

● Contributed Paper

NMR IMAGING EXPERIMENTS FOR THE VERIFICATION OF STOCHASTIC TRANSPORT THEORY

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Nuclear magnetic resonance (NMR) imaging techniques were investigated as a means to obtain velocity field measurements for the verification of a stochastic transport theory. A modified stimulated-echo pulse sequence was used to make velocity images in an aperiodic heterogeneous porous medium. Construction of the velocity covariance from measurements of the fluctuating velocity has demonstrated that nuclear magnetic resonance imaging is a valuable tool for obtaining experimental data for the examination of existing stochastic transport theory. © 1998 Elsevier Science Inc.

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INTRODUCTION

The accurate modeling of flow in porous media has proved challenging because of the heterogeneous nature of natural porous systems. It has been shown that mixing in heterogeneous systems is scale-dependent.^{1–6} This scale-dependent behavior prevents the use of the classic advection-dispersion equation for the determination of large-scale flow and transport properties in natural systems. As a result, a stochastic approach to modeling has been adopted in recent years.^{7,8} These models have been extensively developed over the last decade and show promise as a predictive technique.⁸ However, the application of these models often depends on specific assumptions about the nature of the velocity field and on theoretical constitutive relationships between velocity and log-hydraulic conductivity covariance. Thus, experimental determination of the velocity field would allow for the critical evaluation of existing models. Nuclear magnetic resonance (NMR) imaging presents a non-invasive means to obtain specific information about the behavior of the velocity field.⁹ In this study, gradient imaging techniques were investigated as a means to obtain veloc-

ity field measurements for the verification of stochastic transport theory.

THEORY

A stochastic analysis of the contaminant transport problem has been developed by Deng and co-workers.¹⁰ Their approach is based on an Eulerian perturbation solution to the stochastic flow model and has been solved in transform space to yield the following equation for the prediction of mean concentration. The degree symbol is used to denote variables that are both Laplace and Fourier transformed and the circumflex indicates those that are only spatially Fourier transformed:

$$\overline{C}^o(k,s) = \hat{C}_o \left[B^{o^{-1}} + \left(\frac{1}{2\pi} \right)^3 (k_i k_j) (b^{o*}_k \hat{R}_{ij}) \right]^{-1} \quad (1)$$

where:

$$B^{o*}_k \hat{R}_{ij} = \int_{R^3} B^o(k - k', s) \hat{R}_{ij}(k') dk' \quad (2)$$

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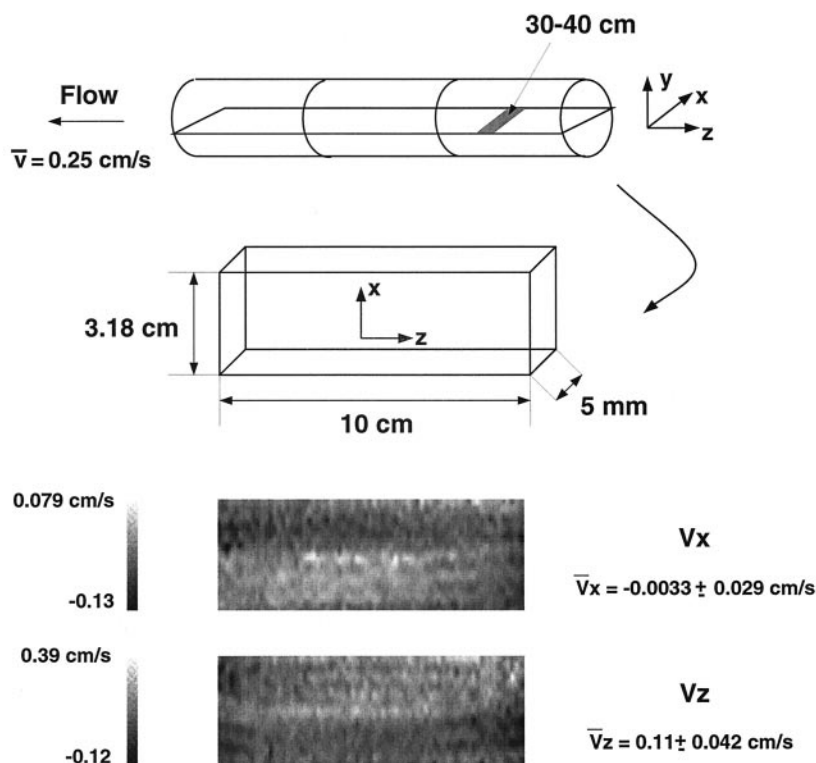


Fig. 1. Images of the x- and z-components of the velocity field in a 10-cm section in the first layer of the aperiodic heterogeneous model. The image resolution is 1.27 mm in the x-direction, 1.43 mm in the z-direction with a slice thickness of 5 mm in the y-direction.

and:

$$B^o(k,s) = [s + ik_i\bar{V}_1 + D_i k_i^2]^{-1}. \quad (3)$$

To evaluate Eq. (1), one must know the velocity covariance function, R_{ij} , the initial concentration distribution, C_0 , the mean velocity, $\bar{V}_1$, and the local dispersivities. The velocity covariance function is typically obtained from the covariance function of the log-hydraulic conductivity fluctuation.¹¹ By directly measuring R_{ij} , the applicability of this constitutive relationship and the validity of the stochastic nonlocal model can be examined.

EXPERIMENT

A modified stimulated-echo pulse sequence¹² was used to make velocity images in an aperiodic heterogeneous model. Three identical columns, each filled with a homogeneous distribution of unconsolidated spherical glass beads (Cataphote Inc., Jackson, MS, USA), were threaded together to create a single column with aperiodic heterogeneity. The resulting three-layer model was 183 cm long with an inside diameter of 3.18 cm. The properties of each layer are given in Table 1. In each

layer, the ratio of tube to particle diameter was equal to or greater than 45, values sufficiently large to neglect wall effects.¹³

The model was saturated with a dilute, 0.125% by volume, aqueous solution of gadopentetate dimeglumine contrast agent (Magnevist, Berlex Imaging) to reduce the water spin-lattice relaxation time from 2 to 0.3 s. Flow was controlled with a syringe pump assembly that was constructed by attaching a motorized shaft to the piston of a plastic syringe. The motor was driven by a controller (Model RP240 from CompuMotor) and the flowrate was adjusted by setting the motor velocity. The experiments were performed at a flowrate of 0.655 mL/s. This flowrate corresponds to an average linear velocity of 0.250 cm/s. While this velocity is not typical of groundwater flow, it was chosen in order to maximize velocity sensitivity, but still maintain flow in the region of mechanical dispersion ($50 \leq Pe \leq 10^4$).

The imaging experiments were performed on a versatile VAX-based (Digital Equipment) imaging spectrometer (Quest 4400 from Nalorac Cryogenics Corporation, Martinez, CA, USA) interfaced to a 31 cm horizontal bore superconducting magnet (Oxford Instruments, UK) at a field of 1.89 T. The Larmor frequency for ^1H at this field was 80.34 MHz. The magnet

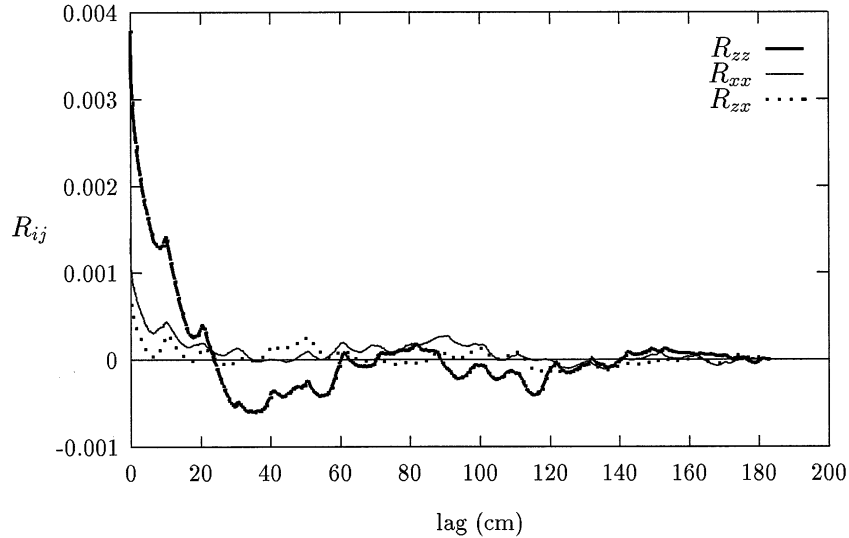


Fig. 2. Plot of the longitudinal velocity covariance determined according to Eq. (4).

contained actively shielded gradient coils (Magnex Scientific, Abingdon, UK) and a high-pass bird cage radio frequency (rf) probe. Velocity images were made in 10 cm sections along the length of the model. Copper foil was used to isolate each section for imaging. Because of the size of the model, a total of 18 velocity images were required to produce a single realization of the velocity field.

RESULTS

Images of the x- and z-components of the velocity field were made in 10-cm sections along the length of the aperiodic heterogeneous model (see Fig. 1 for a definition of the coordinate system). A representative set of images is presented in Fig. 1. The image resolution is 1.27 mm in the x-direction, 1.43 mm in the z-direction with a slice thickness of 5 mm in the y-direction. At this resolution, the measured velocity in each voxel represents an average over 40 to 1000 beads. The resolution was chosen such that the porosity in each voxel would match the overall porosity of the model. It has been argued that resolution below the pore scale is necessary

in order to obtain reliable data.¹⁴ However, an average measurement over several pores was considered appropriate since the stochastic nonlocal model is based on a Darcy-scale or superficial velocity. The velocity covariance function can be determined from these measurements according to:¹⁵

$$R(u) = \frac{1}{N} \sum_{i=1}^{N-u} (W_i - \bar{W})(W_{i+u} - \bar{W}), \quad (4)$$

where $\bar{W}$ is the mean of the process, u is the lag at which correlation is estimated, i is the location at which W has been measured, and n is the total number of observations. Estimates of the covariance, $R(u)$, are reliable for values of $n \geq 50$ and $u \leq n/4$. The longitudinal velocity covariance is plotted as a function of lag in Fig. 2. For these experiments, $n = 1278$ and $u = 320$ (46 cm). The irregularities that are observed every 10 cm are the result of combining data from separate images. This artifact can be avoided in future experiments by making images in overlapping sections.

CONCLUSION

Nuclear magnetic resonance imaging techniques were successfully applied to measure the velocity field in an aperiodic heterogeneous model. Direct measurement of the velocity covariance in this model has demonstrated that nuclear magnetic resonance imaging is a valuable tool for obtaining experimental data for the examination of existing stochastic transport theory.

Table 1. Properties of each layer in three-layer model

Property	First layer	Second layer	Third layer
Bead diameter range (μm)	125–150	300–425	600–850
Pore size range (μm)	25–50	75–100	150–200
Porosity	0.336	0.324	0.339
Permeability (Darcy)	10.3	53.2	320

REFERENCES

1. Matheron, G.; de Marsily, G. Is transport in porous media always diffusive? A counterexample. *Water Resour. Res.* 16:901–917; 1980.
2. Smith, L.; Schwartz, F.W. Mass transport. 1. A stochastic analysis of macroscopic dispersion. *Water Resour. Res.* 16:303–313; 1980.
3. Sudicky, E.A.; Cherry, J.A.; Frind, E.O. Migration of contaminants in groundwater at a landfill: A case study. *J. Hydrol.* 63:81–108; 1983.
4. Freyburg, D.L. A natural gradient experiment on solute transport in a sand aquifer. 2. Spatial moments and the advection and dispersion of nonreactive tracers. *Water Resour. Res.* 22:2031–2046; 1986.
5. Garabedian, S.P.; LeBlanc, D.R.; Gelhar, L.W.; Celia, M.A. Large-scale natural gradient tracer test in sand and gravel, Cape Cod, Massachusetts. 2. Analysis of spatial moments for a nonreactive tracer. *Water Resour. Res.* 27:911–924; 1991.
6. Gelhar, L.W.; Welty, C.; Rehfeldt, K.R. A critical review of field-scale dispersion in aquifers. *Water Resour. Res.* 28:1955–1974; 1992.
7. Dagan, G. *Flow and Transport in Porous Formation*. New York: Springer-Verlag; 1989.
8. Cushman, J.H. (Ed). *Dynamics of Fluids in Hierarchical Porous Media*. San Diego: Academic; 1990.
9. Caprihan, A.; Fukushima, E. Flow measurements by NMR. *Phys. Rep.* 198:195–235; 1990.
10. Deng, F.-W.; Cushman, J.H.; Delleur, J.W. A fast Fourier transform stochastic analysis of the contaminant transport problem. *Water Resour. Res.* 29:3241–3247; 1993.
11. Gelhar, L.W.; Axness, C.L. Three-dimensional stochastic analysis of macrodispersion in aquifers. *Water Resour. Res.* 19:161–180; 1983.
12. Waggoner, R.A.; Fukushima, E. Velocity distribution of slow fluid flows in Bentheimer sandstone: An NMRI and propagator study. *Magn. Reson. Imaging* 14:1085–1091; 1996.
13. Chu, C.F.; Ng, K.M. Flow in packed tubes with a small tube to particle diameter ratio. *AIChE* 35:148–158; 1989.
14. Nesbitt, G.J.; Fens, T.W.; van den Brink, J.S.; Roberts, N. Evaluation of fluid displacement in porous media using NMR microscopy. In: B. Blumich, W. Kuhn, (Eds). *Magnetic Resonance Microscopy: Methods and Application in Materials Science, Agriculture, and Biomedicine*. New York: VCH Publishers, 1992: pp. 86–307.
15. Hassan, A.E. Monte Carlo simulations of flow and transport in heterogeneous porous media: An evaluation of first- and second-order theories and the importance of porosity variability. Ph.D. Thesis. West Lafayette, IN: Purdue University; 1995.