

● Short Communication

A NEW METHOD FOR ESTIMATING T_2 DISTRIBUTIONS FROM NMR MEASUREMENTS

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A semi-continuous relaxation model is constructed using sums of gamma functions and non-negative least squares for the inversion of Carr–Purcell–Meiboom–Gill (CPMG) echo data. No regularization is necessary for this approach, and yet the solution is stable even with noisy data. Test results derived from 60 echo trains are presented. Computational advantages of the method are presented. © 1998 Elsevier Science Inc.

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INTRODUCTION

Typical T_2 distributions of fluid-saturated porous media extend from below 1 ms to over 1 s because of the heterogeneous nature of the pore system and/or variation of surface relaxivity of wetting and non-wetting fluids. Estimation of T_2 distributions from Carr–Purcell–Meiboom–Gill (CPMG) echo-train data:

$$M(nTE) = \int P(T_2) \exp(-nTE/T_2) dT_2 + \text{noise} \quad (1)$$

is known to be an extremely ill-conditioned problem and is particularly onerous for well log data because of their poor signal-to-noise ratio (S/N). Noise not only results in uncertainty (broadening) of the intrinsic T_2 spectra, it also causes oscillatory behavior of the solution. Usually, a regularization technique¹ is applied to dampen the oscillations, which further smear the spectrum. Figure 1 illustrates the estimated T_2 spectra using 20-bin multiexponential analysis from two echo trains constructed from a single exponential and a bi-modal distribution with different levels of Gaussian noise added to the echo trains. Singular value decomposition (SVD) regularization is applied.²

Varying the degree of regularization from depth to depth in well log data is challenging because S/N varies

constantly. Oversmoothing not only smears the structure, but may also introduce added uncertainty in porosity.

The relaxation distribution model we used in this paper is a semi-continuous model, that does not require the commonly used regularization technique. The noise-induced broadening is naturally considered in the model. Therefore, we directly used the negative least squares algorithm^{3,4} for the matrix inversion.

To describe our model, we start by rewriting the T_2 distribution as $P(T_2) = \sum A_i F_i(T_2)$, where i can be varied but typically 10–40 is sufficient. The basis function $F_i(T_2)$ is continuous; it could be either a Gaussian, a B-splines, or some other function with a variable width. Then:

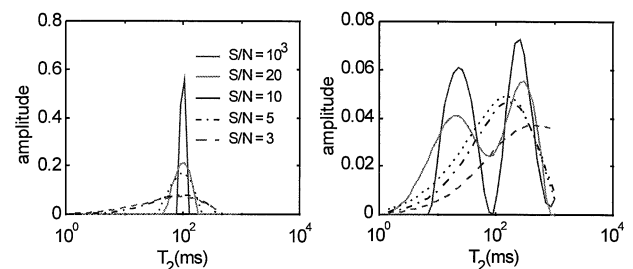


Fig. 1. T_2 spectrum broadening due to random noise.

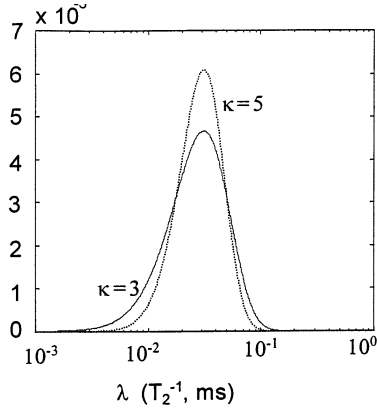


Fig. 2. Gamma density function.

$$M(nTE) = \sum A_i \int F_i(T_2) \cdot \exp(-nTE/T_2) dT_2 = \sum A_i I_i. \quad (2)$$

We used gamma functions and constructed the individual gamma densities:

$$F_i(\lambda) = (\kappa \lambda / \lambda_i)^\kappa \cdot \exp(-\kappa \lambda / \lambda_i) / (\lambda \cdot \Gamma(\kappa)), \quad (3)$$

where $\lambda = T_2^{-1}$ is an exponential decay rate, λ_i is the mean of individual distribution, and $1/\sqrt{\kappa}$ is the coefficient of variation and is defined to be the standard deviation divided by the mean. As κ is increased, the spread decreases. Figure 2 shows an example of $F_{1/32\text{ms}}(\lambda)$ with $\kappa = 3$ and 5, respectively.

The use of a decay rate λ and a gamma density results in a simple analytical form of the integrals, I_i :

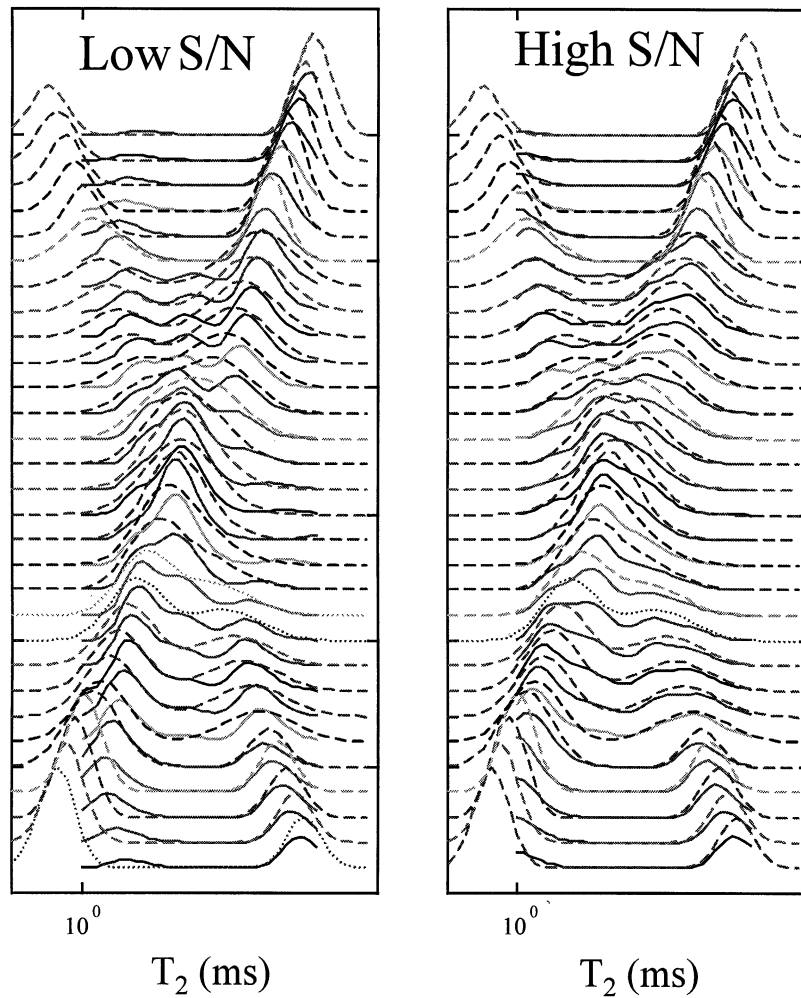


Fig. 3. Comparison of input model and inversion results (dotted line indicates input model; solid line indicates fitting result).

$$I_i(n \times TE) = \left(1 + \frac{\lambda_i \times n \times TE}{\kappa}\right)^{-\kappa} \quad (4)$$

thereby avoiding numerical integrals in Eq. (2). By plugging Eq. (4) into Eq. (2), the problem reduces to solving a set of linear equations. The linear problems are solved with the negative least squares algorithm of Lawson and Hanson,³ and no regularization is applied.

It is well known that fluid-saturated porous media generally exhibit uni-, bi-, or tri-modal distribution patterns. Furthermore, the resolvability of T_2 modes from an CPMG echo train, even with the S/N level comparable to laboratory measurements, is quite limited. Thus, in general, no more than 30 gammas are required to represent T_2 distributions extending over 3–4 decades.

Because the effect of noise broadening is naturally incorporated in the selection of κ , wild oscillations between adjacent T_2 components due to noise artifacts in under-regularized multiexponential solution are absent. Moreover, it is possible to vary κ from one gamma to another, thus offering a means to account for varying S/N within a single echo train. The latter is important, because the uncertainty of short T_2 component is greater than that of the longer ones.

We performed tests on 60 synthetic CPMG echo trains provided by Shell International (The Hague, Netherlands). Each consists of 300 echoes with an echo time of 1.2 ms. The S/N of 30 data sets (Fig. 3a) is comparable to well log data, and that of the other 30 sets are comparable to laboratory core measurements (Fig. 3b). Twenty-one gammas with $\lambda_i = 2^{-s}$, $s = 0, 0.5,$

$1, \dots, 11$ were used, which correspond to $T_2 = 2^s$ ms, and the range is typical of nuclear magnetic resonance logs. A κ of 3 was used for all of the data. The synthetic data have broader T_2 distributions than typical log data. Because data are sampled with an echo time of 1.2 ms, short T_2 components are inevitably unrecoverable. The results indicate that the new method can satisfactorily estimate T_2 spectra from CPMG echo trains with different levels of S/N.

The usual SVD method requires the computation of the SVD of a large number of subsets of matrices with a size of number of echoes by number of exponentials. These can be pre-computed, but it requires a very large amount of storage. In contrast, in the present approach, the QR factorization of the initial number of echoes by number of exponentials matrix is computed once, and all further calculations use only subsets of a small upper triangular matrix with number of exponentials rows and columns. This feature makes the technique ideal for real-time log processing.

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