A lecture on Associative Algebraic Geometry

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Abstract

This short note is a complete and condensed presentation of the book "Associative Algebraic Geometry" which is in the press at Word Scientific right now. Apart from the (fundamental) contribution from deformation theory, the hart of the construction can be given in a self content and straightforward, technical though, way.

We start with the elementary theory of representable functors with inductive (direct) and projective (inverse) limits as examples. Then we give the needed results from deformation theory: Let A be an associative $\Bbbk$-algebra, algebraically closed of characteristic zero, and let $M = \{M_1, ..., M_r\}$ be a set of r simple (finite dimensional) modules. Then there is an A-algebra $\eta_M : A \to A_M$ which has M as a set of simple modules, and if right multiplication by $\eta(f)$ acts injective on each $M_i \in M$, then $\eta(f)$ is a unit in A_M.

On this basis, we let $\operatorname{Simp} A$ be the set of (isomorphism classes) of right simple A-modules, we prove that the sets $\{D(f)\}$ is a basis for a topology on $\operatorname{Simp} A$ so that we can define the variety $\operatorname{Simp} A$ with a sheaf of associative rings $\mathcal{O}_{\operatorname{Simp} A}$ given as the limit of the local rings A_M, and in general an associative variety X as one covered by these. We use results from Buan et. al. to define the spectrum of right A-modules in general, defining a general natural (meaning that it is a functor) scheme $\operatorname{SSimp}(A)$ which coincides with $\operatorname{Spec} A$ when A is commutative.

The main result is that any associative scheme is $\operatorname{SSimp} U$ for an associative ring U. This solves a pedagogical issue for reading "Mathematical Models in Science," the up to now last book published by O.A. Laudal.