

● *Contributed Paper***PERMEABILITY RELATION FOR PERIODIC STRUCTURES**

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The permeability relation for periodic porous media is studied with respect to other petrophysical parameters such as formation factor, porosity, surface-to-volume ratio, and nuclear magnetic resonance (NMR) relaxation time. All these quantities were computed for periodic structures of simple, body-centered, and face-centered cubic arrays of touching and overlapping spheres. The formation factors were calculated by using a method which is based on a Fourier-space representation of an integral equation for the electric potential in a two-component composite. The nuclear magnetic resonance relaxation time for the case where surface-enhanced relaxation plays a dominant role is known to be $V_p/\rho S$ (V_p is the pore volume, S is the pore surface, is the surface relaxation strength) when ρ is not too large. Previously calculated permeabilities for these structures from the literature were used for correlation studies with other petrophysical parameters. Various correlation schemes among these quantities, such as $k = aT^b F^c$, and $k = aT^b \phi^c$, were investigated, where k is permeability, T is the NMR relaxation time, ϕ is the porosity, and F is the formation factor. © 1998 Elsevier Science Inc.

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INTRODUCTION

Permeability is one of the most important parameters of porous media. In the petroleum industry, it is used as the determining factor for making production decision. Because direct measurements of permeability in boreholes as a function of depth is often impractical, well-log measurements of other petrophysical parameters, such as porosity, formation factor, nuclear magnetic resonance (NMR) relaxation time, sonic attenuation, mineral compositions, . . . , etc., are used to estimate permeability. These estimators are based largely on empiric correlations.^{1–3}

In this paper, we study the correlation between the theoretically computed petrophysical parameters of permeability, porosity, formation factor, and NMR relaxation time, for periodic porous structures. All these quantities are computed for simple cubic, body-centered cubic, and face-centered cubic arrays of touching and overlapping spheres. The computation of porosity for these structures is straightforward. The formation factor is calculated using a method proposed by Bergman and Dunn.⁴ The NMR relaxation time for the case where the surface relaxation plays a dominant role is given by V_p/S

(V_p is the total volume of the pore space, S is the total area of the pore-matrix interface, and ρ is the relaxation strength), and can be easily calculated. The values of permeability are taken from Larson and Higdon.⁵

The purpose of correlating theoretically computed petrophysical quantities for periodic porous structures is to investigate any systematic relationship between these parameters which may still be valid for disordered porous media. The advantage of using theoretically computed data are that it provides us the opportunity of independently varying physical parameters, such as size scale, surface relaxation strength, and degree of consolidation, and allows us to study how these changes affect the fitting parameters of the empiric correlation schemes. This is expected to provide a basis for improved interpretation of real rock data, as well as lead to insights regarding the relationship between the empiric permeability predictors and the microstructure of real rocks.

FORMATION FACTOR

We use the method described in Ref. 4 to compute the formation factor for periodic porous media. Moments of

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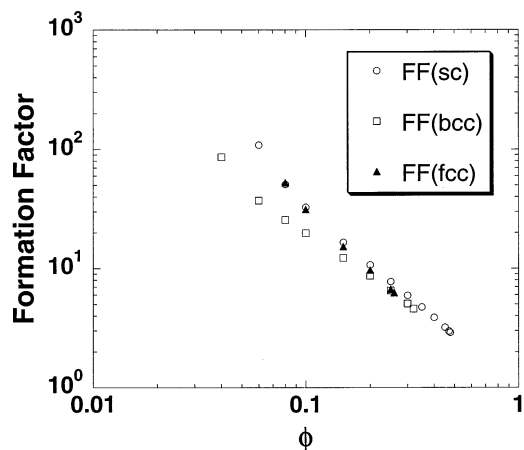


Fig. 1. Formation factor as a function of porosity.

the pole spectrum of a two-component composite are calculated to produce a continued fraction expansion for the bulk effective dielectric constant or electrical conductivity, and rigorous bounds are used to determine that quantity for the composite.

The computation involves Fourier expansion in reciprocal lattice space up to $N = 21$, where N defines the size of the reciprocal space of such an expansion. The reciprocal lattice vectors $g = 2\pi(n_x, n_y, n_z)/a$ have integers n_x , n_y , and n_z ranging from $-N$ to $+N$. The formation factors computed for three different periodic structures, i.e., simple cubic, body-centered cubic, and face-centered cubic arrays of identical touching and overlapping spheres are shown in Fig. 1. Comparing our results with those of Schwartz and Kimminau⁶ and Shen et al.,⁷ we find results of the present calculation are quite consistent with those of Ref. 6, which used the network simulation and minimum area approximation approaches. The formation factors calculated in Ref. 7 tend to be smaller at low porosity values. This is understandable because they used a Fourier expansion method with $N = 1$ only. The convergence of Fourier expansion method is known to be reasonably fast for porosities where spheres just touch or slightly overlap with each other, but becomes quite slow for lower porosities where many higher order Fourier expansion terms are needed.

PERMEABILITY

The problem of theoretically computing permeability for periodic porous media has been studied extensively in the past by expanding in spherical harmonics. Here, we use Larson and Higdon's⁵ results for our analysis. The permeability values from Ref. 5 are normalized by length² and are, hence, dimensionless. To convert them to millidarcy, we have assumed a lattice size of $a = 10 \mu\text{m}$.

The NMR relaxation times are calculated assuming

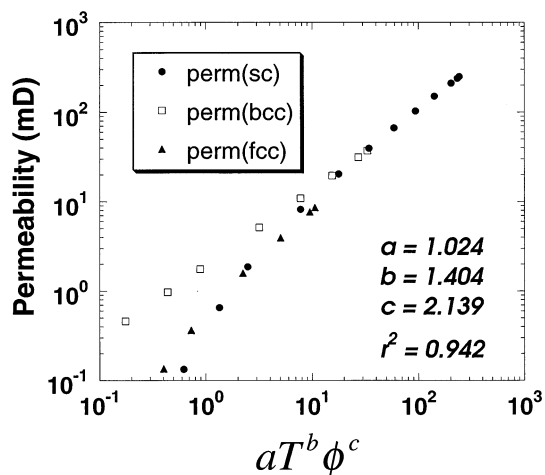


Fig. 2. Permeability for periodic structures for an edge length for the Bravais lattice of $10 \mu\text{m}$.

that surface relaxation is the dominant mechanism for the polarization decay and that bulk relaxation can be ignored. For a small, finite surface absorption strength (i.e., $\rho a/D < 1$), the relaxation time T_1 is simply given by $Vp/\rho S$. We computed the relaxation times in milliseconds by assuming a surface relaxation strength of $\rho = 10^{-3} \text{ cm/s}$ and evaluating Vp and S directly for the case of $a = 10 \mu\text{m}$. Here we do not make the distinction between the longitudinal relaxation time T_1 (for which ρ_1 should be used) and the transverse relaxation time T_2 (for which ρ_2 should be used)⁸. In general, as the Larmor frequency gets lower, T_2 is quite close to T_1 , except that T_2 may be diffusionally shortened by induced internal field gradients and externally applied field gradient. The strength of the induced internal gradients depends on the magnetic susceptibility contrast between solid and pore fluid.⁹

Figure 2 shows the result of using the form $aT^b \phi^c$ to represent permeability. Here we have included only porosities for which the spheres are touching or overlapping. The permeability is computed using an edge length for the Bravais lattice of $10 \mu\text{m}$. The fitting constants are $a = 1.024$, $b = 1.404$, and $c = 2.139$, with a correlation coefficient $r^2 = 0.942$. Note that the data sets we used are different packings of identical spheres where the porosity reduction is achieved by increasing the amount of overlap between neighboring spheres. This grain consolidation process reduces permeability and provides the dynamic range for the permeability shown in Fig. 2. However, such a large variation for permeability can also be achieved at a fixed porosity by simply varying the overall size scale for the spheres. In the latter case, the permeability would simply vary as the square of the length scale.

Figure 3 shows the results of such a change of size scale where the data sets include those of Fig. 2 and the

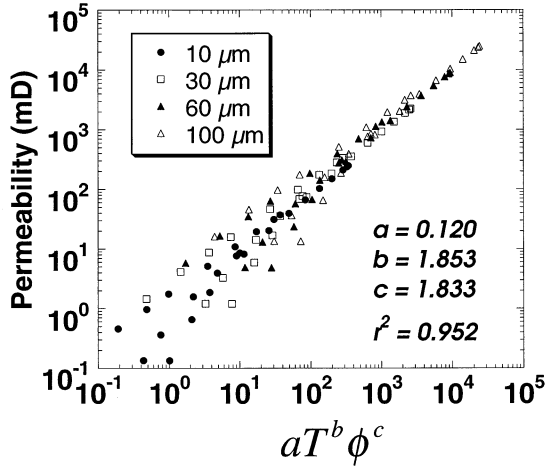


Fig. 3. Permeability versus $aT^b\phi^c$ for periodic structures with different scales.

scaled data sets for edge lengths 30, 60, and 100 μm . Using the same functional form, $aT^b\phi^c$, the fitting constants now become $a = 0.120$, $b = 1.853$, and $c = 1.833$, with a correlation coefficient $r^2 = 0.952$. Notice that the constant b changes from 1.404 to 1.853, from a data set of pure grain consolidation to a data set of both grain consolidation and scale changes. In the functional form, $aT^b\phi^c$, T is the only parameter that depends on the length scale (T is simply proportional to that scale). If the permeability change were entirely due to the scale change, the exponent for T would be exactly 2. Naturally, the b value for the pure consolidation case depends on our model for the periodic structures. The b value for the real rock systems may be different.

When the commonly used fitting constants^{2,3} $a = 1.0$, $b = 2.0$, and $c = 4.0$ are used, the correlation coefficient r^2 becomes 0.913, and the correlation is not as good as that shown in Fig. 3, especially at the higher permeabilities. It is worth pointing out that the change in the exponent c may be quite significant. Such a large change is tolerable because the correlation coefficient is only changed very slightly: the reduction in the factor ϕ^c by increasing c is mostly offset by the modest increase of the exponent b in T^b . This highlights the fact that T and ϕ are not independent parameters in characterizing the permeability.

Estimates of the surface relaxivity ρ vary from 10 to 1 $\mu\text{m/s}$.^{2,8} Figure 4 shows the results of varying ρ where we use the result of Fig. 2 for a lattice constant of 10 μm but with varying strength of surface relaxation from 10 to 1 $\mu\text{m/s}$. Because there is no scale change, the permeability value remains the same as the surface relaxivity changes. This has the effect of broadening the range of scatter even in the high permeability region, where our model showed very good data collapse. Hence, we would

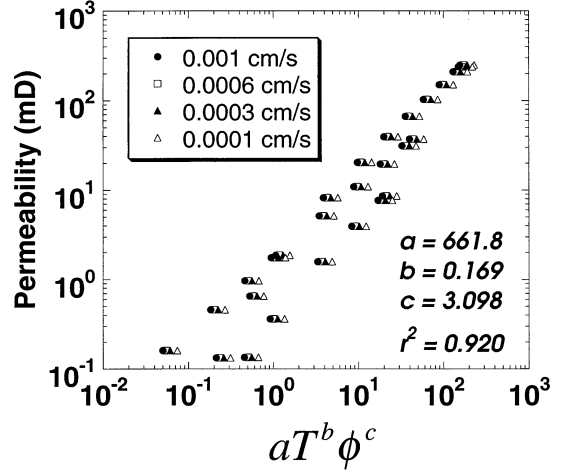


Fig. 4. Permeability versus $aT^b\phi^c$ for periodic structures with different surface relaxivities.

like to argue that in real rock systems of clastic sediment the breadth of scatter of the permeability data in the high permeability region may be in large part due to the wide range of values for the surface relaxivity.

Figure 5 shows the results when we take the data set of Fig. 3, and plot permeability versus $aT^b F^c$ instead of $aT^b\phi^c$. The exponents b and c used in Fig. 5 are quite consistent with those used in Fig. 3, if we apply the relation $F \approx \phi^{-1.5}$, as observed in Fig. 1 for large porosities. However, the use of formation factor F instead of porosity ϕ is more than just a simple substitution because the Archie's exponent is often different for different structures at low porosities. As a result, using F instead of ϕ significantly increases the correlation with permeability data, as evidenced by the increase of correlation coefficient r^2 in Fig. 5 as compared with that in Fig. 3.

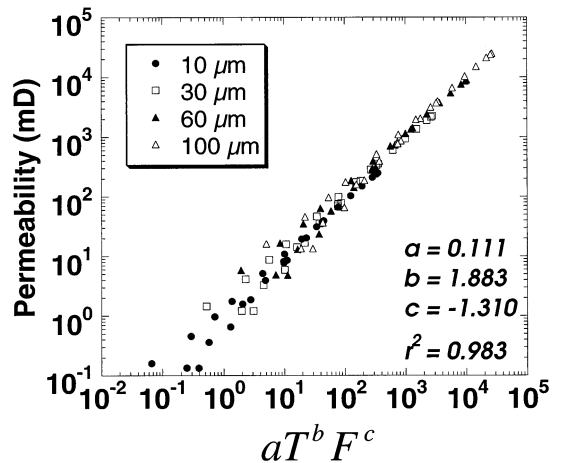


Fig. 5. Permeability versus $aT^b F^c$ for periodic structures with different scales.

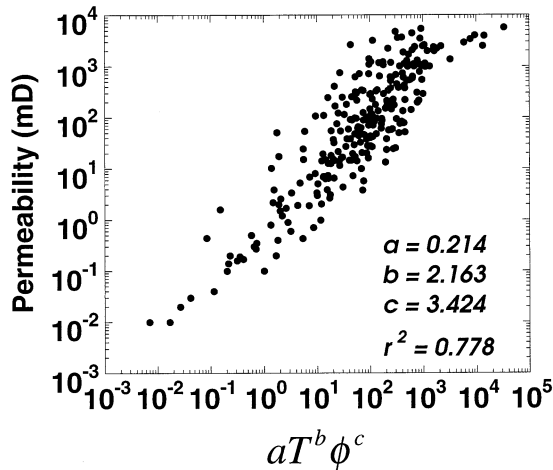


Fig. 6. Permeability versus $aT^b\phi^c$ for sandstones.

REAL ROCK DATA

The real rock data present many more uncertainties and unknown factors. The model systems we considered so far were made of identical spheres. Although the sizes of the spheres (or grain size) can be varied so as to have a large dynamic range for permeability, it is not trivial to estimate the permeability of a composite where there is a distribution of grain sizes. The permeability of the composite will depend on how these grains of different sizes are packed together.

Figure 6 shows the results for permeability of sandstones of 260 core samples from different parts of the world, where the NMR relaxation times are T_{1G} , the geometric averages of T_1 over the distribution of relaxation times of each sample between 1 ms to 10 s measured at 10 MHz. With such a large data scattering, it is very difficult to extract any underlying feature. The fact that the exponent b is close to 2 leads us to conclude that the data set is composed mainly of grains of different sizes and with very little differences in the degree of grain consolidation. This is indeed shown to be the case as most of the core porosities fall in a narrow range of 0.20 to 0.25 even though the permeability values span a large range.

The large horizontal scatter of data points in these figures may be an indication that there is a large variation in surface relaxation strength among different samples. This can easily increase the value of the exponent for ϕ as demonstrated in Fig. 4.

CONCLUSIONS

We have studied the correlation between theoretically computed permeability, porosity, formation factor, and

NMR relaxation times for periodic porous media of identical touching and overlapping spheres. Although the systems that we consider are extremely idealized, this approach has the advantage of isolating each physical parameter, such as grain consolidation (porosity reduction), size scaling (size of the grain), and surface relaxation strength, and studying its implication and effects individually on the fitting constants in the various correlation schemes. We have arrived at the following conclusions: 1) The form aT^bF^c generally provides better correlation to permeability than $aT^b\phi^c$; and 2) the exponent b for the relaxation time T varies from 1.4 for pure grain consolidation systems to 2 for pure scaling change systems. Sedimentary rock systems often have a value near 2 indicating that the changes are mainly due to grain size change.³ The variation of surface relaxation strength can cause the data for permeability versus $aT^b\phi^c$ or aT^bF^c to scatter horizontally, increasing the value for the exponent c .⁴ The form $aT^b\phi^c$ is relatively tolerant, especially for some-what large variation of c . This happens because large increase in c can be easily offset by modest increase in b .

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