

● Short Communication

ESTIMATES OF PERMEABILITY AND IRREDUCIBLE WATER SATURATION BY MEANS OF A NEW ROBUST COMPUTATION OF FRACTIONAL POWER AVERAGE RELAXATION TIMES

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In a suite of water-saturated sandstones, we have recently demonstrated that irreducible water saturation can be well estimated using relaxation time *only*, in the form of any of several “averages” giving more emphasis to short times than does the geometric mean time. The best estimate of permeability came from fits giving more emphasis to slightly longer times. In this paper we present estimates of irreducible water saturation and permeability using approximations (here called *Robusta* approximation) to the fractional power average relaxation time $\langle T^p \rangle^{1/p}$. The advantage of this approximation is that it does not involve previous computation of exponential components; therefore, it does not depend on the choice of the inversion method, and it permits a very fast computation. The *Robusta* approximation gives some of the best correlations using $p = -0.55$ for irreducible water saturation and $p = +0.18$ for permeability. © 1998 Elsevier Science Inc.

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INTRODUCTION

In the last decade great efforts have been made to improve nuclear magnetic resonance (NMR) relaxation estimates of permeability κ and irreducible water saturation S_{wi} of porous media from NMR measurements on samples fully saturated with water. An intriguing problem is to decide what single relaxation time T should be used to represent a complex relaxation curve or distribution of relaxation times, when relaxation times can be distributed over a range of a factor of a thousand or more. The choice of a form of “average” time can give emphasis to either the larger or else the smaller pores. We will show that S_{wi} and κ can be estimated better from different forms of average relaxation times. For example, for either purpose, the arithmetic average time T_{aa} will weight a few large pores too heavily, while the rate average (harmonic mean) time T_{ra} will be overly influenced by a few small pores. The middle ground is a range of average times, whether computed from distributions of relaxation times or by some other means, that is in some sense between T_{ra} and T_{aa} . If we consider the

p -power average $T^{(p)} = \langle T^p \rangle^{1/p}$, then $T_{rms} = T^{(2)}$, $T_{aa} = T^{(1)}$, and $T_{ra} = T^{(-1)}$. The neutral average $T^{(0)}$ between $T^{(-1)}$ and $T^{(1)}$ is the geometric mean relaxation time, $T_{gm} = T^{(0)} = \exp(\ln T) = \lim_{\epsilon \rightarrow 0} \langle T^\epsilon \rangle^{1/\epsilon}$.

In a recent report¹ on a set of sandstone samples we were able to find an estimator of S_{wi} that gives a smaller error factor δ ($= 1.26$) by using relaxation times *only* that could be obtained by using a cutoff relaxation time. By far the best estimator that we tried was the double exponential $S_{wi} \approx 1 - \exp(1 - \exp(\tau/T))$, where the characteristic time τ depends on the specific definition and computation of the relaxation time T . We have seen that average relaxation times T that give *significantly more emphasis to short times* than does the geometric mean were optimum for estimating the irreducible water saturation. These include fractional p -power averages computed from distributions of relaxation times, with a negative exponent $p \approx -0.4$. This negative exponent seems to give the correct emphasis to the short times, that is, to the high surface-to-volume ratio pores, where the wetting phase remains trapped. We also observed that estimates of S_{wi} deteriorated when the shortest data time

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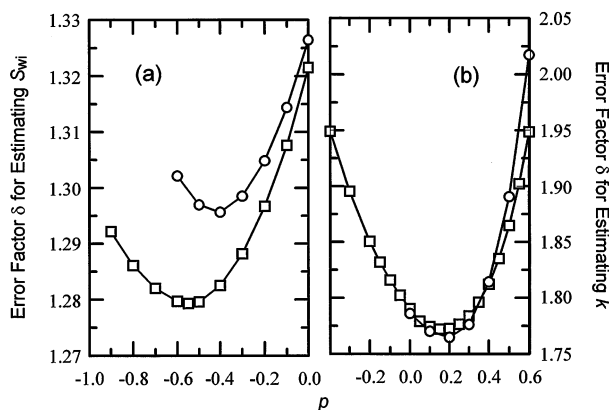


Fig. 1. Error factors δ of the fit to $\ln S_{wi}$ by $\ln [1 - \exp (1 - \exp (\tau/T))]$ (a) and to $\ln (\kappa/\phi)$ by $A + B \ln T$ (b), by using as T the fractional p -power average of computed exponential components (circles) and the RA (squares). κ is defined by the Darcy law, $\mathbf{u} = (\kappa/\mu) \nabla P$, where $\mathbf{u}$ is the flow rate per equipotential unit area, μ is the viscosity, ∇P is the pressure gradient. S_{wi} is the fraction of the wetting phase (in this case water) that remains trapped in the solid matrix after displacement with a non-wetting phase. Here and in the following correlations κ is in milli-Darcy units (a Darcy $\approx 1 \mu\text{m}^2$), times are in milliseconds, S_{wi} is expressed as fraction of pore space, and ϕ is the fractional porosity.

for T_1 measurements started at more than a few tenths of a millisecond. For estimating the permeability κ from some form of relaxation time *alone*, the best T for our data set was precisely the geometric mean, giving an error factor $\delta = 1.94$. A significantly better estimate ($\delta = 1.76$) could be obtained for fit to the logarithm of the Kozeny parameter κ/ϕ and by using several forms of T that gave slightly *less emphasis to short times* than does the geometric mean. The best fractional power average time to use has a positive exponent $p \approx +0.2$. As an alternative to increased weighting of longer relaxation times ($p > 0$), we tried fitting $\ln (\kappa/\phi)$ using geometric mean times computed without extrapolation of signal beyond the shortest time used. The best fits were obtained by not using data before a few milliseconds, whether the power of porosity used is fixed at 1.0 or allowed to vary. It is noteworthy that changing the method of computing the relaxation times has mild effects on the position of the minimum, but the presence of a minimum at a few milliseconds persists. It is to be noted that the fraction of the signal not used when starting at a few milliseconds is *not negligible* for some samples. For a few samples, from 10–25% of the signal has decayed by 2 ms.

These results are compatible with the reasonable picture, where high surface-to-volume pores, giving signal components with short relaxation times but not contributing to the permeability, are important in determining

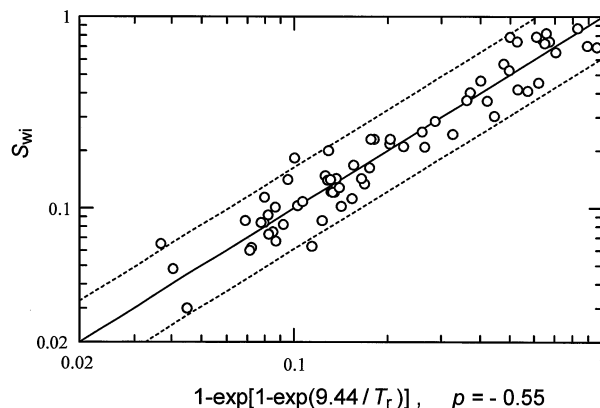


Fig. 2. Measured irreducible water saturation S_{wi} for 67 fully brine-saturated sandstone samples vs. the estimated value, $1 - \exp (1 - \exp(9.44/T_r))$, where T_r is the RA for $p = -0.55$. The error factor is $\delta = 1.28$; the dashed lines are at factors of $\delta^{\pm 2}$.

the fraction of the wetting phase that remains trapped in the solid matrix after displacement with a nonwetting phase.

In this paper we demonstrate that these results can be obtained also by using computations that do not need to invert multiexponential relaxation data. The algorithm is a generalization of the *Robusta* approximation (RA) for T_{gm} , which does not involve previous computation of exponential components.²

NMR T_1 measurements were made by inversion recovery at 41°C and 20 MHz on 106 clean (clay-free) sandstone samples saturated with brine. For experimental

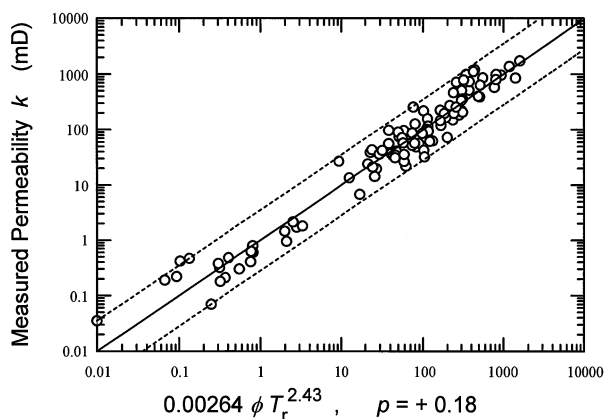


Fig. 3. Measured κ vs. the estimated value, $0.00264 \phi T_r^{2.43}$ for $p = +0.18$. The error factor is $\delta = 1.77$; the dashed lines are at factors of $\delta^{\pm 2}$.

details see Ref. 1. The quality of the fit for the correlations between conventionally measured quantities and NMR-estimated quantities is represented by the error factor $\delta = \exp \epsilon$, where ϵ is the rms logarithmic scatter. The average times we used are the fractional p -power average $T^{(p)} = \langle T^p \rangle^{1/p}$, mentioned above, and its RA, $T_r^{(p)}$, or simply T_r if p is specified separately. $T^{(p)}$ is here derived from a distribution of relaxation times obtained by inverting the relaxation data without extrapolating components beyond the shortest data times used. The algorithm to obtain $T_r^{(p)}$ is derived in Ref. 3.

In Fig. 1 the error factors δ for the estimate of S_{wi} and κ are plotted against p for estimates using $T^{(p)}$ and for estimates using T_r . For $p \ll 0$, T_r is not very close to $T^{(p)}$, but T_r appears nevertheless to be a good form of T for estimation of S_{wi} . By using T_r we get a minimum error for the estimation of S_{wi} at $p = -0.55$ instead of $p = -0.40$, but the errors and their behavior are comparable. The approximation is very close for p values closer to zero. We get the minimum error for the estimate of κ from $T^{(p)}$ at $p = +0.20$ and from T_r at $p = +0.18$, substantially the same. Figures 2 and 3 show S_{wi} and κ , respectively, versus their NMR estimates.

In summary, we see that the irreducible water saturation S_{wi} and the permeability κ of a suite of sandstone samples can be estimated well from NMR relaxation data from the samples saturated with brine. The estimates are

at least as good using certain average relaxation times computed without inversion of the relaxation data as those using times derived from computed distributions of relaxation times. The best estimate of S_{wi} uses an average time giving substantially more emphasis to short times (and small pores) than does the geometric mean. The best estimate of κ uses an average time giving slightly more emphasis to long times (and large pores) than does the geometric mean.

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REFERENCES

1. Borgia, G.C.; Brown, R.J.S.; Fantazzini, P. Different “average” nuclear magnetic resonance relaxation times for correlations with fluid-flow permeability and irreducible water saturation in water-saturated sandstones. *J. Appl. Phys.* 82: 4197–4204; 1997.
2. Borgia, G.C.; Bortolotti, V.; Brown, R.J.S.; Fantazzini, P. A robust method for calculating geometric mean times from multiexponential relaxation data, using only a few data points and only a few elementary operations. *Magn. Reson. Imaging* 14:895–897; 1996.
3. Borgia, G.C.; Bortolotti, V.; Brown, R.J.S.; Fantazzini, P. A method for approximating fractional-power average relaxation times without inversion of multiexponential relaxation data. *Magn. Reson. Imaging* 16:625–627; 1998.