Deforming Fano varieties of lines

Let $X \subset \mathbb{P}^n$ be a hypersurface of degree d and $F_1(X) \subset \mathbb{G}(1,n)$ the Fano variety of lines on X. In general, assume a reductive group G acts on an algebra S and Y is a good quotient of an open subset of $\operatorname{Spec} S$. I wish to apply criteria Jan Kleppe and I found for comparing the invariant deformations of the algebra S with the deformations of the scheme Y to the deformations of $F_1(X)$. A (hopefully partial) result I can prove is: If $F_1(X)$ has expected dimension, $d \leq n$ and $n \geq 6$ then $\mathrm{Def}_{F_1(X)} \simeq \mathrm{Def}_X$. I will begin the talk with definitions and introductory remarks on deformation theory.