Combinatorics and Hodge theory

Nordfjordeid Summer school

June 18–22, 2018

Arrival: Sunday evening or Monday morning
Departure: Friday evening or Saturday morning
Note that dinner is served on Sunday (arrival day) and breakfast is served on Saturday (departure day).

Speakers

Petter Brändén (KTH)
June Huh (IAS/Princeton)
Nicholas Proudfoot (Oregon)

Abstracts

Petter Brändén

Geometry and combinatorics of hyperbolic polynomials

Recently methods using hyperbolic and stable polynomials have seen several spectacular applications in combinatorics, computer science, probability theory and other areas. Hyperbolic and stable polynomials are generalizations of univariate real-rooted polynomials as well as multivariate determinantal polynomials. I will give an introduction to the theory of stable and hyperbolic polynomials and give/discuss applications such as the existence of infinite families of Ramanujan graphs of each degree, the Kadison-Singer problem and the van der Waerden conjecture.

June Huh

Hodge theory in geometry, algebra, and combinatorics

I will give a broad overview of the Hard Lefschetz theorems and the Hodge-Riemann relations in the theory of polytopes, complex manifolds, reflection groups, algebraic and tropical varieties, in a down-to-earth way. Several applications to the elementary combinatorics of graphs and matroids will be introduced.

Nicholas Proudfoot

The algebraic geometry of Kazhdan-Lusztig-Stanley polynomials

Kazhdan-Lusztig-Stanley polynomials are general combinatorial gadgets that include, as special cases, classical Kazhdan-Lusztig polynomials, toric g-polynomials, and Kazhdan-Lusztig polynomials of matroids. In each of these cases, many of the polynomials can be realized as intersection cohomology Poincare polynomials of certain algebraic varieties. I will describe a general geometric framework for

providing this kind of cohomological interpretation of Kazhdan-Lusztig-Stanley polynomials, with an emphasis on the examples coming from matroids. (No previous familiarity with intersection cohomology will be assumed.)
I will also discuss the conjectural log concavity and real-rootedness of matroidal Kazhdan-Lusztig polynomials and Z-polynomials. Almost nothing has been proved in general, so this will consist mostly of working through some very concrete examples.

References

[1] P. Brändén. Lecture notes written for Interlacing families.
`https://people.kth.se/~pbranden/Interlacing-notes.pdf`

[2] P. Brändén's. Geometry of zeros and applications.
`https://people.kth.se/~pbranden/Stability-lecturenotes.pdf`

[3] Wagner, David G. Multivariate stable polynomials: theory and applications. Bull. Amer. Math. Soc. (N.S.) 48 (2011), no. 1, 53–84.

[4] Marcus, Adam W.; Spielman, Daniel A.; Srivastava, Nikhil Interlacing families I: Bipartite Ramanujan graphs of all degrees. Ann. of Math. (2) 182 (2015), no. 1, 307–325.

[5] Marcus, Adam W.; Spielman, Daniel A.; Srivastava, Nikhil Interlacing families II: Mixed characteristic polynomials and the Kadison-Singer problem. Ann. of Math. (2) 182 (2015), no. 1, 327–350.

[6] June Huh, Benjamin Schröter, and Botong Wang. Correation bounds for fields and matroids. arXiv:1806.02675

[7] June Huh. Combinatorial applications of the Hodge-Riemann relations. arXiv:1711.11176

[8] Antoine Chambert-Loir. Relations de Hodge-Riemann et matroïdes, d'après Adiprasito, Huh et Katz. Séminaire Bourbaki , Mars 2018.

[9] Matt Baker. Hodge theory in combinatorics. Bulletin of the American Mathematical Society 55 (2018).

[10] Karim Adiprasito, June Huh, and Eric Katz. Hodge theory for combinatorial geometries. Annals of Mathematics 188 (2018).

[11] June Huh and Botong Wang. Enumeration of points, lines, planes, etc. Acta Mathematica 218 (2017).

[12] Nick Proudfoot. The algebraic geometry of Kazhdan-Lusztig-Stanley polynomials.
`http://pages.uoregon.edu/njp/geometry.pdf`

	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
8:00		Breakfast	Breakfast	Breakfast	Breakfast	Breakfast	Breakfast
9:00		Huh 1	Proudfoot 2	Huh 4	Huh 5	Brändén 6	
9:45		Coffee	Coffee	Coffee	Coffee	Coffee	
10:15		Brändén 1	Huh 3	(10:00) Brändén 4	Brändén 5	Proudfoot 6	
11:15		Proudfoot 1	*Problem session*	(11:00) Proudfoot 4	*Problem session*	*Problem session*	
12:30		Lunch	Lunch	(12:00) Lunch	Lunch	Lunch	
13:30		*Problem session*	Brändén 3	(13:00) Excursion	Proudfoot 5		
14:45		(16:00) Huh 2	Proudfoot 3		Huh 6		
15:30		(17:00) Brändén 2	*Problem session*		*Problem session*		
18:30	Dinner	Dinner	Dinner	Dinner	Dinner	Dinner	