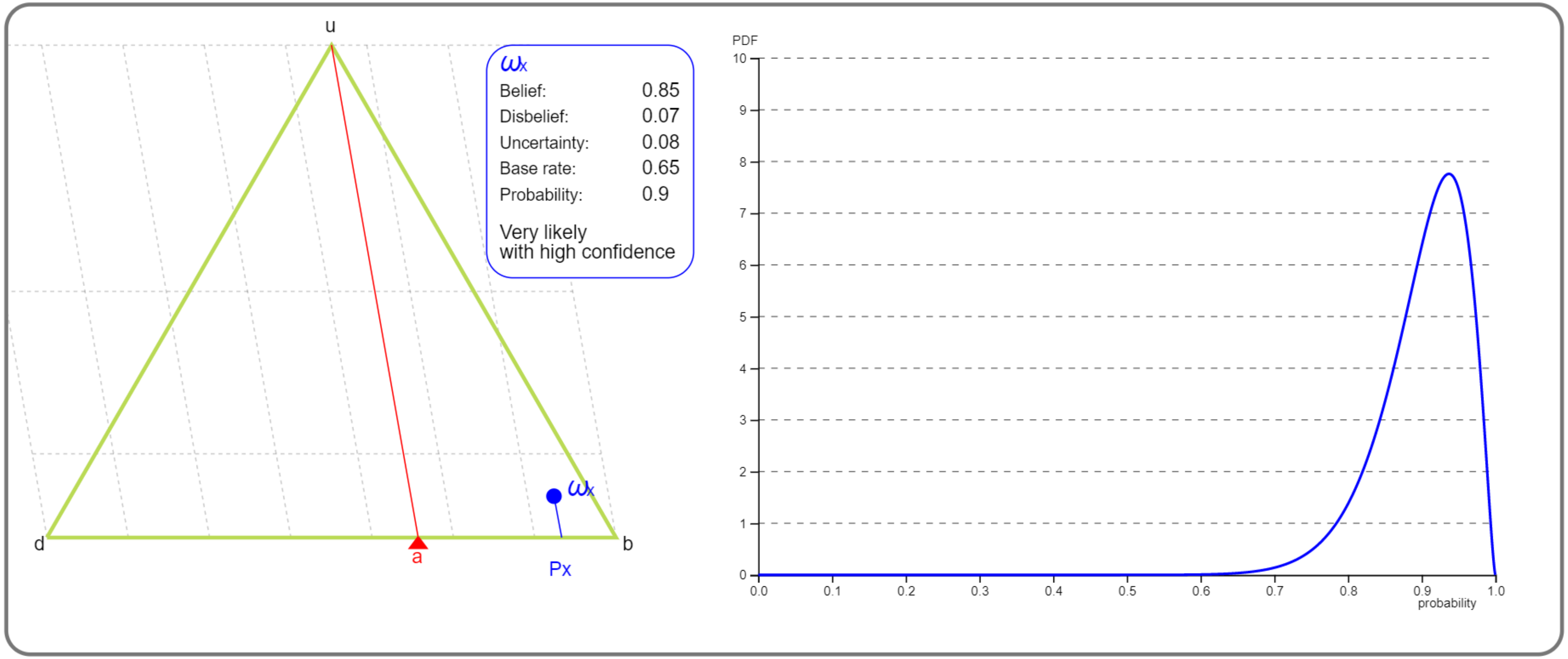


FIG 1: Beta function after 7 positive and 1 negative results



Online demo of opinion visualisation



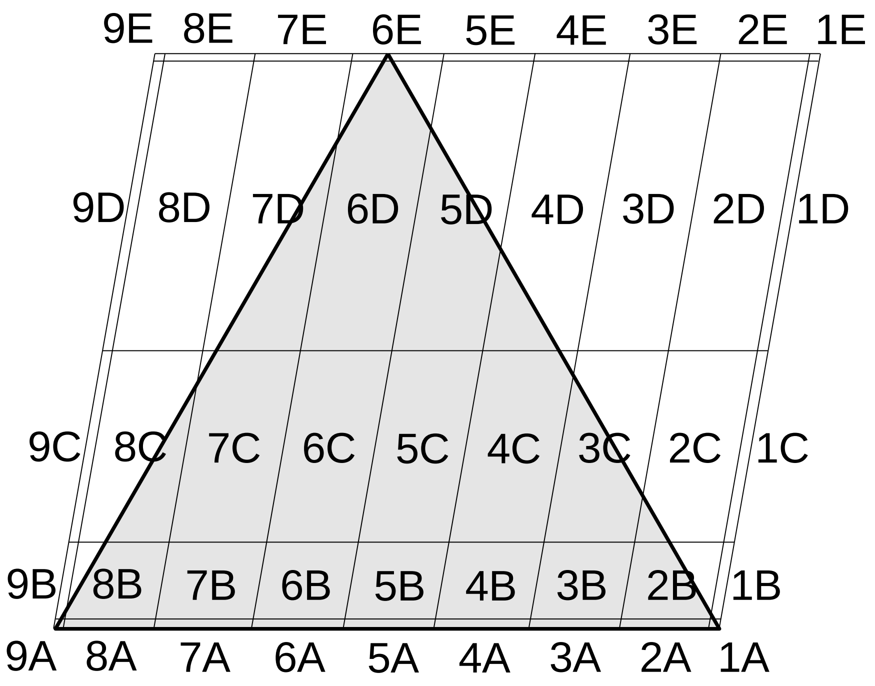
https://folk.universitetetioslo.no/josang/sl/BV.html

Likelihood and Confidence

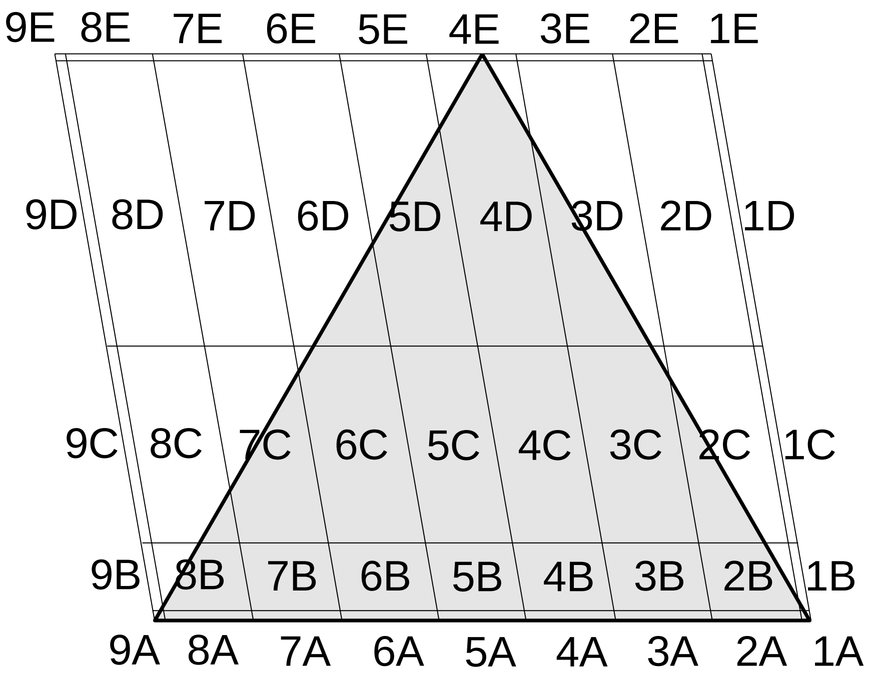
Likelihood (probability) levels:		Absolutely not	Very unlikely	Unlikely	Somewhat unlikely	Chances about even	Somewhat likely	Likely	Very likely	Absolutely
Confidence (certainty) levels:		**9**	**8**	**7**	**6**	**5**	**4**	**3**	**2**	**1**
No confidence	**E**	9E	8E	7E	6E	5E	4E	3E	2E	1E
Low confidence	**D**	9D	8D	7D	6D	5D	4D	3D	2D	1D
Some confidence	**C**	9C	8C	7C	6C	5C	4C	3C	2C	1C
High confidence	**B**	9B	8B	7B	6B	5B	4B	3B	2B	1B
Total confidence	**A**	9A	8A	7A	6A	5A	4A	3A	2A	1A

Mapping qualitative to opinion

- Categories mapped to corresponding field of triangle
- Mapping depends on base rate
- Non-existent categories depending on base-rates



base rate $a = 1/3$



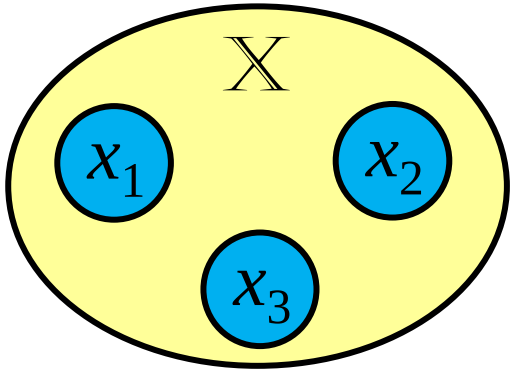
base rate $a = 2/3$

Mapping qualitative to opinions

- Overlay qualitative matrix with opinion triangle
- Matrix becomes skewed as a function of base rate
- Not all qualitative combinations map to opinions
  - For a base rate $a = 1/3$, it is impossible to describe an event as likely with low confidence (3D), but possible to describe it as unlikely with low confidence (7D).
  - E.g. with regard to tuberculosis which has a very low base rate, it would be irrational to say that a patient is likely to be infected, with low confidence (high uncertainty). However, it would be rational to say that a patient is unlikely to be infected, with low confidence (high uncertainty).

Multinomial domains

- Generalisation of binary domains
- Set of exclusive and exhaustive singletons.
- Example ternary domain: $\mathbb{X}=\{x_1, x_2, x_3\}$, $|\mathbb{X}| = 3$.



Multinomial Opinions

- Domain: $\mathbb{X}=\{x_1 \ldots x_k\}$
- Random variable $X \in \mathbb{X}$
- Multinomial opinion: $\omega_X = (b_X, u_X, a_X)$

- Belief mass distribution b_X where $u + \Sigma b_X(x) = 1$

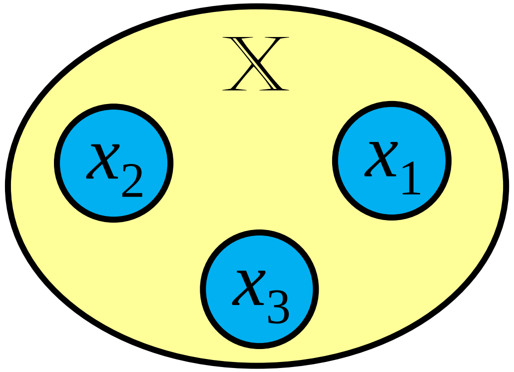
  $b_X(x)$ is belief mass on $x \in \mathbb{X}$
- Epistemic uncertainty mass: u_X is a single value in range $[0,1]$
- Base rate distribution a_X where $\Sigma a_X(x) = 1$

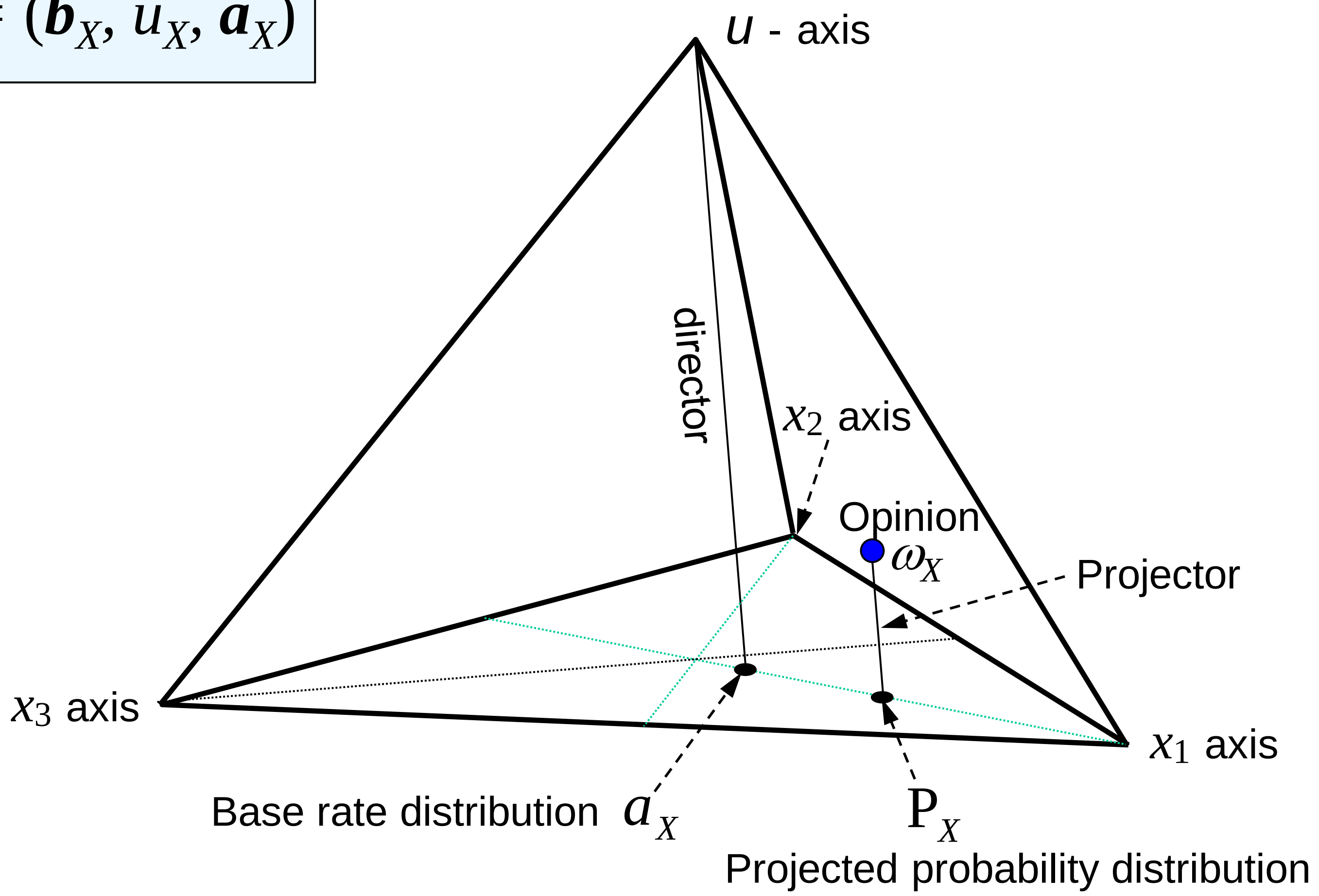
  $a_X(x)$ is base rate of $x \in \mathbb{X}$
- Projected probability: $\mathrm{P}_X(x) = b_X(x) + a_X(x) \cdot u_X$

Opinion tetrahedron (ternary domain)

Multinomial opinion: $\omega_X = (\boldsymbol{b}_X, u_X, \boldsymbol{a}_X)$

$$\boldsymbol{b}_X = \overbrace{\{\boldsymbol{b}_X(x_1), \boldsymbol{b}_X(x_2), \boldsymbol{b}_X(x_3)\}}^{\text{Belief masses}}$$





Dirichlet PDF representation

$$\mathrm{Dir}(p_X) = \frac{\Gamma\left(\sum_{i=1}^{k} \alpha_X(x_i)\right)}{\prod_{i=1}^{k} \Gamma(\alpha_X(x_i))} \prod_{i=1}^{k} p_X(x_i)^{\alpha(x_i)-1}$$

$$\Sigma\, p_X(x_i) = 1$$

$$\alpha_X(x_i) = r_X(x_i) + W \cdot a_X(x_i)$$

$r_X(x_i)$: # observations of x_i

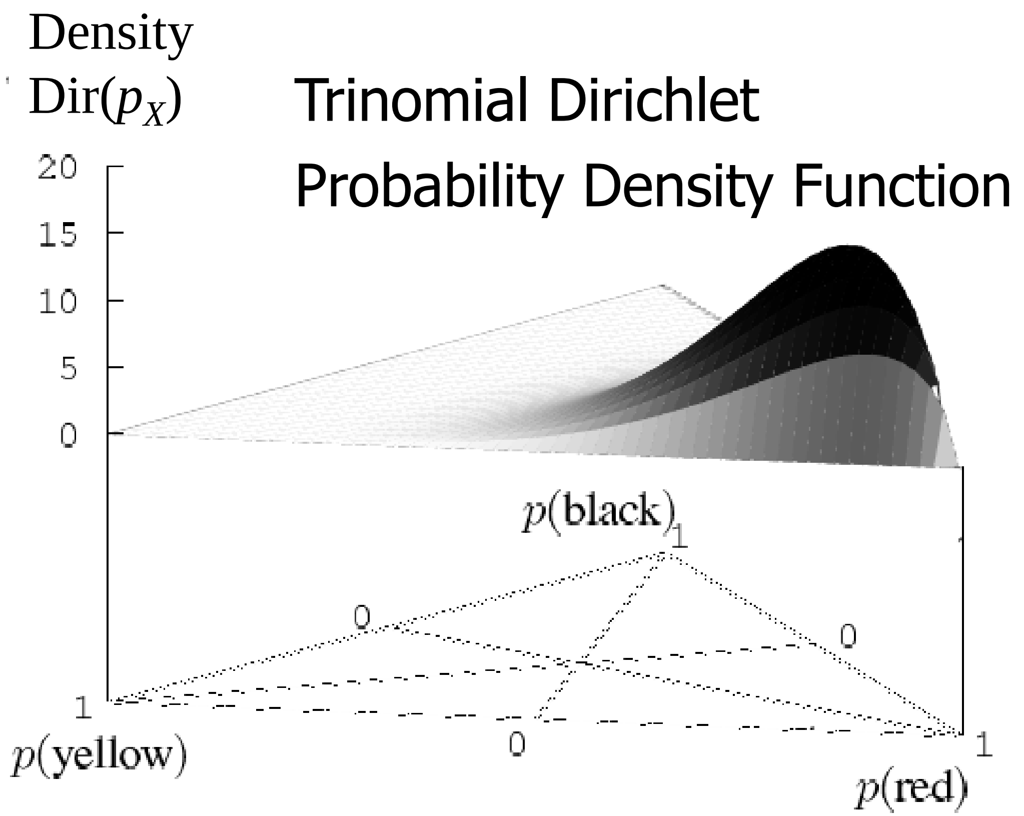
$a_X(x_i)$: base rate of x_i

E_X: Expected proba. distr.

$\mathrm{E}_X = \mathrm{P}_X$

Example:

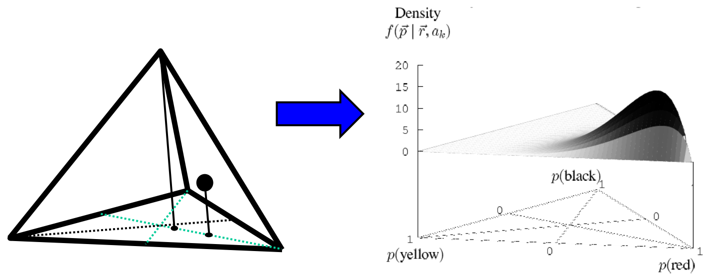
- 6 red balls
- 1 yellow ball
- 1 black ball



Multinomial Opinion $\leftrightarrow$ Dirichlet PDF

- Dirichlet PDF evidence parameters: (r_X, a_X)
- Multinomial opinion parameters: (b_X, u_X, a_X)
- Op $\rightarrow$ Dir:

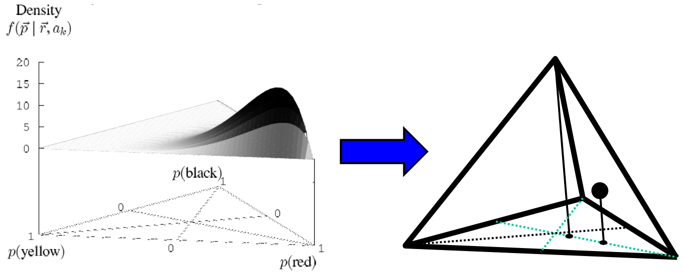
$$\begin{cases} r_X(x) = \dfrac{W \cdot b_X(x)}{u_X} \\[2ex] u_X + \sum b_X(x) = 1 \end{cases}$$



$W = 2$

- Dir $\rightarrow$ Op:

$$\begin{cases} b_X(x) = \dfrac{r_X(x)}{W + \sum r_X(x)} \\[2ex] u_X = \dfrac{W}{W + \sum r_X(x)} \end{cases}$$



Non-informative prior weight: W

- The prior Dirichlet PDF is assumed to be uniform, requiring that W is equal to the frame cardinality k.
- However, for arbitrarily large domains, W would become equally large, making the Dirichlet PDF insensitive to new observations, which would be an inadequate model.
- Solution: dynamic non-informative prior weight, where initially $W=k$, and where W converges to convergence constant C_W .

$$W = \frac{k + C_W k \sum r_X(x)}{1 + k \sum r_X(x)}$$

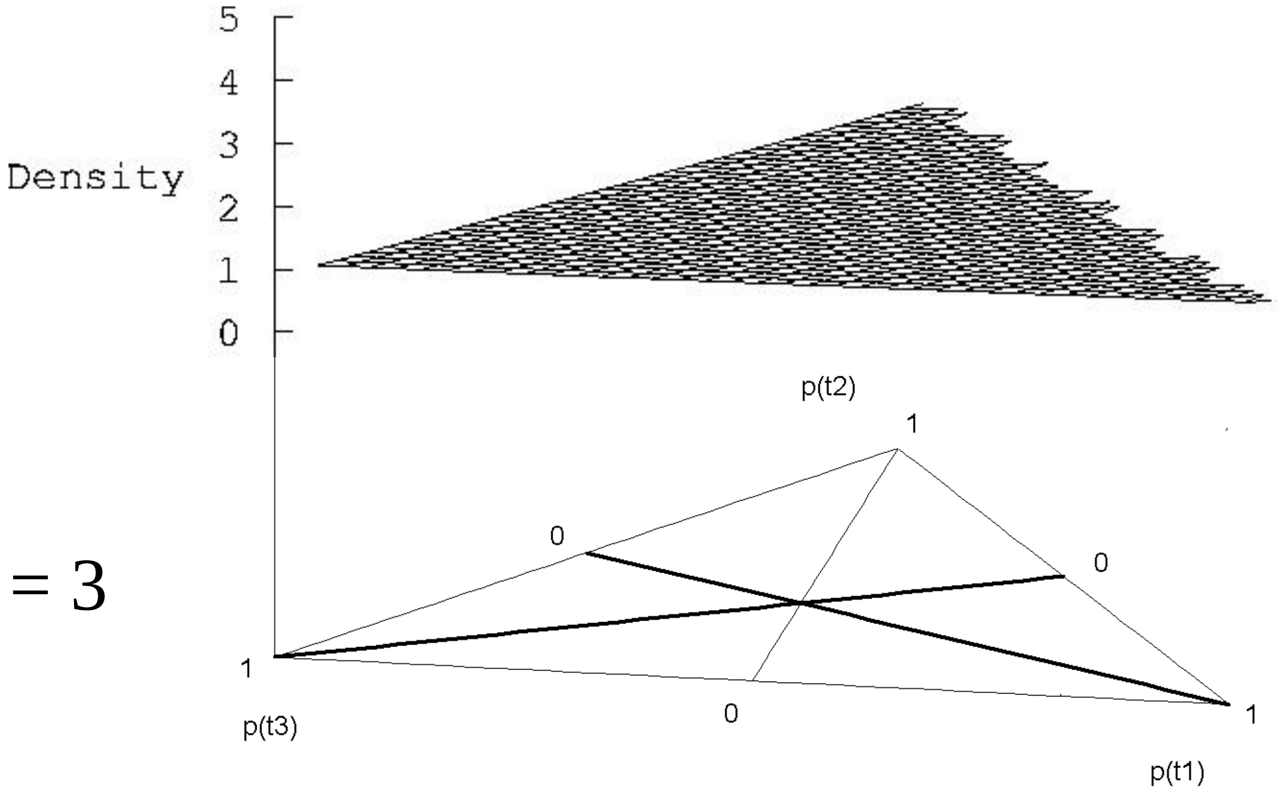
- It is normally assumed that the Beta PDF has the appropriate sensitivity to new observations, which dictates $C_W = 2$.

Prior trinomial Dirichlet PDF, $W = 3$

Example:

Urn with balls of 3 different colors.

- t_1: Red
- t_2: Yellow
- t_3: Black



Cardinality: $k = 3$

No balls have been picket.

Non-informative prior weight: $W = k = 3$

Uniform *prior* probability density.

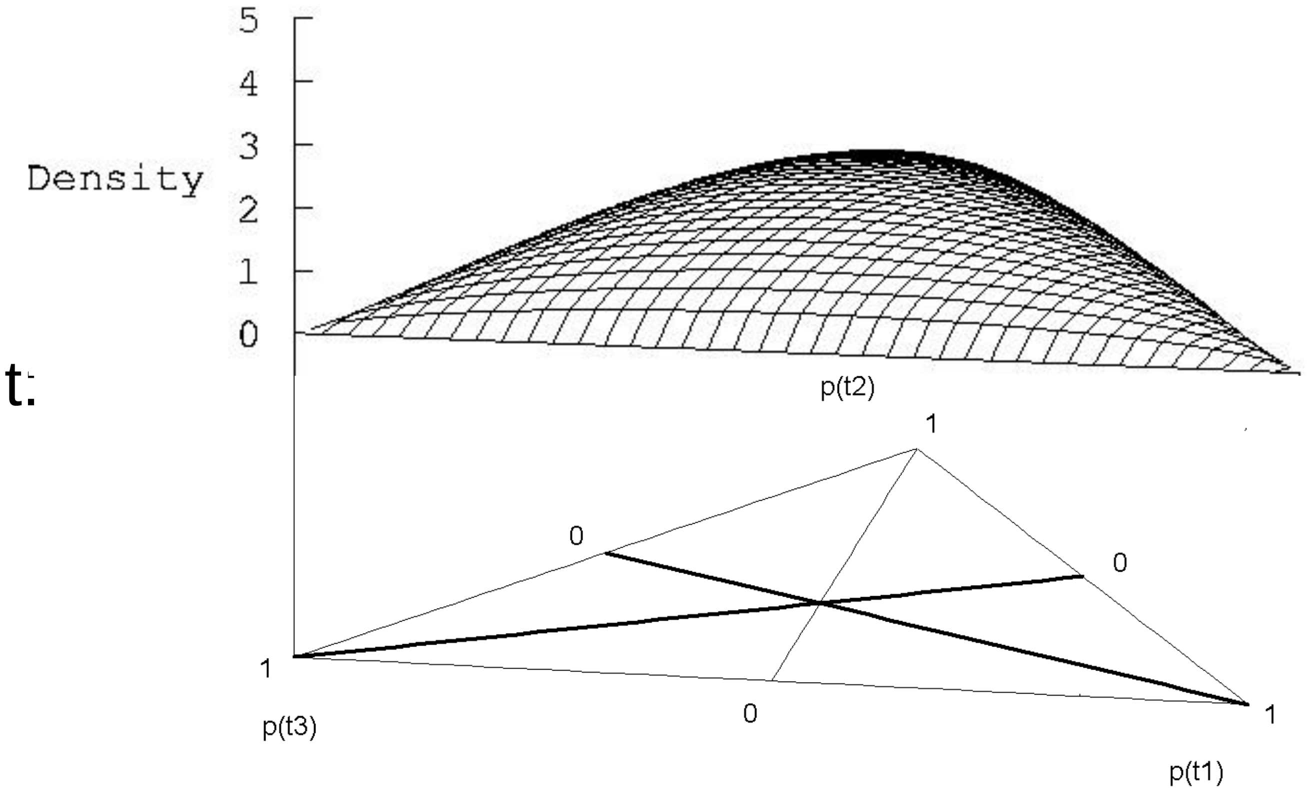
Posterior trinomial Dirichlet PDF

A posteriori probability density after picking:

1 red ball (t_1)
1 yellow ball (t_2)
1 black ball (t_3)

Dynamic non-informative prior weight:

$$W = \frac{k+2k\sum r_X(x)}{1+k\sum r_X(x)} = \frac{21}{10} = 2.1$$

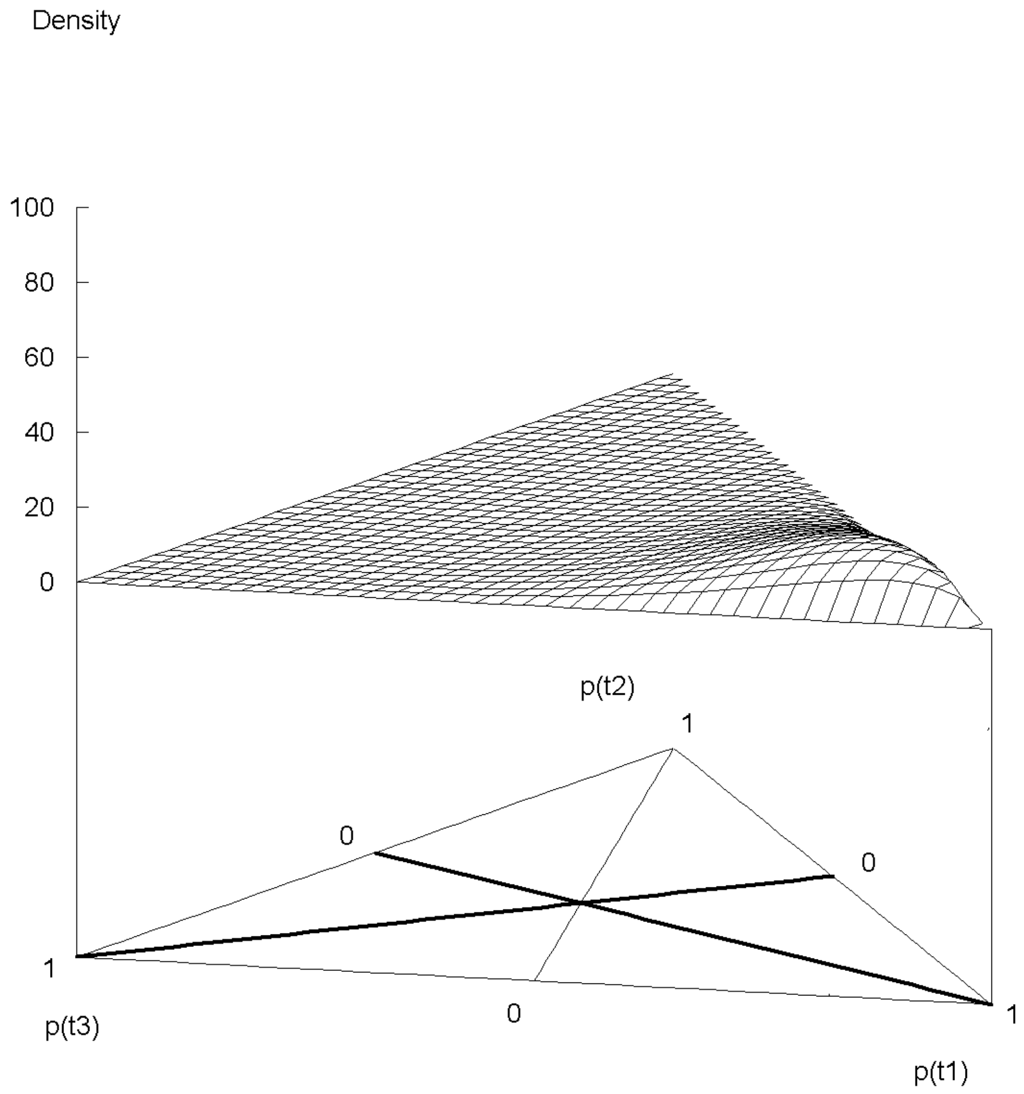


Posterior trinomial Dirichlet PDF

A posteriori probability density after picking:

- 6 red balls (t_1)
- 1 yellow ball (t_2)
- 1 black ball (t_3)

- $W = 2.04$

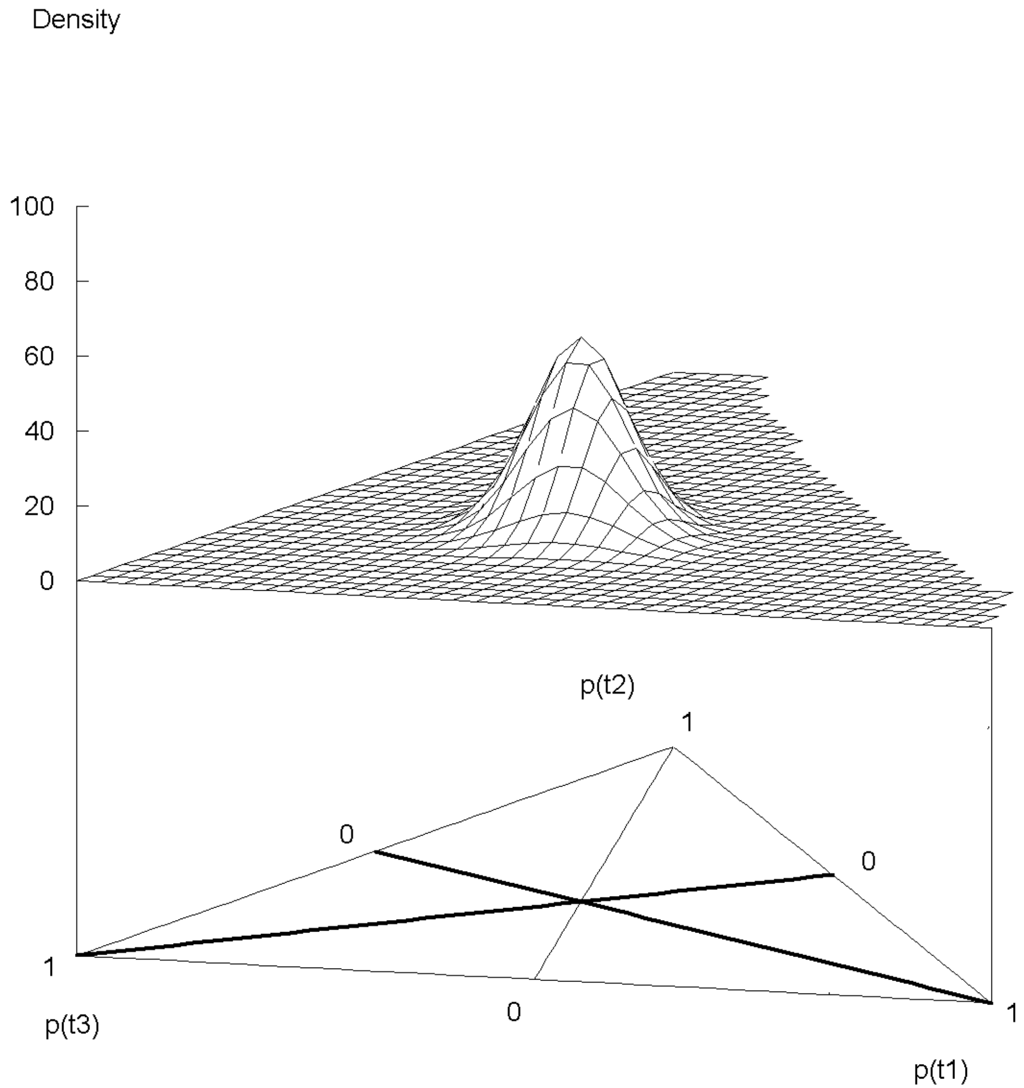


Posterior trinomial Dirichlet PDF

A posteriori probability density after picking:

- 20 red balls (t_1)
- 20 yellow balls (t_2)
- 20 black balls (t_3)

- $W = 2$

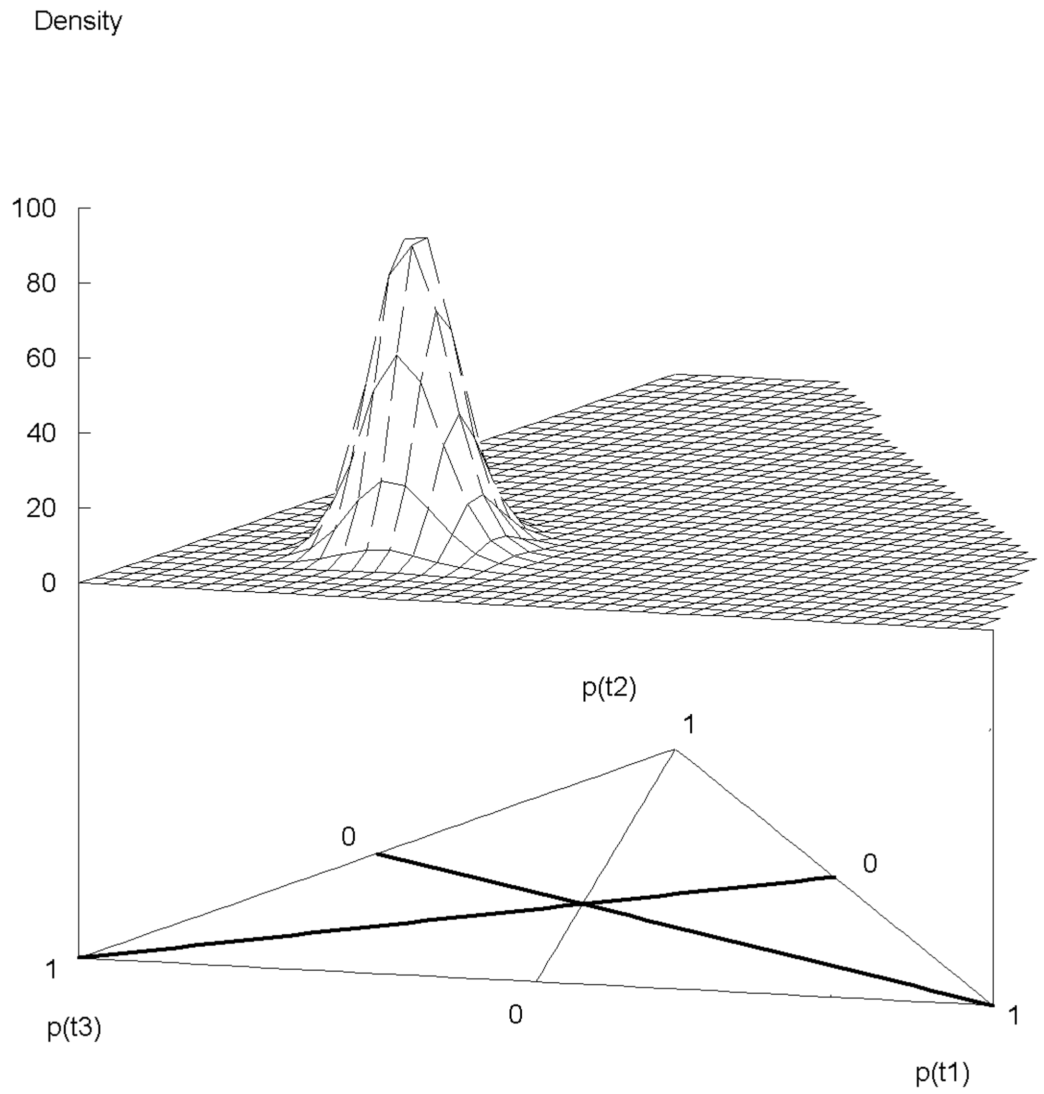


Posterior trinomial Dirichlet PDF

A posteriori probability density after picking:

- 20 red balls (t_1)
- 20 yellow balls (t_2)
- 50 black balls (t_3)

- $W = 2$



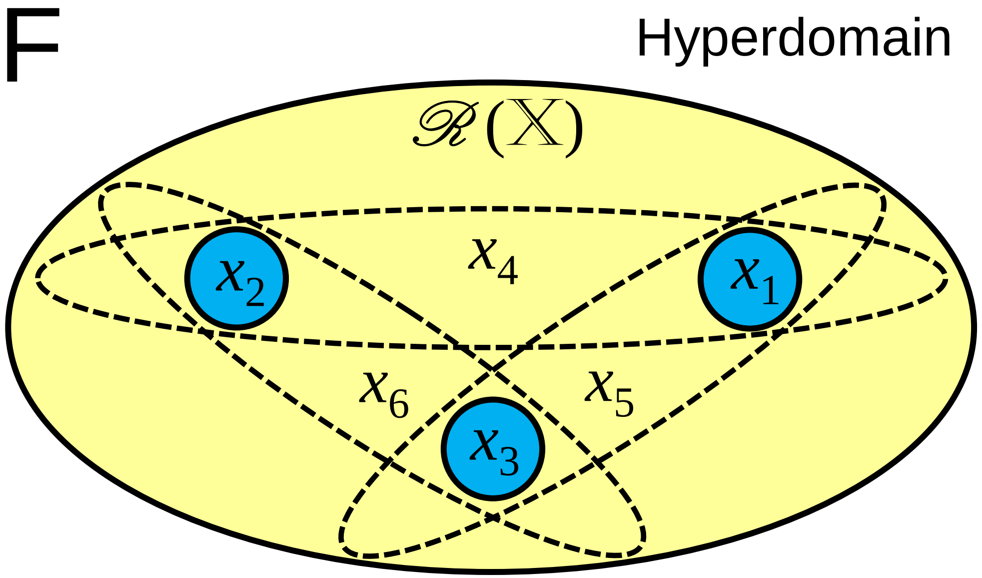
Hyper-Opinions

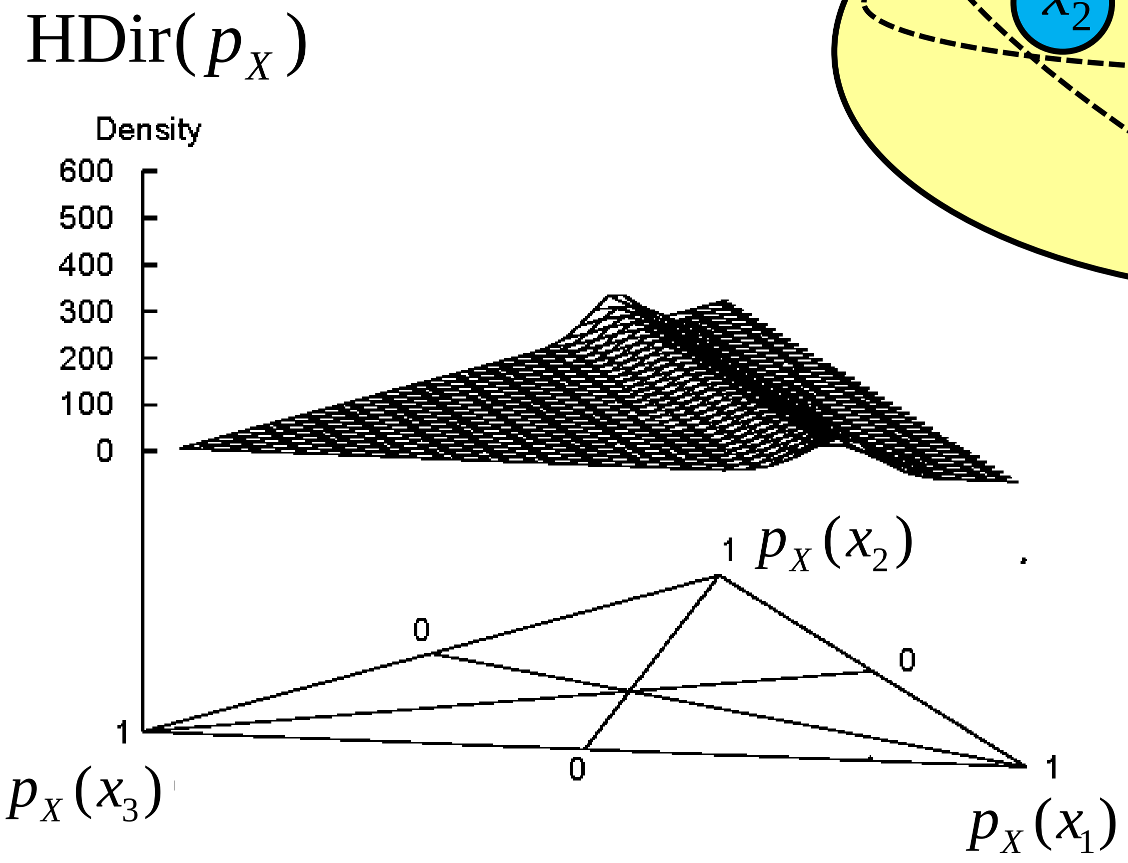
- Domain: $\mathbb{X}=\{x_1 \ldots x_k\}$
- $\mathscr{P}(\mathbb{X})$ is the powerset of $\mathbb{X}$
- Hyperdomain $\mathscr{R}(\mathbb{X}) = \mathscr{P}(\mathbb{X}) \setminus \{\mathbb{X}, \emptyset\}$
- $\mathscr{R}(\mathbb{X})$ is the reduced powerset of $\mathbb{X}$
- Hypervariable: $X \in \mathscr{R}(\mathbb{X})$
- $\boxed{\text{Hyper opinion: } \omega_X = (\boldsymbol{b}_X, u_X, \boldsymbol{a}_X)}$
- Belief mass distribution: $\boldsymbol{b}_X$ where $u_X + \sum_{X \in \mathrm{R}(\mathrm{X})} \boldsymbol{b}_X(x) = 1$

  $b_X(x)$ is belief mass on $x \in \mathscr{R}(\mathbb{X})$
- Base rate distribution: a_X where $\sum_{X \in \mathrm{X}} \boldsymbol{a}_X(x) = 1$

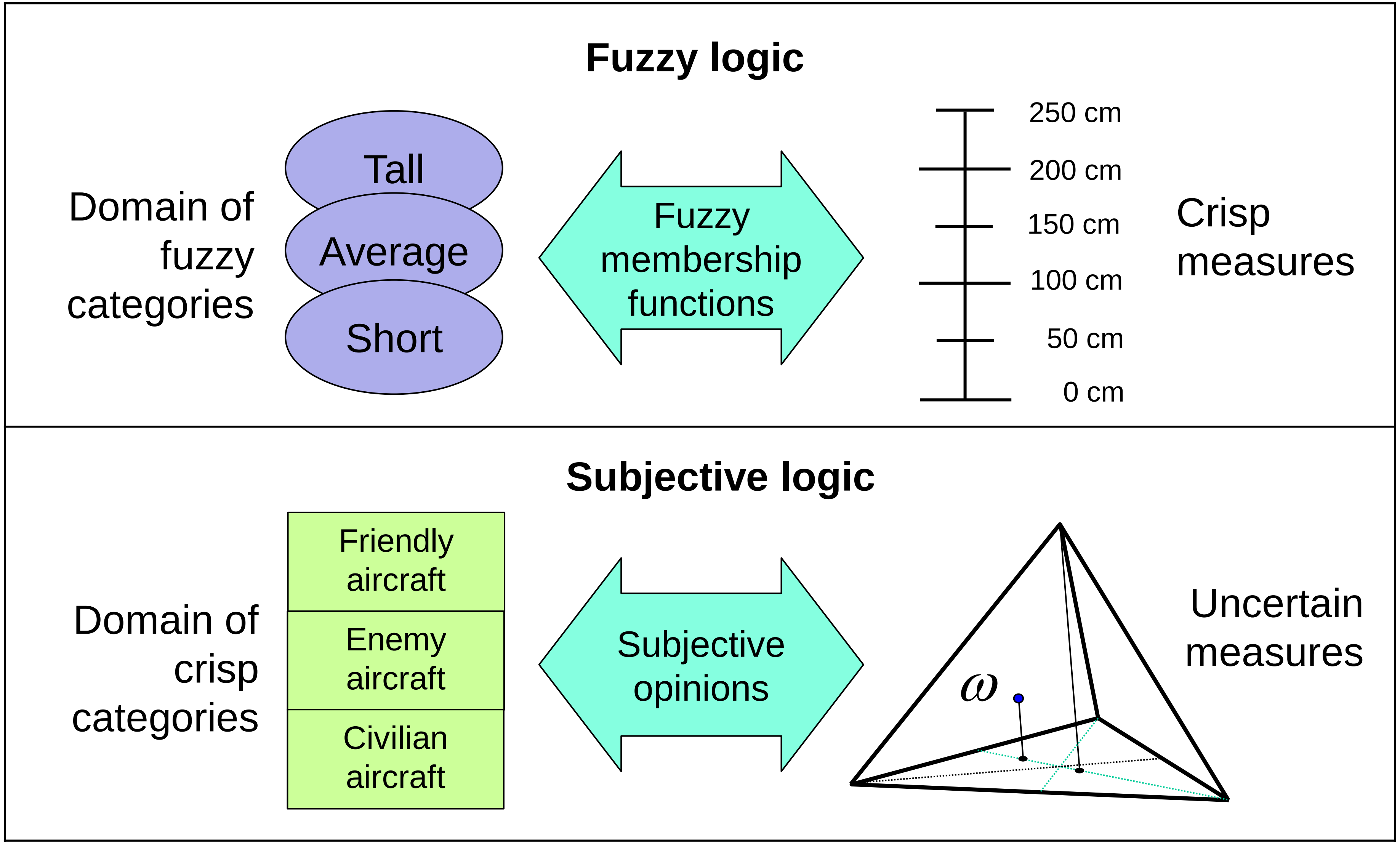
  $a_X(x)$ is base rate of $x \in \mathbb{X}$
- Projected probability: $\mathrm{P}_X(x) = a_X(x) \cdot u_X + \sum_{x_j \in \mathscr{R}(\mathbb{X})} a_X(x \mid x_j) \cdot b_X(x_j)$

Hyper Opinions and Hyper Dirichlet PDF

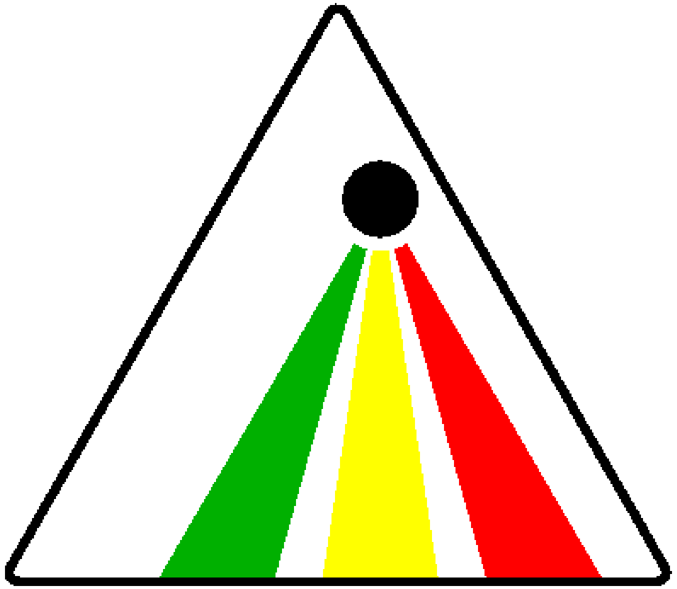




Opinions v. Fuzzy membership functions



Subjective Logic Operators



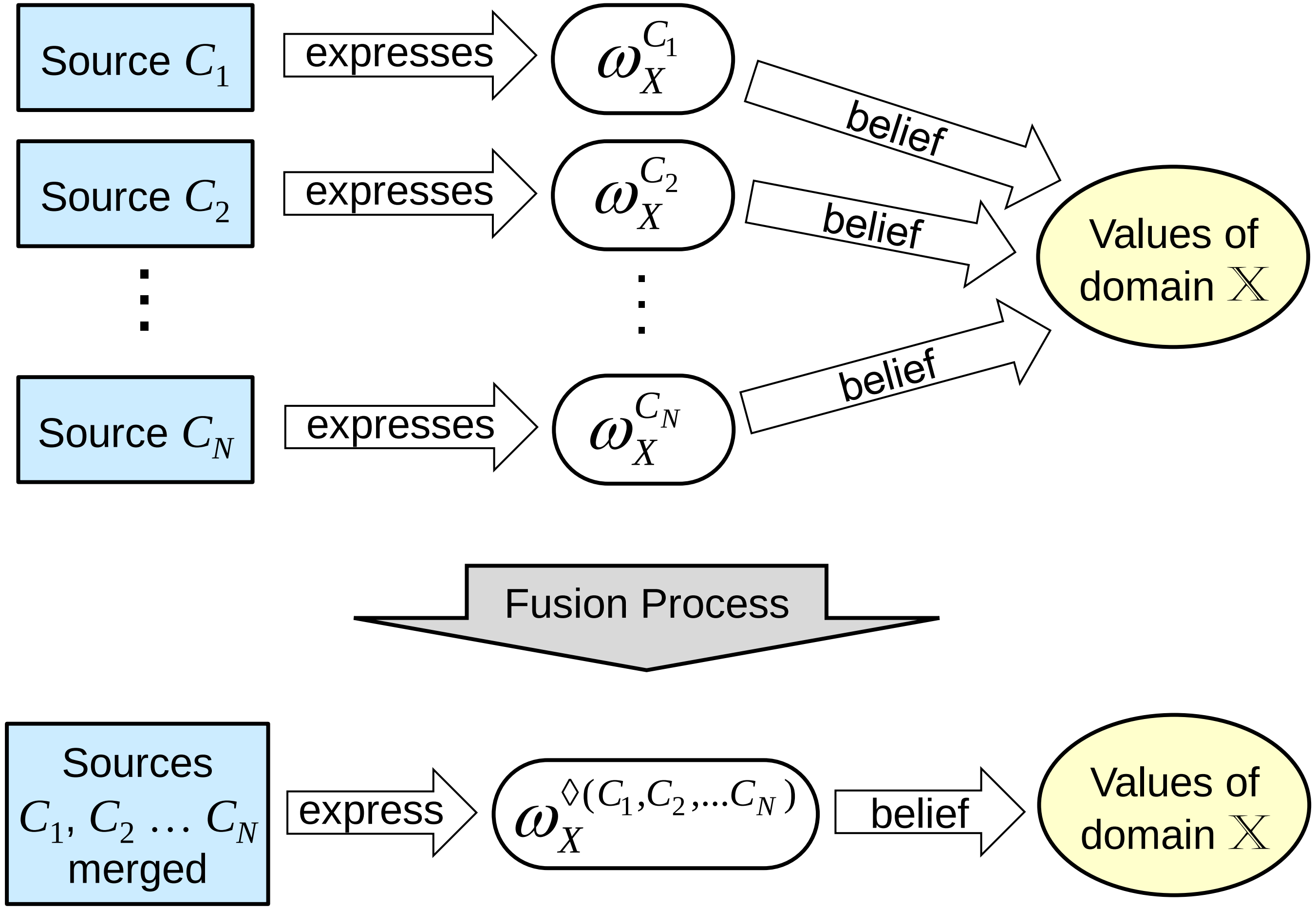
Subjective logic operators 1

Opinion operator name	**Opinion operator symbol**	**Logic operator symbol**	**Logic operator name**
Addition	+	$\cup$	UNION
Subtraction	-	\	DIFFERENCE
Complement	¬	$\overline{x}$	NOT
Projected probability	P(x)	n.a.	n.a.
Multiplication	·	$\wedge$	AND
Division	/	$\overline{\wedge}$	UN-AND
Comultiplication	$\sqcup$	$\vee$	OR
Codivision	$\overline{\sqcup}$	$\overline{\vee}$	UN-OR

Subjective logic operators 2

Opinion operator name	**Opinion operator symbol**	**Logic operator symbol**	**Logic operator name**
Transitive discounting	$\otimes$	:	TRANSITIVITY
Cumulative fusion	$\oplus$	$\Diamond$	n.a.
Averaging fusion	$\underline{\oplus}$	$\underline{\Diamond}$	n.a.
Constraint fusion	$\odot$	&	n.a.
Inversion, Bayes' theorem	$\tilde{\phi}$	$\widetilde{\mid}$	CONTRAPOSITION
Conditional deduction	$\circledcirc$	$\Vert$	DEDUCTION (Modus Ponens)
Conditional abduction	$\tilde{\circledcirc}$	$\widetilde{\Vert}$	ABDUCTION (Modus Tollens)

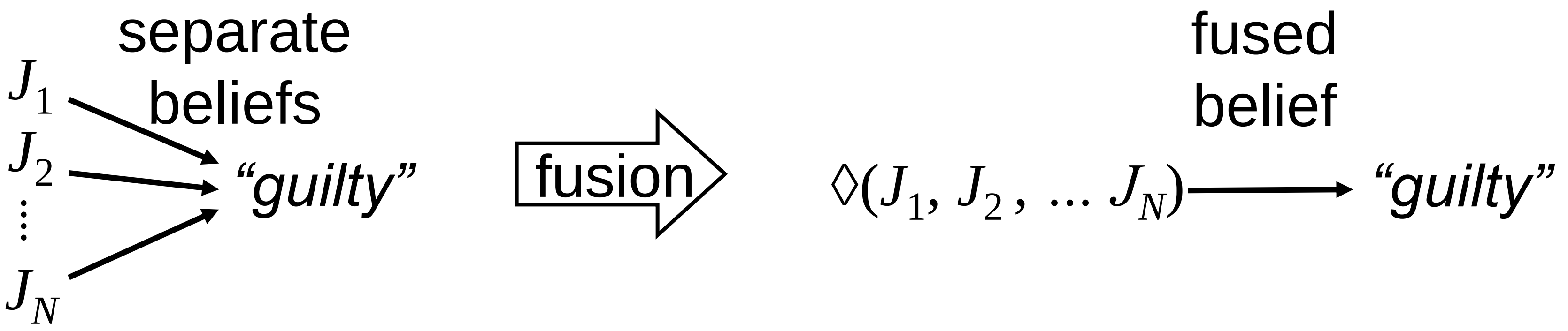
Belief Fusion



- Notation: $\omega_X^{\diamond(C_1,C_2,\dots CN)} = \omega_X^{C_1} \oplus \omega_X^{C_2} \oplus \dots \omega_X^{C_N}$

Example: Reaching a verdict

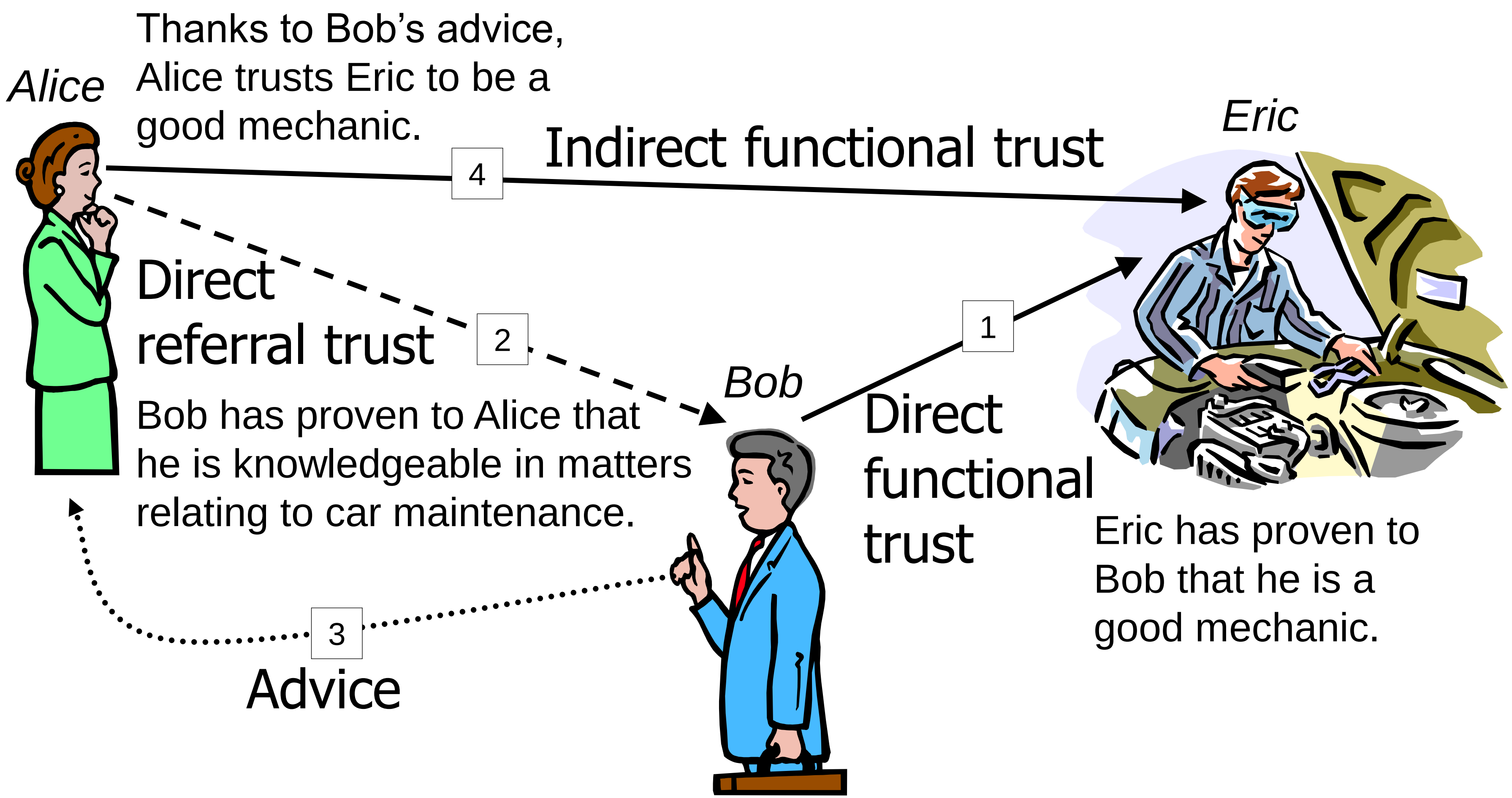
- $J_1, J_2, \ldots J_N$ are N different jury members.
- "guilty" is a binary statement.
- $[J_1, J_2, \ldots J_N]$ denotes the whole jury.
- ω_{BRD} is a politically defined threshold value for *"Beyond Reasonable Doubt"*.



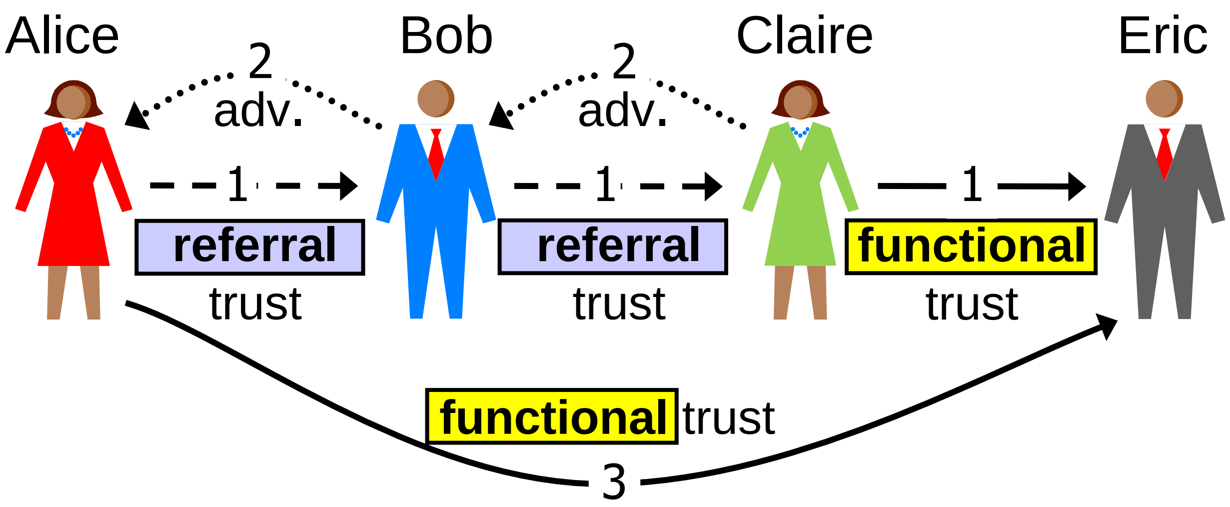
Criterion for guilty conviction: $\omega^{\Diamond(J_1,J_2,\ldots JN)}_{\text{"guilty"}} > \omega_{\text{BRD}}$?

Subjective Trust Networks

Trust transitivity



Functional trust derivation requirement

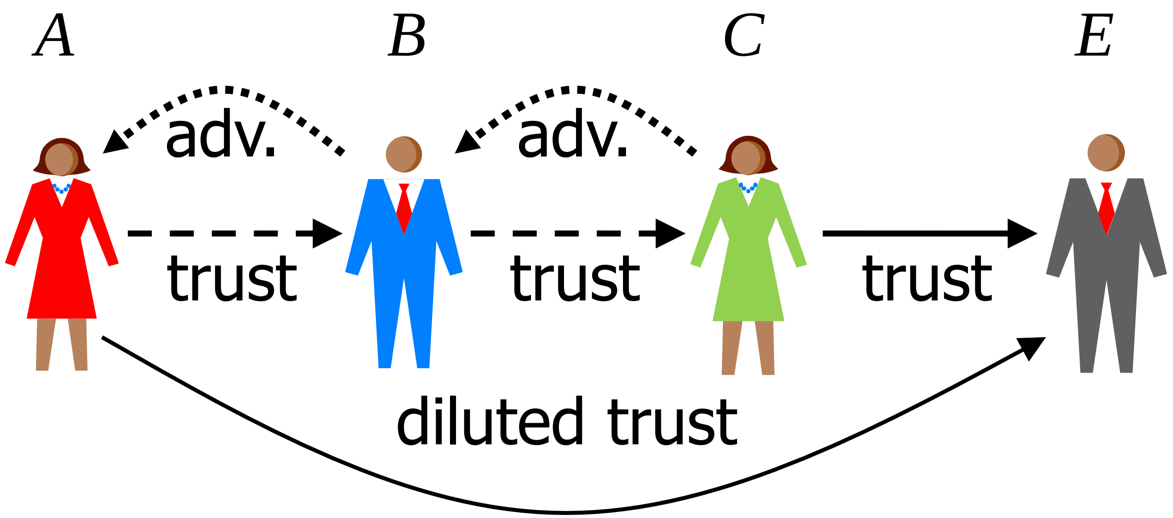


- Functional trust derivation through transitive paths requires that the last trust edge represents functional trust (or an opinion) and that all previous trust edges represent referral trust.

- Functional trust can be an opinion about a variable.

Trust transitivity characteristics

Trust is diluted in a transitive chain.



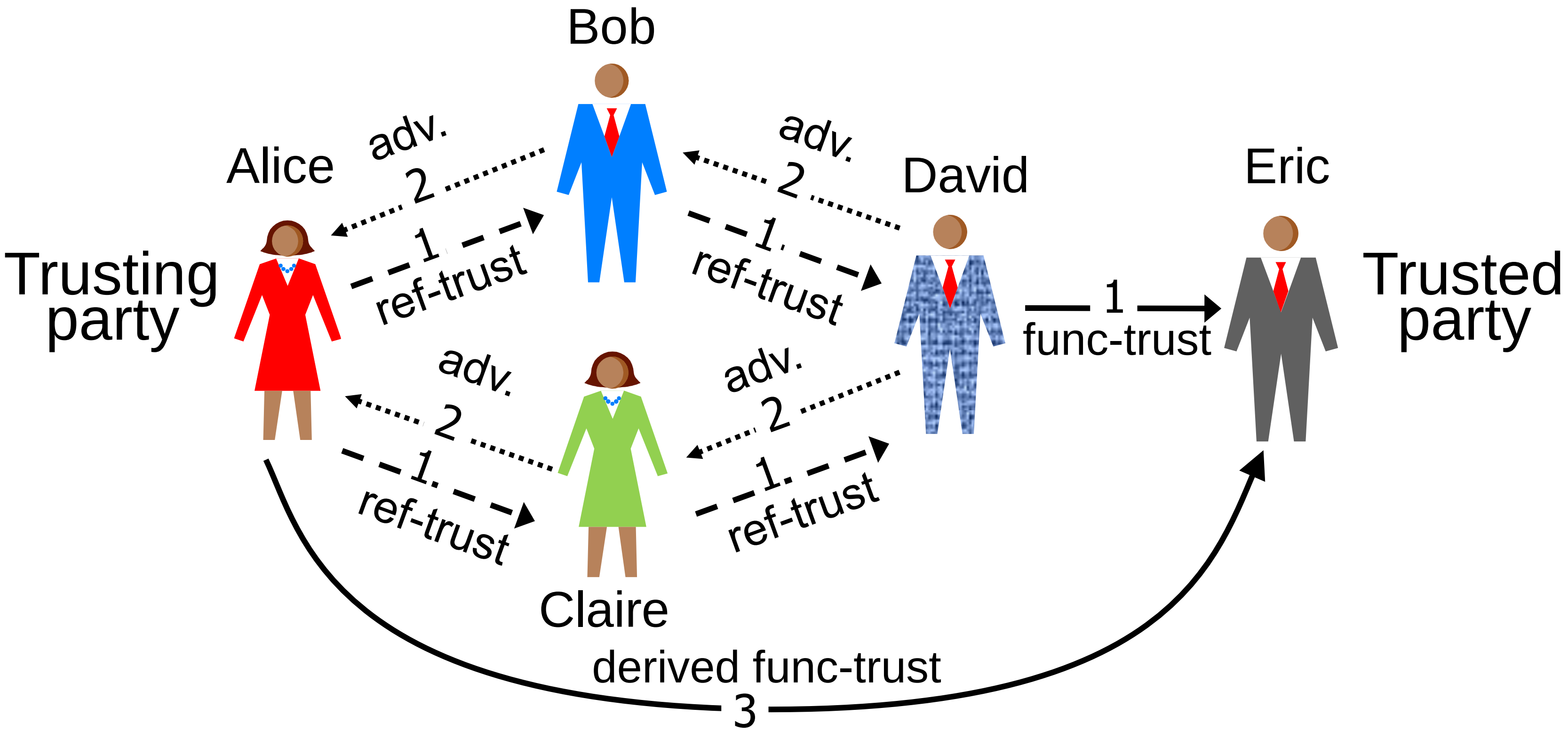
Computed with discounting/transitivity operator of SL

Graph notation: $[A, E] = [A; B] : [B; C] : [C, E]$

SL notation: $\omega_E^{(A;B;C)} = \omega_B^A \otimes \omega_C^B \otimes \omega_E^C$

Trust Fusion

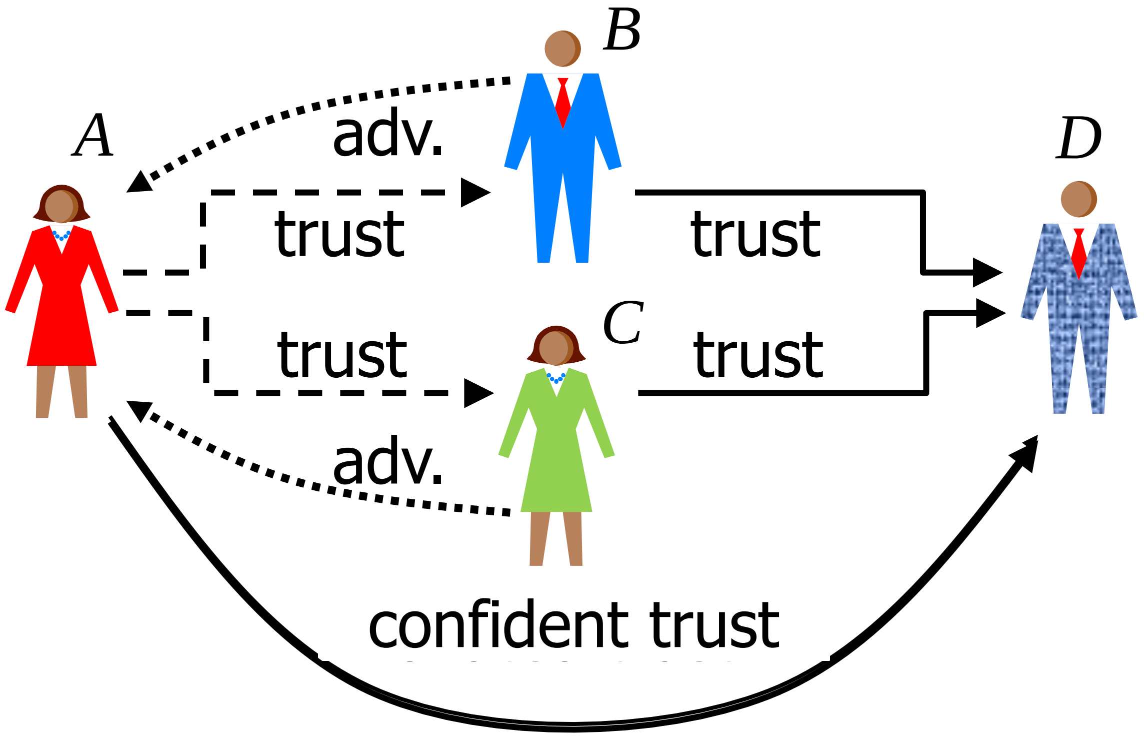
Combination of serial and parallel trust paths



Graph notation: $[A, E] = (([A;B] : [B;D]) \Diamond ([A;C] : [C;D])) : [D,E]$

SL notation: $\omega_E^{[A;B;D]\Diamond[A;C;D]} = ((\omega_B^A \otimes \omega_D^B) \oplus (\omega_C^A \otimes \omega_D^C)) \otimes \omega_E^D$

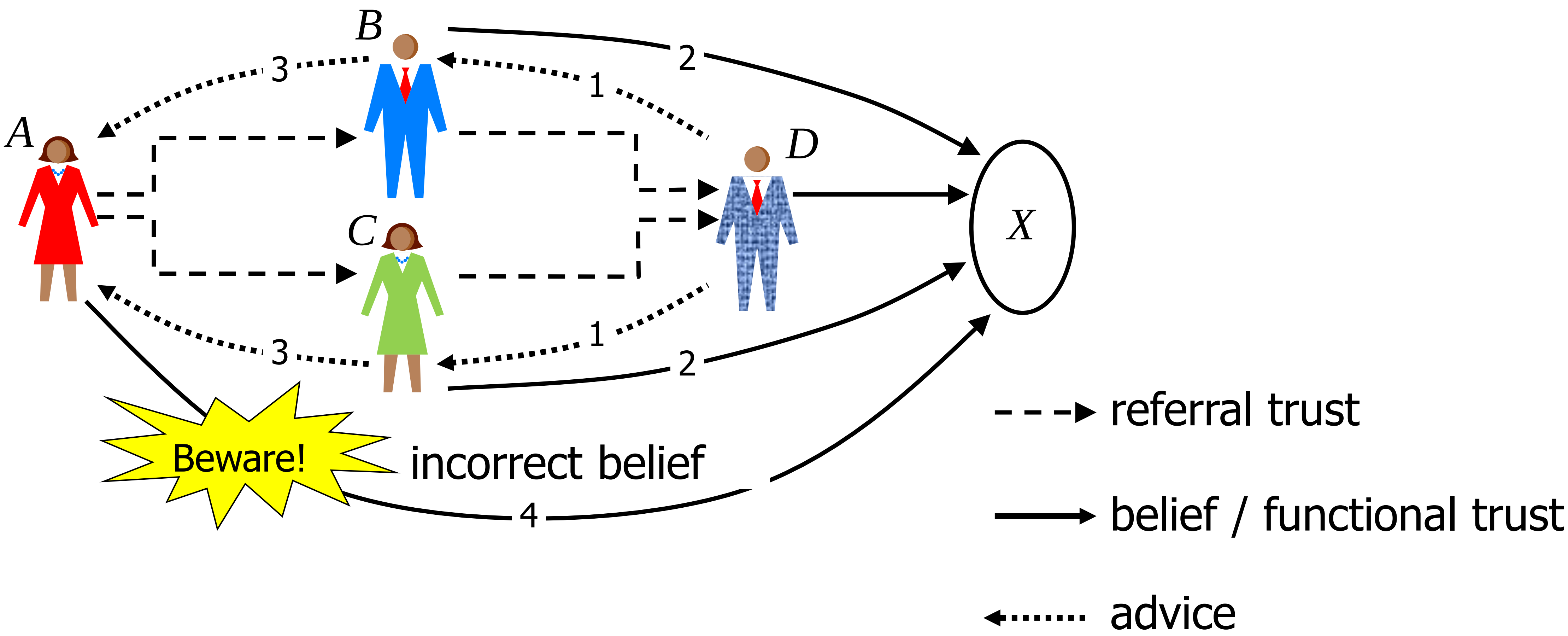
Discount and Fuse: Dilution and Confidence



Discounting dilutes trust confidence

Fusion strengthens trust confidence

Incorrect trust / belief derivation

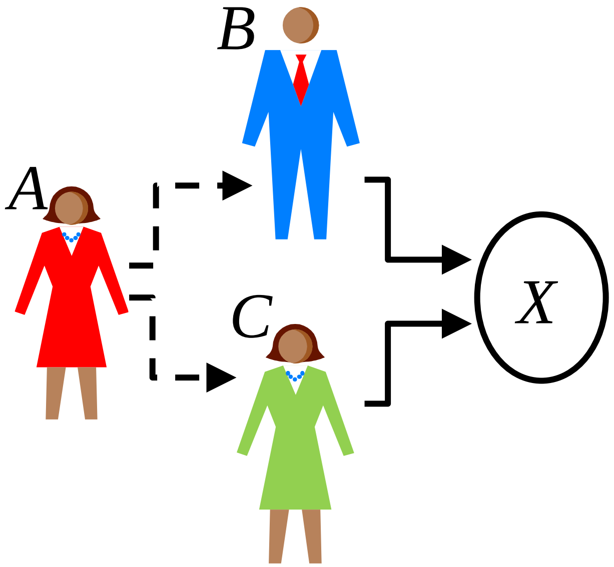


Perceived: $([A, B] : [B, X]) \lozenge ([A, C] : [C, X])$

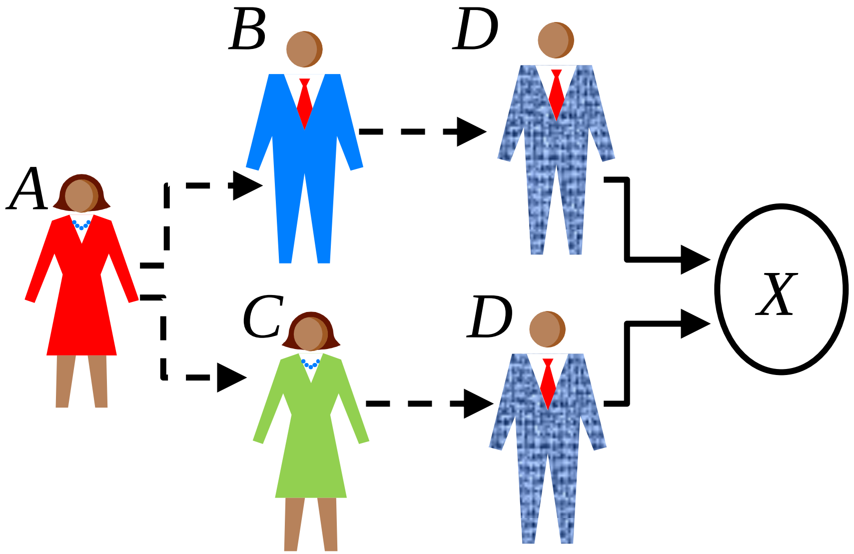
Hidden: $([A, B] : [B, D] : [D, X]) \lozenge ([A, C] : [C, D] : [D, X])$

Hidden and perceived topologies

Perceived topology:



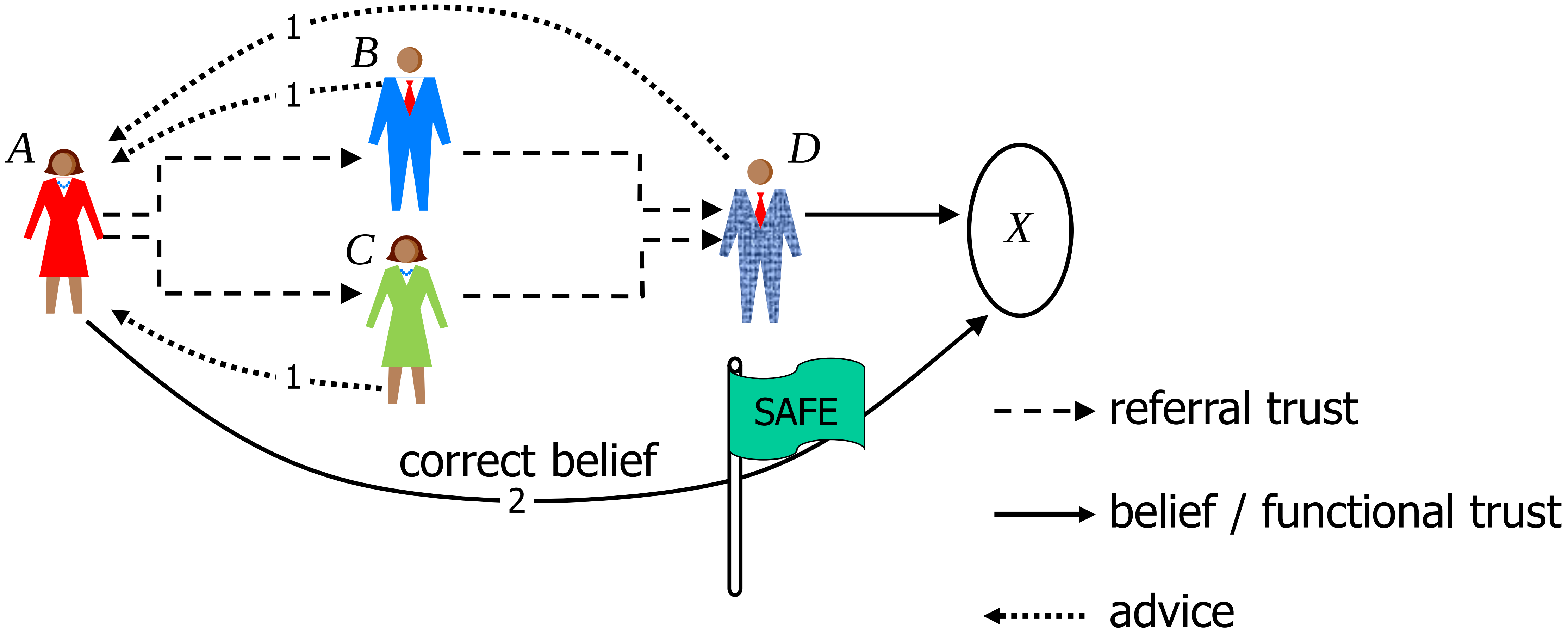
Hidden topology:



$$([A, B] : [B, X]) \;\Diamond\; ([A, C] : [C, X])$$
$$\neq ([A, B] : [B, D] : \mathbf{[D, X]}) \;\Diamond\; ([A, C] : [C, D] : \mathbf{[D, X]})$$

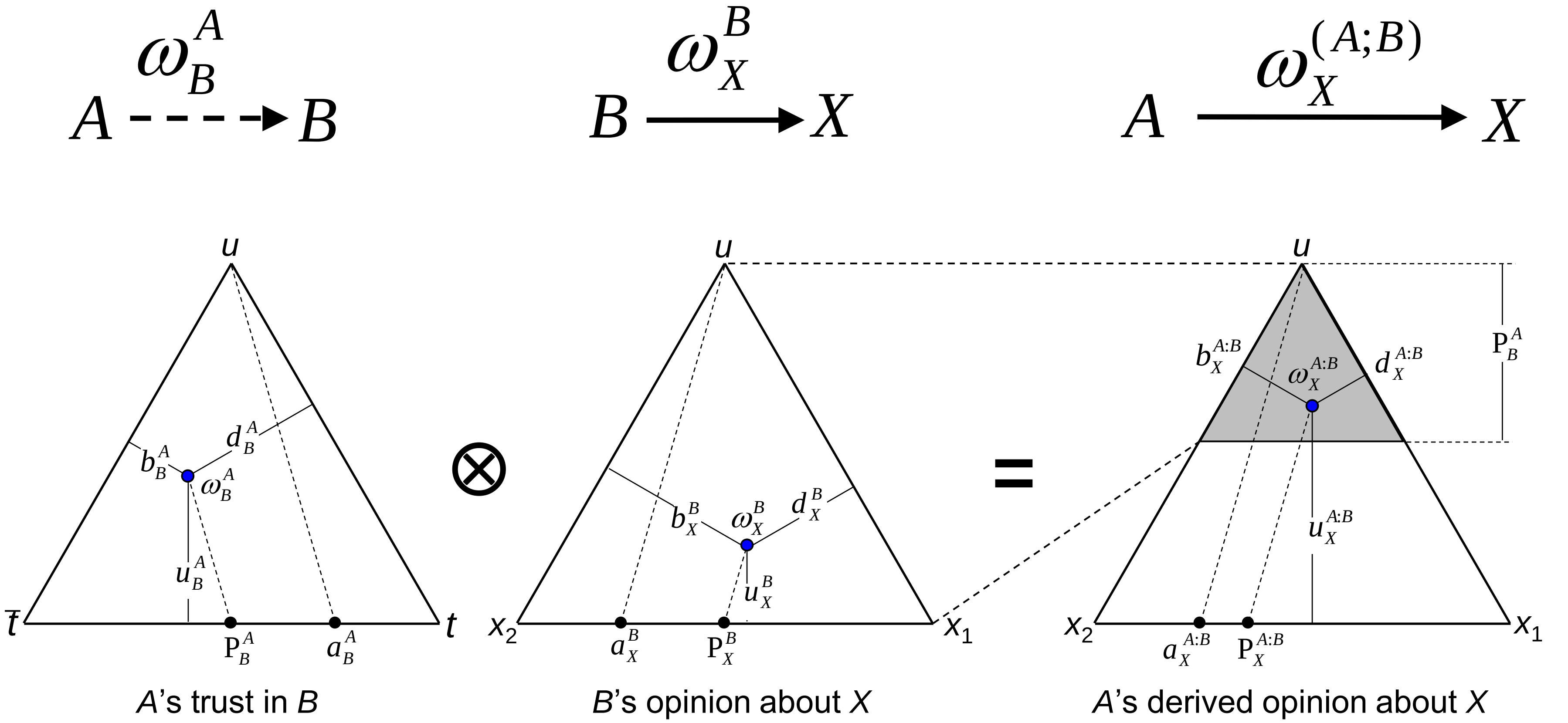
(D, E) is taken into account twice

Correct trust / belief derivation



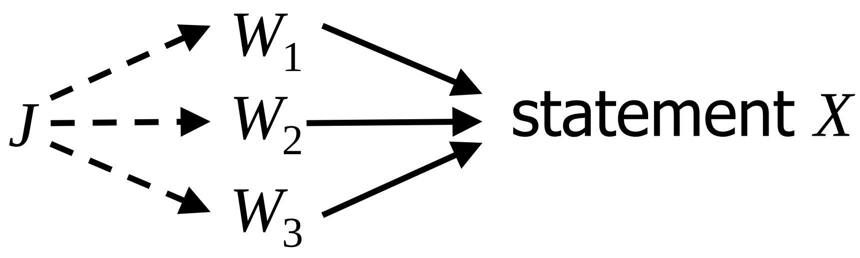
Perceived and_real topologies are equal: $(([A;B]:[B;D]) \Diamond ([A;C]:[C;D])):[D,X]$

Computing discounted trust



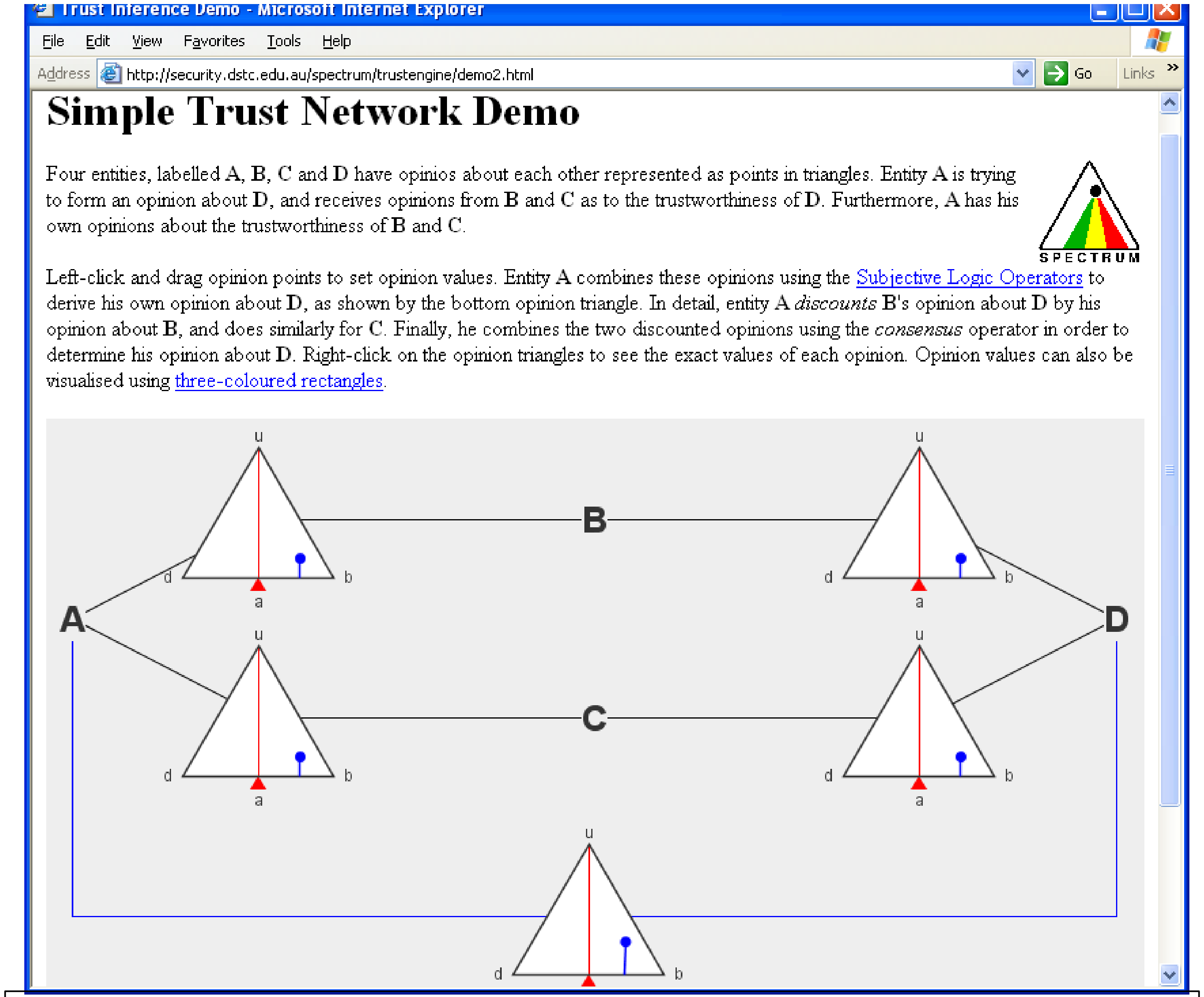
Example: Weighing testimonies

- Computing beliefs about statements in court.
- J is the judge.
- W_1, W_2, W_3 are witnesses providing testimonies.
- X is a statement



Judge's opinion about statement: $\omega_X^{(J;W_1)\Diamond(J;W_2)\Diamond(J;W_3)}$

Computational trust with subjective logic



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Bayesian Reasoning

Bayes' Theorem

Parent node X

Causal conditionals

$\omega_{Y|X}$

Child node Y

Bayes' theorem is used to invert conditionals

Parent node Y

Derivative conditionals

$\omega_{\tilde{X}|Y}$

Child node X

Bayes' Theorem

- Traditional statement of Bayes' theorem:

$$p(x \mid y) = \frac{p(y|x)\,p(x)}{p(y)}$$

- Bayes' theorem with explicit base rates:

$$p(x \mid y) = \frac{p(y|x)a(x)}{a(y)}$$

- Marginal base rates:

$$a(y) = p(y \mid x)a(x) + p(y \mid \bar{x})a(\bar{x})$$

- Bayes' theorem with marginal base rates

$$\begin{cases} p(x \mid y) = \dfrac{p(y|x)a(x)}{p(y|x)a(x)+p(y|\bar{x})a(\bar{x})} \\[2ex] p(x \mid \bar{y}) = \dfrac{p(\bar{y}|x)a(x)}{p(\bar{y}|x)a(x)+p(\bar{y}|\bar{x})a(\bar{x})} \end{cases}$$

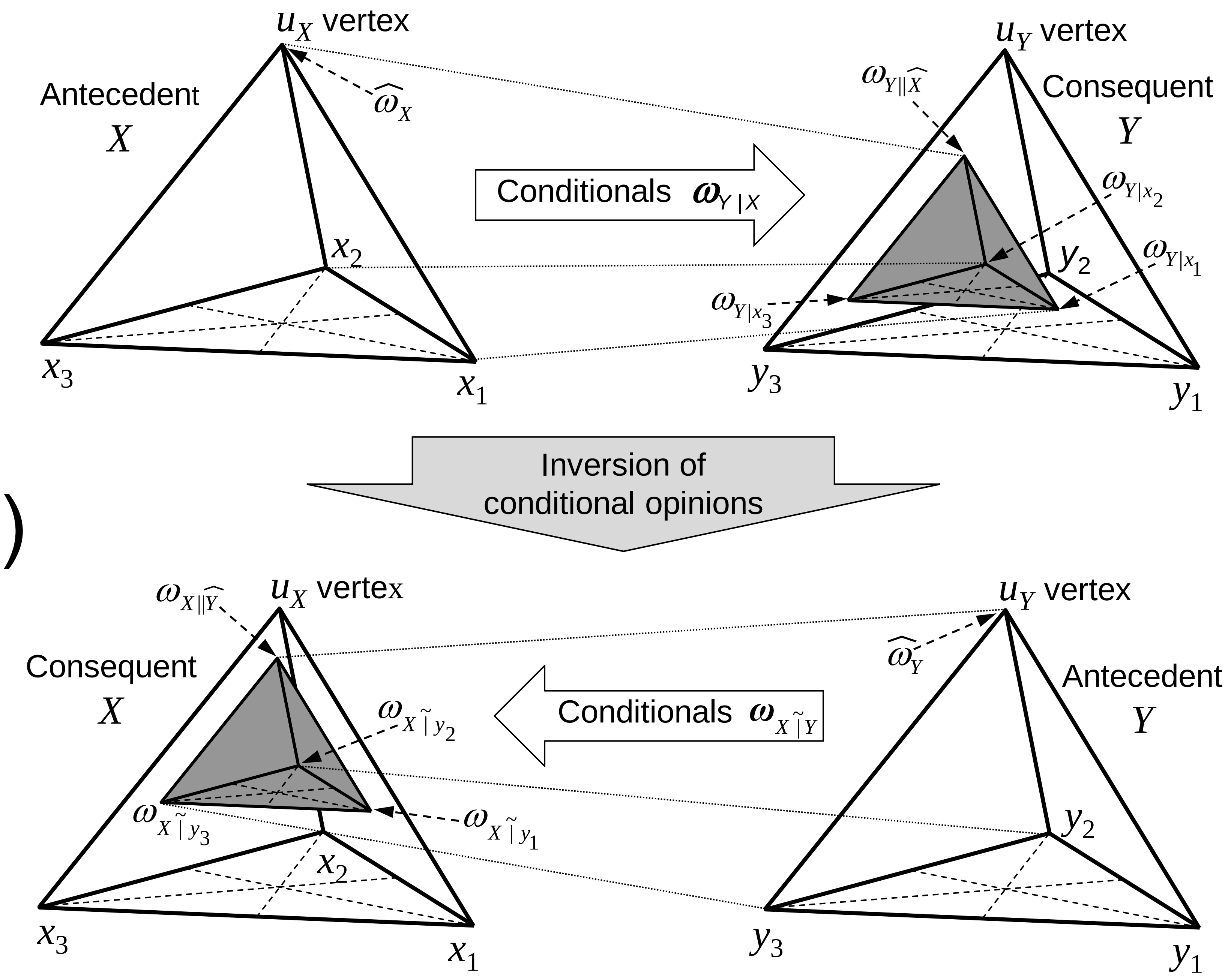
The Subjective Bayes' Theorem
Inversion of conditional opinions

Binomial: $$(\omega_{x \tilde{|} y},\ \omega_{x \tilde{|} \bar{y}}) = \tilde{\phi}\,(\omega_{y|x},\ \omega_{y|\bar{x}},\ a_x)$$

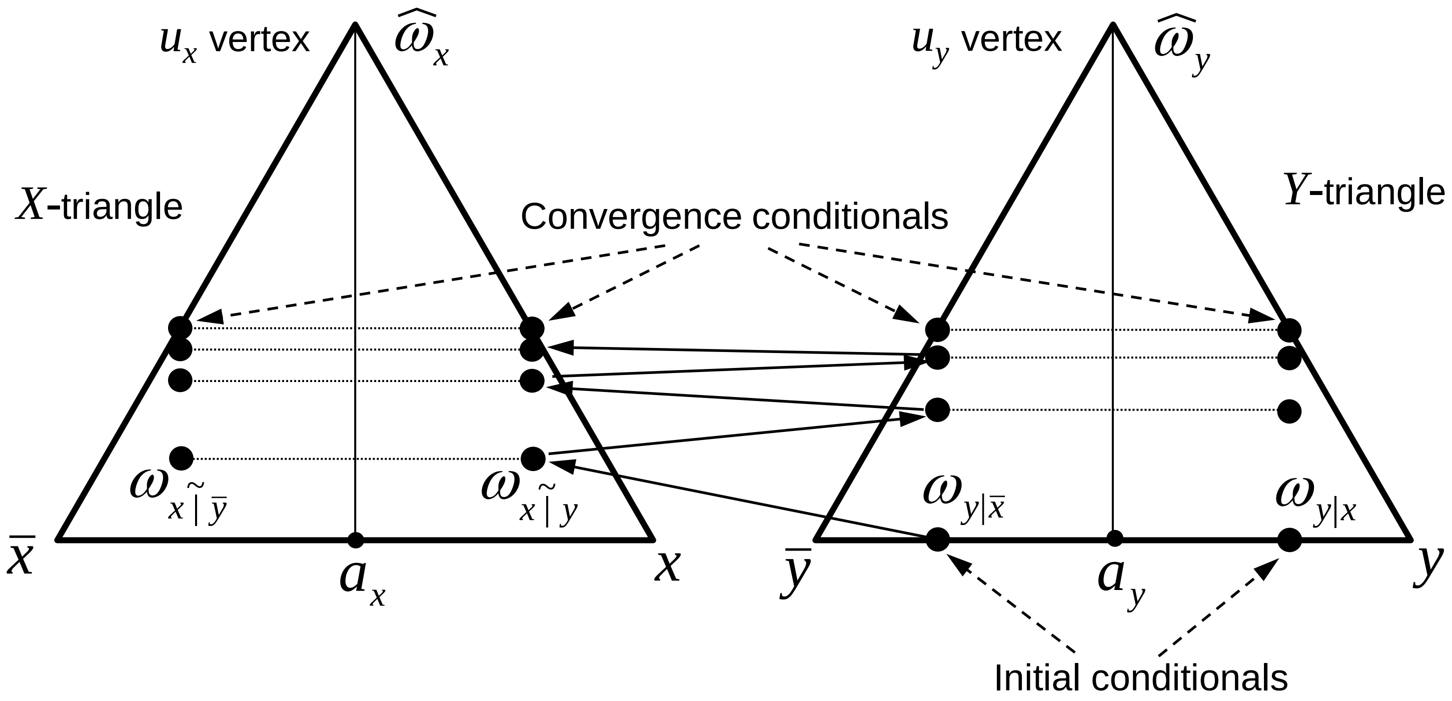
Multinomial: $$\boldsymbol{\omega}_{X \tilde{|} Y} = \tilde{\phi}\,(\boldsymbol{\omega}_{Y|X},\ \boldsymbol{a}_X)$$

Visualising inversion of conditionals

(Subjective Bayes' theorem)

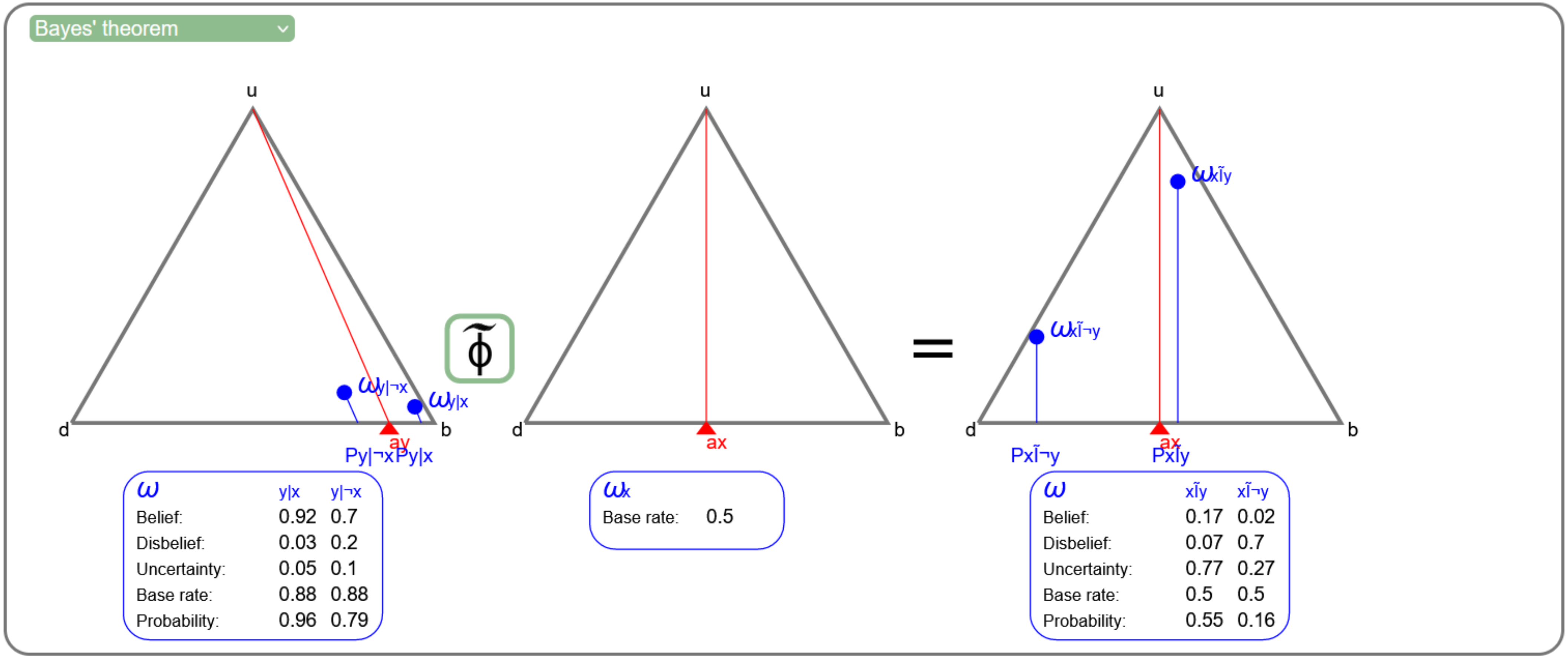


Subjective Bayes' Theorem and Uncertainty



- Figure shows effect of repeated conditional inversion with the subjective Bayes' theorem
- Uncertainty increases and converges to uncertainty-maximised conditional opinions

Bayes' theorem – online operator demo



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Deduction and Abduction

Parent node X

Causal conditionals $\omega_{Y|X}$

Child node Y

ω_X

Deduction

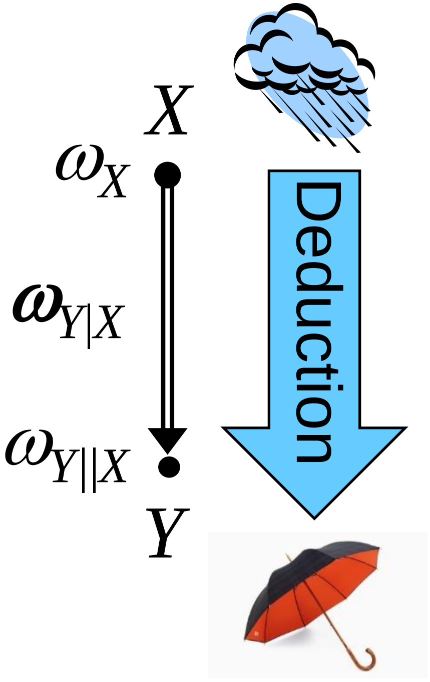
$\omega_{Y\|X}$

$\omega_{X\widetilde{\|}Y}$

Abduction

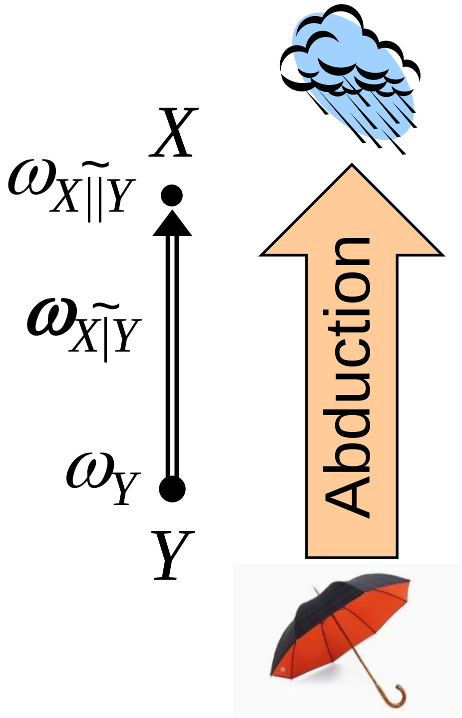
ω_Y

Deduction and abduction notation



$$\omega_{Y\|X} = \boldsymbol{\omega}_{Y|X} \circledcirc \omega_X$$

$$\begin{aligned}\omega_{X\tilde{\|}Y} &= (\boldsymbol{\omega}_{Y|X},\ \boldsymbol{a}_X) \,\tilde{\circledcirc}\, \omega_Y \\ &= \tilde{\phi}(\boldsymbol{\omega}_{Y|X},\ \boldsymbol{a}_X) \circledcirc \omega_Y \\ &= \boldsymbol{\omega}_{X\tilde{|}Y} \circledcirc \omega_Y\end{aligned}$$



Example: Medical reasoning

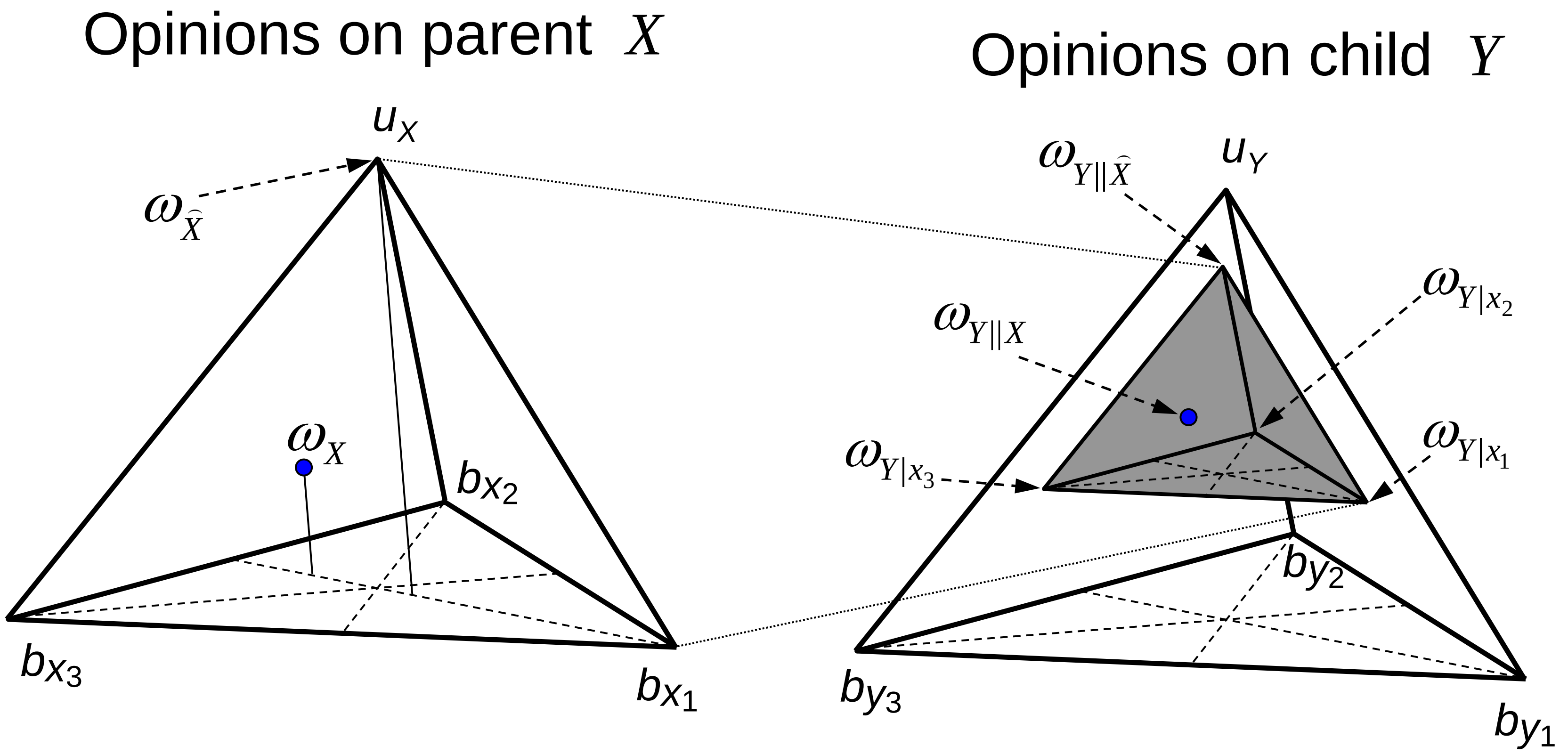
- Medical test reliability determined by:
  - true positive rate $p(y \mid x)$ where x: infected
  - false positive rate $p(y \mid \bar{x})$ y: positive test
- Bayes' theorem: $p(x \mid y) = \dfrac{p(y \mid x)\,p(x)}{p(y)} = \dfrac{p(y \mid x)a(x)}{p(y \mid x)a(x) + p(y \mid \bar{x})a(\bar{x})}$
- Probabilistic model hides uncertainty
- Use subjective Bayes' theorem to determine $\omega_{(\text{infected})}$

$$\omega_{X \tilde{|} Y} = \tilde{\phi}\,(\boldsymbol{\omega}_{Y|X}\,,\ \boldsymbol{a}_X\,)$$

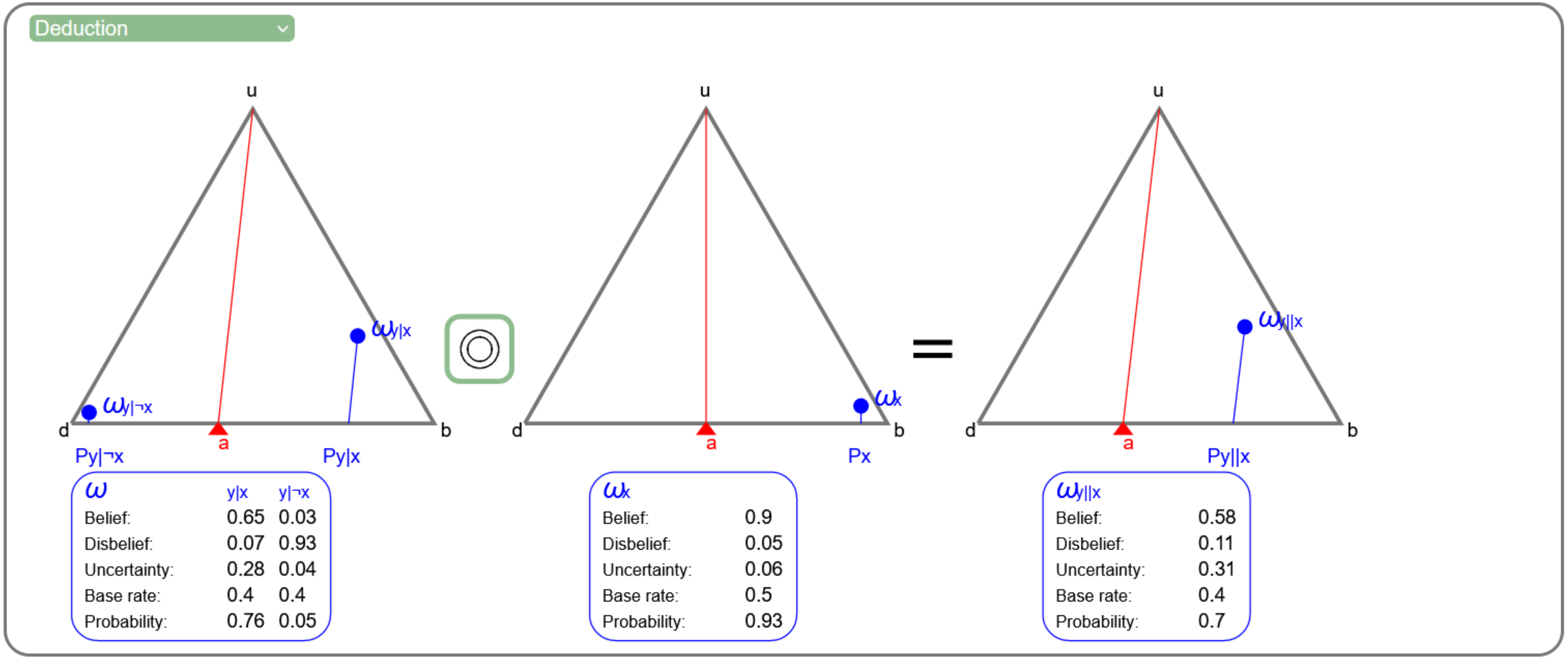
- GP derives $\omega_{(\text{infected}\,\tilde{|}\,\text{positive})}$ and $\omega_{(\text{infected}\,\tilde{|}\,\text{negative})}$
- Finally compute diagnosis $\omega_{(\text{infected}\,\tilde{\|}\,\text{test result})}$
- Medical reasoning with SL reflects uncertainty

Deduction visualisation

- Evidence pyramid is mapped inside hypothesis pyramid as a function of the conditionals.
- Conclusion opinion is linearly mapped



Deduction – online operator demo



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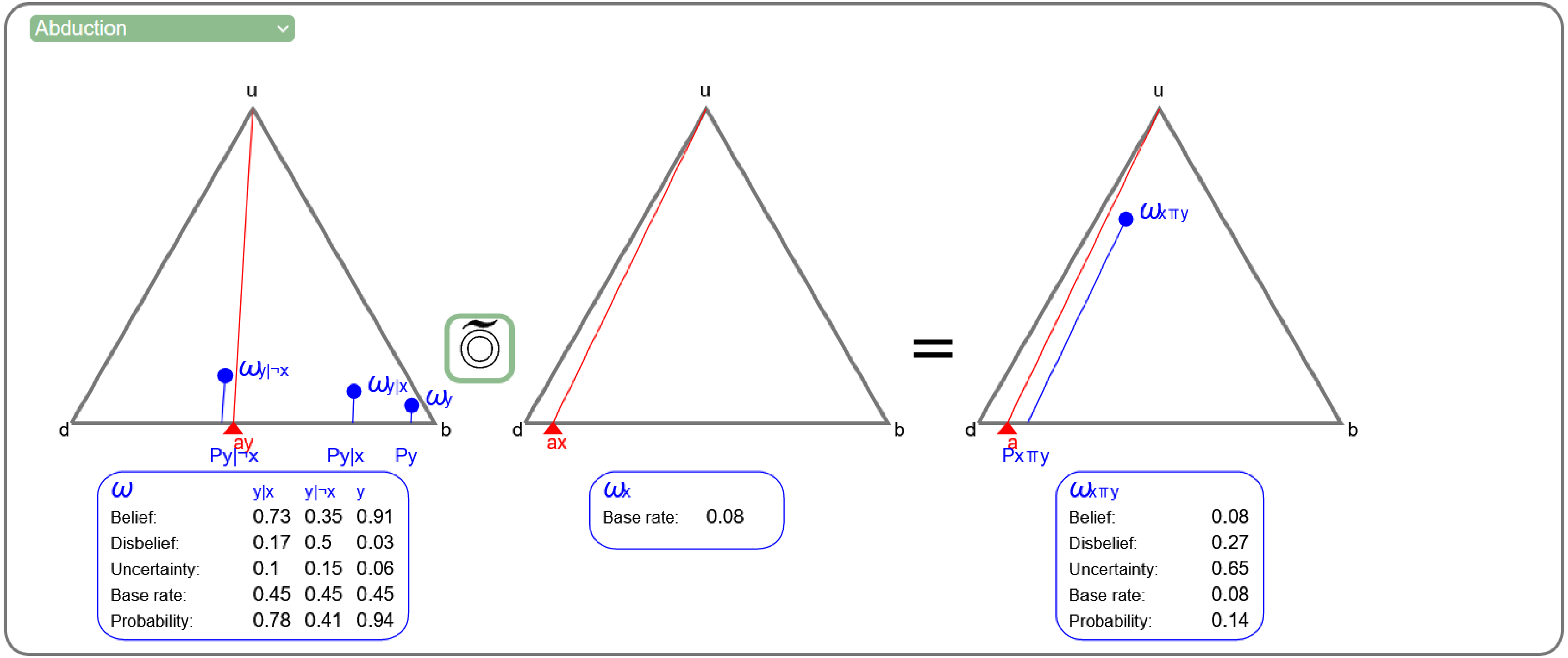
Abduction – Online operator demo



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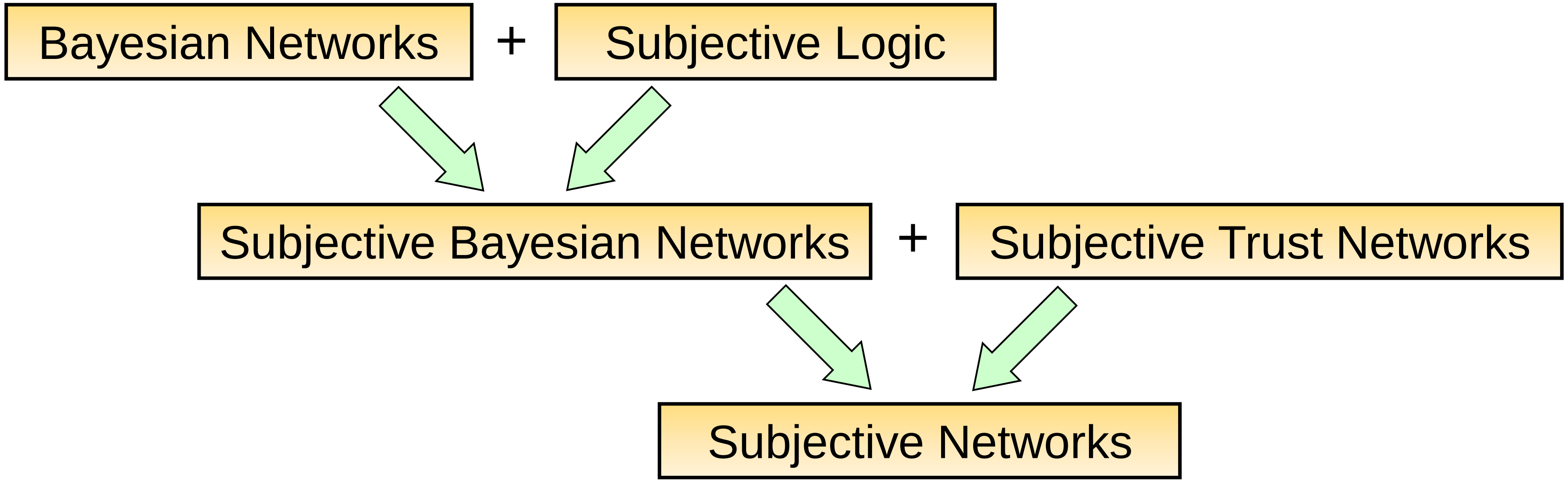
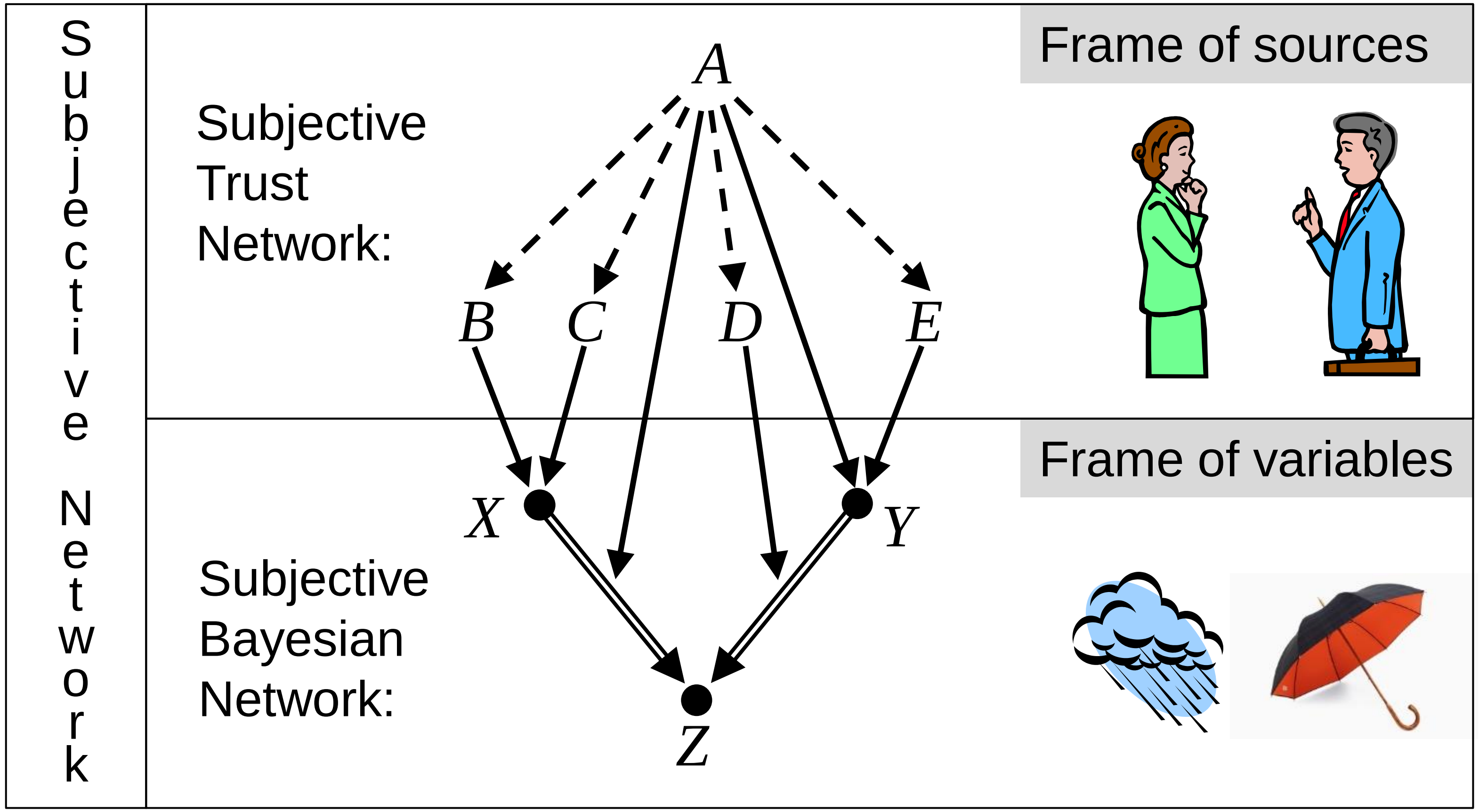
The General Idea of Subjective Networks



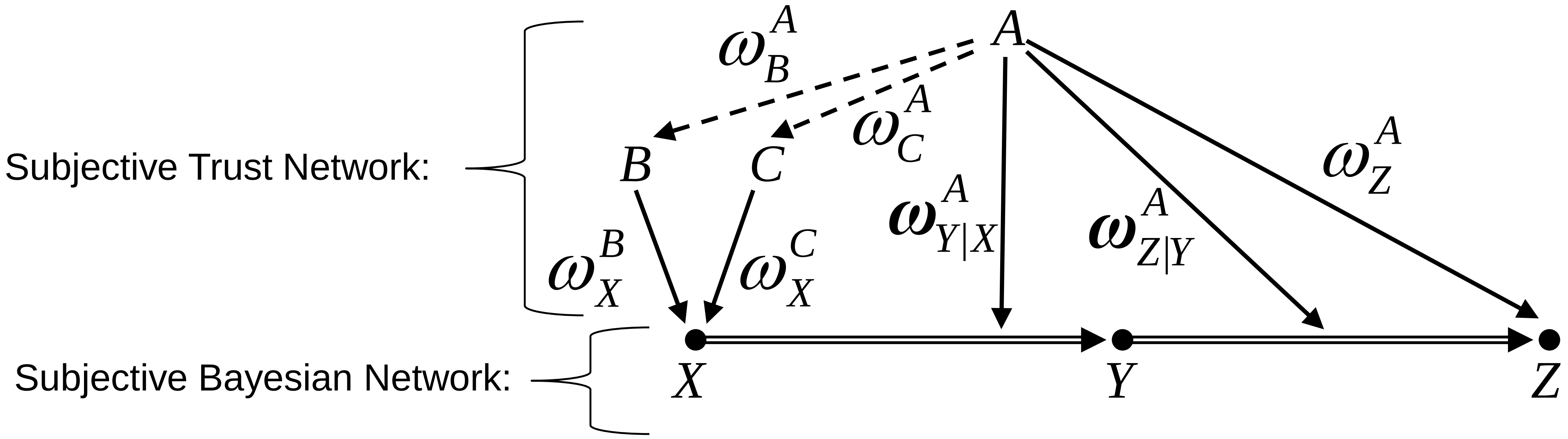
Subjective Networks



Legend:

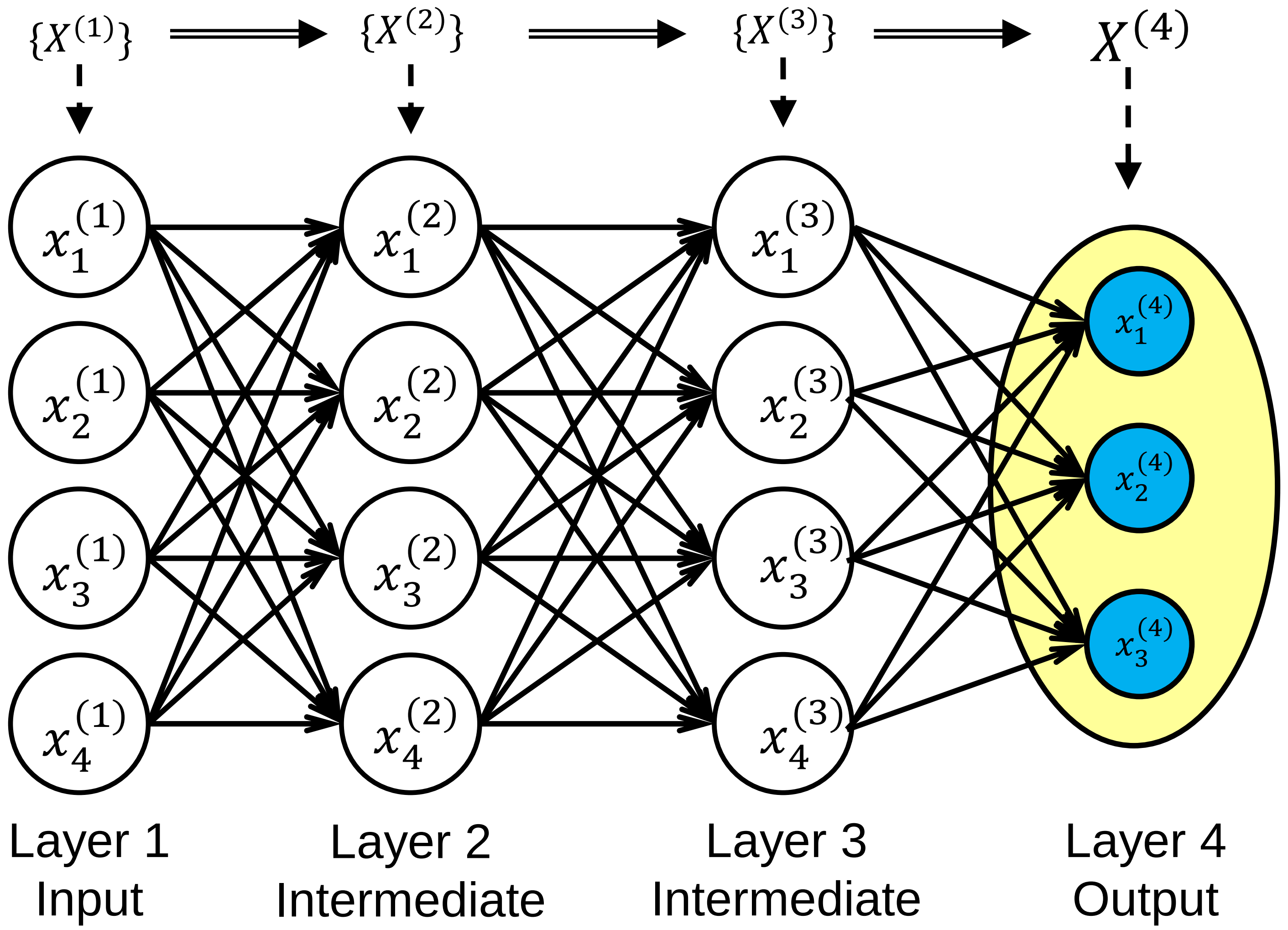
A:	Analyst;	- - - - - →	Trust;
B,*C*,*D*,*E*:	Sources;	——→	Belief;
X, *Y*, *Z*:	Variables;	•══⇒	Conditional dependence.

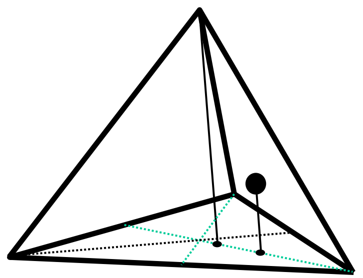
Example Subjective Network Model



$$\omega_Z^A = \boldsymbol{\omega}_{Z|Y}^A \circledcirc (\boldsymbol{\omega}_{Y|X}^A \circledcirc ((\omega_B^A \otimes \omega_X^B) \oplus (\omega_C^A \otimes \omega_X^C)))$$

Trust and Uncertainty in AI





Classification expressed as subjective opinion

MUDL:
Multidimensional Uncertainty-Aware Deep Learning Framework

- Research project during 2021-2025
- Coordinated by Virginia Tech USA
- Collaboration with
  - University of Texas at Dallas
  - US Army Research Lab
  - University of Oslo

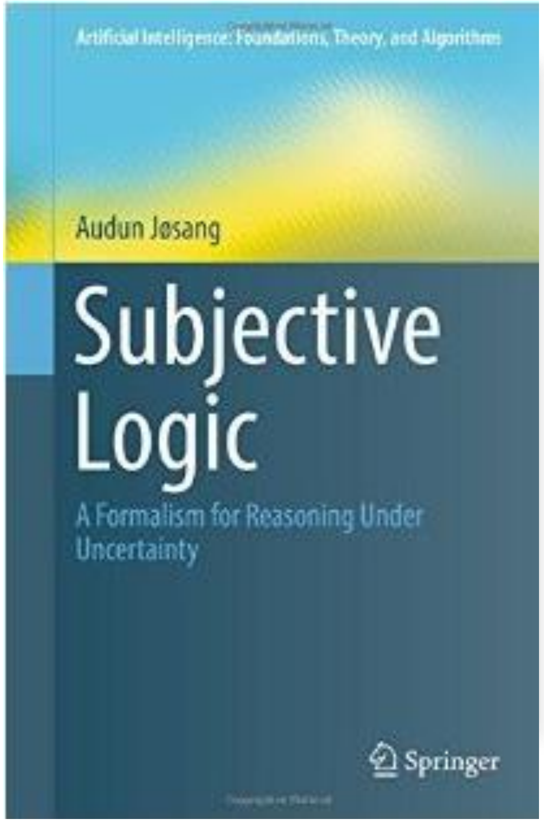
- Funded by NSF

National Science Foundation
WHERE DISCOVERIES BEGIN

Book on Subjective Logic

Springer

springer.com



A. Jøsang

Subjective Logic

A Formalism for Reasoning Under Uncertainty

Series: Artificial Intelligence: Foundations, Theory, and Algorithms

- **A critical tool in understanding and incorporating uncertainty into decision-making**
- **First comprehensive treatment of subjective logic and its operations, by the researcher who developed the approach**
- **Helpful for researchers and practitioners who want to build artificial reasoning models and tools for solving real-world problems**

This is the first comprehensive treatment of subjective logic and all its operations. The author developed the approach, and in this book he first explains subjective opinions,