# An upper bound for polynomial log-volume growth of automorphisms of zero entropy


**Abstract**

Let $X$ be a normal projective variety of dimension $d \geq 2$ over an algebraically closed field and $f$ an automorphism of $X$. Suppose that the pullback $f^*|_{\mathsf{N}^1(X)_{\mathbf{R}}}$ of $f$ on the space $\mathsf{N}^1(X)_{\mathbf{R}}$ of numerical $\mathbf{R}$-divisor classes is unipotent and denote the index of the eigenvalue $1$ by $k+1$. We prove an upper bound for polynomial log-volume growth $\operatorname{plov}(f)$ of $f$, or equivalently, for the Gelfand–Kirillov dimension of the twisted homogeneous coordinate ring associated with $(X, f)$, as follows:

$$\operatorname{plov}(f) \leq (k/2+1)d.$$

In characteristic zero, combining with the inequality $k \leq 2(d-1)$ due to Dinh–Lin–Oguiso–Zhang, we obtain an optimal inequality that

$$\operatorname{plov}(f) \leq d^2,$$

which affirmatively answers questions of Cantat–Paris-Romaskevich and Lin–Oguiso–Zhang. This is joint work with Chen Jiang.