List of Abstracts

Assessment of implicit and explicit methods to compute Gaussian mean and principal curvatures from 3D scalar data

Eric Albin, Ronnie Knikker, Shihe Xin,
Christian Oliver Paschereit, Yves D'Angelo
Université de Lyon, CNRS, France

An implicit method based on diffuse approximation and/or finite differences to determine the mean and Gaussian curvatures from a three dimensional scalar field is presented and assessed. The method also determines normal vectors and principal curvatures. Compared to explicit methods, the implicit approach shows robustness and improved accuracy. The method is applied to compute the curvatures of laminar and turbulent wrinkled flames on large triangular unstructured meshes (namely a 3D iso surface of temperature).

References:

[1] Goldman R. 2005. Curvature formulas for implicit curves and surfaces. Computer Aided Geometric Design 22(7) pp. 632 658.
[2] Popinet S. 2004. The GNU triangulated surface library.

On some nonlinear subdivision schemes and associated multiresolution transforms

Sergio Amat
U. P. Cartagena, Cartagena, Spain

This talk is devoted to the convergence and stability analysis of a class of nonlinear subdivision schemes and associated multiresolution transforms. As soon as a nonlinear scheme can be written as a specific perturbation of a linear and convergent subdivision scheme, we show that if some contractivity properties are satisfied, then stability and convergence can be achieved. This approach is applied to various schemes, which give different theoretical results. Finally, some numerical applications are revisited.

High order approximation to non-smooth multivariate functions

Anat Amir, David Levin
Tel Aviv University, Israel

Approximations of non smooth multivariate functions return low order approxima tions in the vicinities of the singularities. Most prior works solve this problem for uni variate functions. In this work we introduce a method for approximating non smooth multivariate functions of the form $f = g + r_+$ where $g, r \in C^{M+1}(\mathbb{R}^n)$ and the function r_+ is defined by

$$r_+(y) = \begin{cases} r(y), & r(y) \geq 0 \\ 0, & r(y) < 0 \end{cases}, \ \forall y \in \mathbb{R}^n .$$

Given scattered (or uniform) data points $X \subset \mathbb{R}^n$, we investigate approximation by quasi interpolation. We design a correction term, such that the corrected approximation achieves full approximation order on the entire domain. We also show that the correction term is the solution to a Moving Least Squares (MLS) problem, and as such can both be easily computed and is smooth. Last, we prove that the suggested method includes a high order approximation to the locations of the singularities.

CAD and mathematical challenges in CAGD

Christian Arber
Missler Software, Evry, France

CAGD (Computer Aided Geometry Design) is one of the pillar technology used by CAD/CAM (Computer Aided Design and Manufacturing), along with computer and technology sciences. Why finally the names of mathematicians are associated to the daily work of some drafting guy in a study office or a milling operator in a workshop? Would it be possible that using the Cad/Cam Software TopSolid, he could reward and highlight the obscure work of the Math Research community, in a sense that this community probably do not even expect and imagine?

In this presentation we will present a concrete industrial case, where we will speak about repairing some imprecise geometry, namely a flat elliptical arc. This apparently simple problem will allow us to take a walk in the algebra realm, and his beautiful somewhat forgotten province, the ancient projective geometry.

When hunting the missing digits for high precision computing purpose, we will touch some profound Maths and Historical questions, exhume some old technics that have been considered as museum pieces since the Bourbaki era, and the annexing of the geometry field by pure algebra in the mid 20 century. However it may happen that these fascinating geometrical constructions could be better than the pure algebra resolutions when precision is required to treat degeneracies cases.

The conclusion could be that the (good) work of a Mathematician never die and could revive in unexpected occasions and times!

Morphing planar curves using subdivision surfaces and centroidal Voronoi tessellations

Pieter Barendrecht, Jiří Kosinka
University of Groningen, The Netherlands

A widely used approach to morph between a pair of planar curves is to embed these curves in 3D at different z values, construct a surface connecting the two curves, and interpret parallel slices of this surface perpendicular to the z axis as the intermediate curves composing the eventual morph. In this work, we propose to construct the surface connecting the two planar curves using a combination of subdivision surfaces [1], dual mesh operations and centroidal Voronoi tessellations [2]. Considering different curve topologies and bivariate subdivision schemes, we compare the resulting intermediate curves based on various quality measures.

References:
[1] Peters, J. and Reif, U., 2008. Subdivision surfaces. Springer.
[2] Du, Q., Faber, V. and Gunzburger, M., 1999. Centroidal Voronoi tessellations: ap plications and algorithms. SIAM review, 41(4), pp. 637 676.

Towards efficient 5-axis flank CNC machining of free-form surfaces via fitting envelopes of surfaces of revolution

Pengbo Bo, **Michael Bartoň**, Denys Plakhotnik, and Helmut Pottmann
BCAM, Bilbao, Spain

We introduce a new method that approximates free form surfaces by envelopes of one parameter motions of surfaces of revolution. In the context of 5 axis computer nu merically controlled (CNC) machining, we propose a flank machining methodology which is a preferable scallop free scenario when the milling tool and the machined free form surface meet tangentially along a smooth curve. We seek both an optimal shape of the milling tool as well as its optimal path in 3D space and propose an optimization based framework where these entities are the unknowns. We propose two initialization strate gies where the first one requires a user's intervention only by setting the initial position of the milling tool while the second one enables to prescribe a preferable tool path. We present several examples showing that the proposed method recovers exact envelopes, in cluding semi envelopes and incomplete data, and for general free form objects it detects envelope sub patches.

Semi-cardinal interpolation with B-spline functions

Aurelian Bejancu
Kuwait University, Kuwait

We describe a B spline approach for semi cardinal spline interpolation at the set of non negative integers. This construction employs a hierarchy of end conditions expressed in terms of finite differences of B spline coefficients. We obtain the approximation order of the resulting interpolation schemes and discuss an extension of this approach to bivariate box splines.

Unstructured C^1 multi-patches of tensor product B-Splines

Michel Bercovier
Hebrew University of Jerusalem, Israel

Isogeometric Analysis has the ambition of using a unique definition for CAD and PDEs Analysis . Complex real life constructions (cf. for instance the invited talk by G. Elber) necessitates multi patch surfaces or volumes. Hence the interest in the study of multi patch C^1 isogeometric spaces. In [1] this question was limited to Bézier surfaces, where it was completely solved. In [2] and [3] results were extended to B Splines under some geometric restrictions. Here we study the relation between the mesh geometry (without restrictions), the resulting C^1 basis functions along edges and the eventual absence of hierarchy in h refinements. We generalise our approach to 3D solid B Rep volume patches.

References:

[1] M. Bercovier and T Matskewich. Smooth Bézier surfaces over arbitrary quadrilateral meshes, arXiv: 1412.1125v2, 2014.

[2] A. Collin, G. Sangalli, and T. Takacs. Approximation properties of multi patch C^1 isogeometric spaces,arxiv:1509.07619v1, 2015.

[3] M. Kapl, F. Buchegger, M. Bercovier, and B. Jüttler. Isogeometric analysis with geometrically continuous functions on multi patch geometries. Technical report, G+S Report No. 35, Johannes Kepler University, Linz, 2015.

Linear barycentric rational interpolation with guaranteed degree of exactness in several dimensions

Jean Paul Berrut
Université de Fribourg, Switzerland

The talk will address the problem of constructing a surface from an equispaced sample of a smooth function of two variables. Our approach is an extension of linear barycentric rational interpolation (LBRI). In recent years, this scheme, introduced in 1988 and im proved in 2007 by Floater and Hormann, has turned out to be one of the most efficient infinitely smooth interpolants from equispaced data in one dimension (see R.B. Platte, "Algorithms for recovering smooth functions from equispaced data", preprint). However, there does not seem to exist a straightforward way of generalizing it to two dimensional non rectangular domains. In our presentation we shall present an attempt to extend to some two dimensional domains the LBRI with guaranteed degree of exactness introduced last year.

Construction of adaptive blends using rational envelope curves

Michal Bizzarri, Miroslav Lávička
University of West Bohemia, Plzeň, Czech Republic

We present a method for constructing rational blending canal surfaces based on re cently introduced Rational Envelope (RE) curves. The main advantage of this blending technique is its simplicity and also its usefulness for constructing blends fulfilling pre scribed constrains, e.g. when avoiding obstacles or bypassing other objects is demanded. Compared to other methods this algorithm does not require any SOS decompositions or PH/MPH interpolation techniques. It is based only on an arbitrary Hermite interpola tion method (e.g. using Ferguson cubics) and it generates the whole family of rational canal surfaces sharing the same silhouette. The constructed blend obtained by this new method is not generally a surface with rational offsets but it still consists of a pencil of curves (circles) along which the unit surface normals are rational, i.e., the blends of this type still belong among surfaces important in technical practise (mainly from the point of view of machine tooling).

Application of CCC Schoenberg operators on image resampling

Tina Bosner, Bojan Crnković and Jerko Škifić
University of Zagreb, Croatia

Resampling of digital images is an essential part of image processing. The most efficient and sufficiently accurate image resampling techniques can produce spurious os cillations near sharp transitions of color when upsampling. To improve that, we use shape preserving approximations by CCC splines for the histoploation process, applied dimension by dimension, see Reference [1]. For these approximations, instead of using the co monotone and co convex tension spline interpolations, we introduce approximations by CCC Schoenberg operators based on the variable degree polynomial splines. The algorithm is simpler than the one from Reference [1], but it obtains the similar image quality.

References:

[1] T. Bosner, B. Crnković and J. Škifić, *Tension splines with application on image resampling*, Math. Commun. **19** (2014), 517 529.

A local THB-spline approach to scattered data fitting

Cesare Bracco, Carlotta Giannelli and Alessandra Sestini
Università di Firenze, Italy

In the literature the scattered data fitting problem has been approached by using a wide range of techniques. In this talk we present a method which, given a set of scattered points in $\mathbb{R}^2$ with associated values, produces an approximant belonging to a hierarchical spline space adaptively selected. More precisely, we define an approximant by combining the basis of THB splines (see [2] for the definition of this basis) with coefficients obtained by suitable local scattered data approximations. Our approach generalizes the framework for hierarchical quasi interpolation introduced in [3] and adapted in [1] for Hermite type quasi interpolants, to the context of scattered data problems. The adaptive nature of the resulting quasi interpolants allows an accurate approximation (whose order depends on the chosen local approximants) without introducing unnecessary degrees of freedom.

References:

[1] C. Bracco, C. Giannelli, F. Mazzia and A. Sestini, Bivariate hierarchical Hermite spline quasi interpolation, BIT (2016), to appear.

[2] C. Giannelli, B. Jüttler and H. Speleers, THB splines: The truncated basis for hier archical splines, Comput. Aided. Geom. Design 29 (2012), 485 498.

[3] H. Speleers and C. Manni, Effortless quasi interpolation in hierarchical spaces, Numer. Math. 132 (2016),155 184.

Fast Fitting Methods for Surfaces and their Analysis

Karl-Heinz Brakhage
Inst. for Geom. and Appl. Math., RWTH Aachen University, Germany

We present a fast method for scattered data approximation with curves/surfaces. Our technique can be applied to standard Bézier and B spline surfaces as well as for subdivision schemes. The approach can be formulated in such a way that for the iteration we have a standard least squares problem in each step. A regularization term that expresses the fairness of the intermediate and/or final result can be added. Furthermore, adaptivity is easily integrated into our concept. Many methods are based on the minimization of the distance between a point and the corresponding foot point on the target surface (PDM). Another approach is the minimization of a quadratic approximant of the squared distance function. Here the aim is to avoid the parameterization problem and to construct algorithms of second order convergence. Unfortunately in general the second order Taylor approximant does not lead to symmetric positive definite system matrices and thus the existence and uniqueness of the minimum cannot be guaranteed. Modifications destroy the second order and thus the claimed quadratic convergence. Nevertheless such approaches need less iteration but have the main drawback of a large computational overhang.

Our approach is based on the minimization of a linear combination of PDM and the distances of the data points to the linear approximation of the target surface at the projection points. This reduces the number of parameter corrections, avoids the computational overhang and can still be written in the form of a standard least squares problem. We will demonstrate the idea with several examples and present the analytical background.

Experimenting with adaptive spline spaces

Andrea Bressan, Dominik Mokriš
JKU, Linz, Austria

We present a unified implementation scheme for adaptive spline spaces. The proposed strategy allows for experimenting with different spline definitions while having reason able performance. We tested the strategy by implementing *HB*[1], *THB*[2], *TDHB*[3], *PNHB*[4] and the *HLR*[5] spaces. We report on the lessons learned and we describe generalizations for multi patch domains, possibly with *enhanced smoothness* [6,7].

<u>References</u>:
[1] R. Kraft. "Adaptive and linearly independent multilevel B splines" *A. Le Méhauté, C. Rabut, L.L. Schumaker (Eds.), Surface Fitting and Multiresolution Methods*, Vanderbilt University Press, Nashville (1997), pp. 209 218
[2] C. Giannelli, B. Jüttler, H. Speelers. "THB splines: The truncated basis for hierar chical splines", *Computer Aided Geometric Design*, 29, no 7 (2012): 485 498
[3] D. Mokriš, B. Jüttler. "TDHB splines: The truncated decoupled basis of hierarchical tensor product splines", *Computer Aided Geometric Design*, 31, no 7 (2014): 531 544
[4] U. Zore. "Constructions and Properties of Adaptively Refined Multilevel Spline Spaces", PhD diss., University of Linz, (2016)
[5] A. Bressan, B. Jüttler. "A hierarchical construction of LR meshes in 2D", *Computer Aided Geometric Design*, 37, (2015): 9 24
[6] F. Buchegger, B. Jüttler, A. Mantzaflaris. "Adaptively refined multi patch B splines with enhanced smoothness" *Applied Mathematics and Computation*, 272, (2016): 159 172
[7] G. Sangalli, T. Takacs, R. Vázquez. "Unstructured spline spaces for isogeometric analysis based on spline manifolds" *arXiv:1507.08477*, (2015)

Arc spline approximation and high precision digital maps

Stephan Brummer
FORWISS, University of Passau, Germany

Autonomous driving requires highly accurate and highly compressed digital maps in order to efficiently store and process the corresponding data.

One approach to approximate lane courses and road edges with both high accuracy and compact representation is the so called Smooth Minimum Arc Path (SMAP). It provides an arc spline, a curve composed of smoothly joint circular arcs and line segments, with a minimal number of segments with respect to a predefined maximum error.

Such a SMAP itself is by definition not unique and the originally published algo rithmic approach shows numerical problems in some cases. We present a method which overcomes these issues and supplies a SMAP which is especially suited to the require ments of the application.

GPU-based rendering of blending spline surface lattices

Jostein Bratlie, **Tanita Fossli Brustad**, Rune Dalmo
UiT The Arctic University of Norway

The well known recursion formula for B splines [1,2] was extended in [3] by adding a blending function. The extended formula includes a recursive definition of GERBS [4] as a special case.

A scheme for rendering of such blending splines was proposed in [5]. The scheme is founded on common properties of blending splines and the patch type primitive of modern tessellation APIs. Nomenclature connecting the scheme to the spline construction was introduced in the article and a first implementation strategy for quad render blocks with regular loci was discussed.

We present ongoing work on strategies for construction and rendering of render blocks, with emphasis on quad render blocks containing irregular loci (t and star joints).

References:

[1] M. G. Cox. The numerical evaluation of B splines. J. Inst. Maths. Applics., 10:134 149, 1972.

[2] C. de Boor. On calculating with B splines. Journal of Approximation Theory, 6(1):50 62, 1972.

[3] A. Lakså. ERBS surface construction on irregular grids. In AMEE13, volume 1570 of AIP Conference Proceedings, pages 113 120, 2013.

[4] L. T. Dechevsky, B. Bang, and A. Lakså. Generalized expo rational B splines. International Journal of Pure and Applied Mathematics, 57(6):833 872, 2009.

[5] J. Bratlie, R. Dalmo, and B. Bang. Evaluation of smooth spline blending surfaces using GPU. In Curves and surfaces. 8th International Conference, volume 9213 of Lecture Notes in Computer Science, pages 60 69. Springer, 2015.

Planar Multi-Patch Domain Parameterization via Patch Adjacency Graphs

Florian Buchegger
MTU Aero Engines, Munich, Germany

As a remarkable difference to the existing CAD technology, where shapes are repre sented by their boundaries, FEM based isogeometric analysis typically needs a parame terization of the interior of the domain. Due to the strong influence on the accuracy of the analysis, methods for constructing a good parameterization are fundamentally impor tant. The flexibility of single patch representations is often insufficient, especially when more complex geometric shapes have to be represented. Using a multi patch structure may help to overcome this challenge.

In this talk we present a systematic method for exploring the different possible pa rameterizations of a planar domain by collections of quadrilateral patches. Given a domain, which is represented by a certain number of boundary curves, our aim is to find the optimal multi patch parameterization with respect to an objective function that captures the parameterization quality. The optimization considers both the location of the control points and the layout of the multi patch structure. The latter information is captured by pre computed catalogs of all available multi patch topologies. Several numerical examples demonstrate the performance of the method.

This talk is based on joint work with B. Jüttler (JKU).

Interpolation and design with hyperbolic spaces

J. M. Carnicer, E. Mainar, J. M. Peña
Universidad de Zaragoza, Spain

Exponential polynomials as solutions of differential equations with constant coeffi cients are widely used for approximation purposes. Recently, mixed spaces containing algebraic, trigonometric and exponential functions have been extensively considered for design purposes. The analysis of these spaces leads to constructions that can be reduced to Hermite interpolation problems. In this paper we focus on hyperbolic spaces generated by algebraic polynomials, hyperbolic sine and hyperbolic cosine. We present classical in terpolation formulae, such as, Newton and Aitken Neville formulae and a suggestion of implementation. We explore another technique, expressing the Hermite interpolant in terms of polynomial interpolants and derive practical error bounds for the hyperbolic interpolant. Applications to shape preserving curve design in hyperbolic spaces are also shown.

Bézier motions with end-constraints on speed

M. Hunt, G. Mullineux, **R. J. Cripps**, B. Cross
University of Birmingham, UK

A free form motion can be considered as a smoothly varying rigid body transfor mation. Motions can be created by establishing functions in an appropriate space of matrices and can be constructed to optimize metrics defined on these spaces. While a smooth motion is created, the significance of these metrics in terms of the geometry of the motion itself is not immediately clear. In a geometric algebra environment, motions can be created using extensions of the ideas of Bézier and B spline curves and the geomet ric significance of the construction is clearer. A motion passing through given precision poses can be obtained by direct analogy with the curve approach. This paper considers the more difficult problem of dealing additionally with velocity constraints at the ends of the motion: here the analogy is less obvious. A geometric construction for the end pairs of control poses is established and is demonstrated by creating motions satisfying given pose and velocity constraints.

Machinability of surfaces via motion analysis

B. Cross, R. J. Cripps, M. Hunt and G. Mullineux
University of Birmingham, UK

The machinability of a surface describes its ability to be machined and the factors which affect this. These are independent of any material properties or cutting parameters but instead reflect an ability to replicate a desired tool path motion with sufficient control of the material removal process. Without this control there is a potential for surface defects and costly finishing stages.

Five axis CNC milling machines are commonly used for machining complex free form shapes. The processes required to obtain CNC instructions for a machine tool, starting from a target surface, are presented. An overview is first given and later formalized with mathematical methods. Specifically, a moving cutting tool is characterised by a tool path motion. Interpreting the moving cutter in terms of moving machine axes provides a diagnostic tool for detecting machining errors.

Examination of two case studies reveals different types of errors, machine dependent and machine independent. The contribution of geometry to machine independent errors is discussed and related back to the machinability of a surface.

Approximating FEM and DEM simulation results using LR-splines

Heidi Dahl, Oliver Barrowclough
SINTEF ICT, Department of Applied Mathematics, Norway

Increases in computing power and improvements in the quality and performance of sensors means that the production of large and complex high quality datasets is becoming easy and inexpensive. With this abundance of data comes a new set of challenges: When your data is too big to download to your computer and too big to load into memory, what can you actually do with it? So the ability to produce big data has triggered a need for new methods and tools for visualizing, interrogating, and interacting with data in a distributed manner.

In this presentation we will share some of our experiences from our current European research projects on geometrical modelling of Big Data. The main focus will be the approximation and visualization of large datasets from Finite Element Method (FEM) and Discrete Element Method (DEM) simulations using Locally Refined (LR) splines. We approximate continuous scalar and vector fields from point based data, as a compact volumetric LR spline model allowing efficient evaluation, and well suited for visualization on the GPU. Using this model we visualize features such as iso surfaces, cut planes, and streamlines directly and interactively.

Recent Progress on C^∞-smooth GERBS-based PUM

Lubomir T. Dechevsky
University of Tromsø / The Arctic University of Norway

This is a continuation of [1] and [2].

References:

[1] L.T. Dechevsky. Recent Progress on C^∞ smooth GERBS based PUM as Intrinsic Tool of Iso geometric Analysis on General Splits and Covers of Multidimensional Do mains. Communication at the 8th International Conference on Curves and Surfaces, Paris, France, 12 18 June 2014.

[2] L.T. Dechevsky. Communication at the 15th International Conference on Approxi mation Theory, San Antonio, Texas, 22 25 May 2016 (to appear).

Additive Manufacturing: New challenges to CAGD

Tor Dokken
SINTEF, Norway

Current Computer Aided Design systems are boundary structure based, and represent objects by their inner and outer hulls. This approach is well suited for object with homogenous material. Additive manufacturing allows the manufacturing of objects with variable material, and complex inner structures. Such objects are impossible to represent by boundary structures. In additive manufacturing cost and manufacturing time is linked to the amount of material used. Designs reducing material use, e.g., weight reducing inner structures, are very attractive. A new generation of simulation and optimization based design tools is needed to support design for additive manufacturing. This relates both to design methods and the shape representations. There is a need for moving from a 2 variate representation of the surfaces of the objects, to a 3 variate representation that includes variable properties of the interior of the object, possibly combined with a procedural type representation of the nonhomogeneous object interior. These representations have to go beyond the volumetric spline representations of Isogeometric Analysis. The talk will include the approaches selected in the Horizon 2020 R&I project www.caxman.eu addressing computer aided technologies of additive manufacturing.

Computer Aided Geometric Design - Analysis, Manufacturing The Next Generation

Gershon Elber, Fady Massarwi, Annalisa Buffa,
Giancarlo Sangalli, and Massimiliano Martinelli
Technion, Israel

The needs of modern (additive) manufacturing technologies can be satisfied no longer by B reps, as they require the representation and manipulation of interior fields as well. Further, while the need for a tight coupling between the design and analysis stages has been recognized as crucial almost since geometric modeling (GM) was conceived, contemporary GM systems only offer a loose link between the two, if at all.

For about half a century, (trimmed) Non Uniform Rational B spline (NURBs) sur faces has been the B rep of choice for virtually all the GM industry. Fundamentally, B rep GM has evolved little during this period. In this work, we seek to examine an ex tended volumetric representation (V rep) that will successfully confront the existing and anticipated design, analysis, and manufacturing foreseen challenges. We extend all fun damental B rep GM operations, such as primitive and surface constructors and Boolean operations, to trimmed trivariate V reps. This enables the much needed tight link to (Isogeometric) analysis on one hand and the full support of additive manufacturing on the other.

In this talk, we will present the necessary tools to make this V rep GM complete, and discuss primitive and higher order volumetric constructors, and V rep Boolean oper ations. Further, special abilities to support Isogeometric analysis will also be presented, that enable robust queries over the V reps, including precise contact analysis, maximal penetration depth, and accurate integration over trimmed domains. Examples and other applications of V rep GM will be demonstrated.

Partially Nested Hierarchical Spline Refinement

Nora Engleitner
Johannes Kepler Universität, Linz, Austria

The established construction of hierarchical B splines [1] starts from a given sequence of nested spline spaces, and hence it is not possible to pursue independent refinement strategies in different parts of a model. In order to overcome this limitation, we generalize the selection mechanism that defines hierarchical B splines to sequences of *partially* nested spline spaces. We identify assumptions that enable us to define a hierarchical spline basis, to establish a truncation mechanism, and to derive a completeness result. In addition we discuss a refinement algorithm for such spline spaces and demonstrate its application to least squares approximation. This is joint work with B. Jüttler and U. Zore.

References:
[1] R. Kraft Adaptive and linearly independent multilevel B splines. in: A. Le Méhauté, (ed.) et al., Surface fitting and multiresolution methods. Vanderbilt University Press, Nashville, TN, 1997, pp. 209 218.

Analysis of Geometric Subdivision Schemes

Tobias Ewald
TU Darmstadt, Germany

In the context of non linear $\mathbb{R}^d$ valued curve subdivision, geometric subdivision schemes are characterized by commutation of the refinement rules with similarities. If such schemes reproduce linear polygons, as Dodgson Sabin's circle preserving scheme and many others do, $C^{1,\alpha}$ regularity can be established. This is done automatically and rig orously by an algorithm using numerical computations. For an important subclass, also $C^{2,\alpha}$ regularity is inherited from a related linear scheme, see [1]. If linear polygons are not reproduced, there is still a framework for checking convergence, see [2]. In parts, this talk is based on a joint work with Ulrich Reif and Malcolm Sabin.

References:
[1] T. Ewald, U. Reif, M. Sabin, *Hölder regularity of geometric subdivision schemes*, Constructive Approximation 42 (3), 2015, pp 425 458
[2] T. Ewald, *Convergence of geometric subdivision schemes*, Applied Mathematics and Computation 272 (1), 2016, pp 41 52

Recent Insights into Computing with Positive Definite Kernels

Greg Fasshauer
Department of Applied Mathematics, IIT, Chicago, U.S.A.

In this talk I will discuss recent joint work with Mike McCourt (SigOpt, San Francisco) that has led to progress on the numerically stable computation of certain quantities of interest when working with positive definite kernels to solve scattered data interpolation (or kriging) problems.

In particular, I will draw upon insights from both numerical analysis and modeling with Gaussian processes which will allow us to connect quantities such as, e.g., (deter ministic) error estimates in terms of the power function with the kriging variance. This provides new kernel parametrization criteria as well as new ways to compute known criteria such as MLE. Some numerical examples will illustrate the effectiveness of this approach.

References:
[1] G. E. Fasshauer and M. J. McCourt, *Kernel based Approximation Methods using* MATLAB, Interdisciplinary Mathematical Sciences Vol. 19, World Scientific Publishers, Singapore, 2015.
[2] M. J. McCourt and G. E. Fasshauer, Stable likelihood computation for Gaussian random fields, submitted for publication.

Applications of spaces of spheres, Bézier curves and massic points: Dandelin spheres and subdivisions of Dupin cyclides

Lionel Garnier, Jean Paul Bécar, Laurent Fuchs, Lucie Druoton
University of Burgundy, France

Dupin cyclides are algebraic surfaces introduced for the first time in 1822 by the French mathematician Pierre Charles Dupin. A Dupin cyclide can be defined as the en velope of an one parameter family of oriented spheres, in two different ways. R. Martin is the first author who thought to use these surfaces in CAD/CAM and geometric mod eling. Using the space of spheres, a Dupin cyclide is represented by two conics which can be modeled by rational quadratic Bézier curves. Using Bézier curve properties and a homographic fonction, we can give some algorithms to subdivide Dupin cyclide. We give too an application of the representation of Dupin cyclides by Bézier curve in the space of spheres: the determination of the Dandelin spheres.

Algebraic surfaces of revolution and algebraic surfaces invariant under scissor shears: similarities and differences

Juan G. Alcázar, **Ron Goldman**
Rice University, USA

Scissor shears are space transformations sharing certain properties with rotations in 3 space. In fact, the formulas for scissor shears are, up to sign, the same as the formulas for rotations with sines and cosines replaced by hyperbolic sines and hyperbolic cosines. Thus one might consider scissor shears as *hyperbolic* versions of 3D rotations. While algebraic surfaces of revolution, which are well known in Computer Aided Geometric Design, are algebraic surfaces invariant under all the rotations about a fixed axis (the axis of revolution of the surface), algebraic *scissor shear invariant surfaces* (or SSI for short) are invariant under all the scissor shear transformations about a fixed axis. Hence, both types of surfaces can be constructed from an axis, and an algebraic space curve.

Interestingly, there are a number of analogies, but also differences, between these two types of surfaces. In both cases, the intersections of the surface with a plane normal to the axis are curves of the same nature, circles in the case of surfaces of revolution, and hyperbolas in the case of SSI surfaces. While surfaces of revolution can have either one axis or infinitely many axes (when the surface is a union of spheres), SSI surfaces can have one, three, or infinitely many axes (when the surface is the union of hyperboloids of one sheet and cones with the same axis, or the union of hyperboloids of two sheets and cones with the same axis). Furthermore, in both cases the form of highest degree of the implicit equation of the surface has a special structure, where again circles are replaced by hyperbolas in the case of SSI surfaces. Finally, the axis (or axes) can be detected by similar methods in both cases: factoring the form of highest degree, and contracting the tensor corresponding to the highest degree form.

References:

[1] Alcázar J.G., Goldman R., (2016), *Finding the axis of revolution of a surface of revolution*, to appear in IEEE Transactions in Visualization and Computer Graphics.
[2] Alcázar J.G., Goldman R., (2016), *Algebraic surfaces invariant under scissor shears*, submitted.

Degree reduction of composite Bézier curves

Przemysław Gospodarczyk, Stanisław Lewanowicz, Paweł Woźny
University of Wrocław, Poland

In recent years, the problem of degree reduction of Bézier curves with parametric continuity constraints has been extensively studied (see, e.g., [1] and the list of references given there). Most of the papers give methods of multi degree reduction of a *single Bézier curve* with respect to L_2 norm. In order to solve the problem of multi degree reduction of *composite Bézier curves*, one could apply such a procedure to one segment of the composite curve after another with properly chosen endpoints continuity constraints. However, in general, the obtained solution does not minimize the distance between two composite curves.

We present a novel approach to the problem of multi degree reduction of composite Bézier curves with parametric continuity constraints at the endpoints of the segments. In contrast to other methods, the new one minimizes the L_2 error for the whole composite curve instead of minimizing the L_2 errors for each segment separately. As a result, an additional optimization is possible. We use the properties of the so called *constrained dual Bernstein polynomials* to solve the problem efficiently. Examples show that the new method gives much better results than multiple application of degree reduction of a single Bézier curve. Moreover, merging of several unconnected Bézier curves into a smooth composite Bézier curve is also possible.

References:

[1] P. Woźny, S. Lewanowicz, Multi degree reduction of Bézier curves with constraints, using dual Bernstein basis polynomials, Computer Aided Geometric Design 26 (2009), 566 579.

TBA

Thomas Grandine
The Boeing Company, Seattle

Adaptive Geometric Modeling within an Industrial Environment

David Großmann
MTU Aero Engines AG, Munich, Germany

Industrial digital products are usually designed through (commercial) Computer Aided Design (CAD) systems based on the B spline technology and its non uniform rational extension (NURBS). To overcome the limitations of their tensor product structure, the recently developed truncated hierarchical B splines (THB splines) have been integrated into the geometric modeling environment of MTU Aero Engines AG.

The mini symposium talk will introduce the use of THB splines for the design and re design of aircraft engine components: Within the surface fitting part of the process for reconstructing CAD models they lead to significant improvements with respect to the quality of the resulting geometric shape compared to the existing tensor product spline technology. Using them for the adaptive approximation of three dimensional simulation data results in highly efficient processes since the 'curse of dimension' makes global refinement even more prohibitive.

Moreover, bridging the gap between a new (spline) technology and industrial applications generally involves additional conditions to be met. We will demonstrate how to combine THB splines with commercial geometric modeling kernels and their exact conversion into CAD geometry. As a consequence, the adaptive modeling tool can be fully integrated into CAD systems that comply with the current NURBS standard and provide modeling operations like blending, thickening and trimming. This paves the way for the introduction of this new technology into the design of real world products.

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A Discrete Variational Approach for Solving Monge-Ampere Equation

David Gu
State University of New York at Stony Brook, USA

Both optimal mass transportation problem and Alexandrov problem in convex ge ometry can be converted to solving Monge Ampere equation. A discrete variational approach is presented, which solve the equation by optimizing a convex energy with ge ometric meaning. The method can be easily converted to computing power Voronoi dia gram and Delaunay triangulation in conventional computational geometry. The method has been applied for area preserving mesh parameterization in computer graphics, shape classification based on Wasserstein distance in digital geometry and medical imaging.

Approximation with Gaussians using varying shape parameters

Thomas Hangelbroek
University of Hawaii, USA

The tuning of shape parameters in kernel approximation is a challenging and impor tant task. Perhaps the most prominent instance of this appears when approximating with Gaussian radial basis functions. By tuning the shape parameter to one extreme, the approximation problem is very stable, (e.g., with interpolation matrices nearing the identity), but approximation power is lost. By tuning to the other extreme, spectral approximation orders are possible, but at the cost of conditioning. A common problem is to find a tuning parameter which provides an "optimal" trade off between these two extremes. In this talk we discuss nonlinear Gaussian approximation schemes, developed in conjunction with Amos Ron, which feature shape parameters that vary spatially and are selected in response to local smoothness/roughness of the target function.

On foundational computational problems in sparse regularization

Anders Hansen
University of Cambridge, UK

Sparse regularization techniques have become popular tools in inverse problems over the last decades. The key to the success of these techniques are robust algorithms than can compute minimizers to convex optimization problems efficiently. Although these algorithms, in many cases, work very well, they can also fail substantially. This failure is not accidental and can be rigorously explained through the classification of such op timization problems in the so called Solvability Complexity Index Hierarchy. This is a classification hierarchy addressing the foundations of computational mathematics. We will discuss how new and potentially surprising classification results of convex optimiza tion problems in this hierarchy show how we may have to revise some of the theory for recovery guarantees in fields such as compressed sensing. The theoretical discussion will be accompanied with many numerical examples.

Graph-Based Methods for Computing Discrete Geodesics

Ying He
Nanyang Technological University, Singapore

Measuring geodesic distances on polyhedral surfaces plays an important role in computer graphics and digital geometry processing. The existing methods are based on either computational geometry or PDE. The former is able to compute exact geodesic distances, however, it is computationally expensive. The latter is fairly efficient but produces only the first order approximation, whose quality is highly sensitive to mesh triangulation. In this talk, we will introduce a new graph theoretic algorithm, called discrete geodesic graph (DGG), for efficient computation of highly accurate geodesic distances. Let M be a manifold triangle mesh with n vertices and $\varepsilon > 0$ the given accuracy parameter. We prove that the DGG associated to M has $O(\frac{n}{\sqrt{\varepsilon}})$ edges and the shortest path distances on the graph approximate geodesic distances on M with relative error $O(\varepsilon)$. Taking advantage of DGG's unique features, we develop a DGG tailored label correcting algorithm that computes geodesic distances in empirically linear time. DGG scales well due to its linear time and space complexities, and the computed distances are guaranteed to be true distance metrics, which is highly desired in many applications. We observe that DGG significantly outperforms saddle vertex graph (SVG)   another graph based method for discrete geodesics   in terms of graph size, accuracy control and runtime performance. As a general computational framework, DGG works for arbitrary triangle meshes and can be extended to other geometric domains, such as point clouds and implicit surfaces, whereas SVG is designed for triangle meshes with sufficient number of saddle vertices. We demonstrate how DGG provides a faster and more accurate approach to geometry processing applications, including intrinsic girth function and farthest point sampling.

Implicitly Modelled Stratigraphic Surfaces with Radial Basis Functions

Michael Hillier, Eric de Kemp and Ernst Schetselaar
Geological Survey of Canada, Ottawa, Canada

A generalized interpolation method using Radial Basis Functions (RBF) on scattered multivariate data is presented to simultaneously model multiple 3D stratigraphic sur faces in geology using the implicit approach. Surfaces are constrained by four types of data: inequality data, multiple interface data, as well as planar and linear orientation data. Resultant modelled surfaces can exhibit topological problems when using hetero geneously sampled data from structurally complex domains. Variably scaled kernels [1] in combination with a greedy algorithm [2] are used improve the topology of modelled surfaces. Many examples are shown to illustrate the applicability of the method.

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Polygonal mesh entropy

Stefan Dantchev and **Ioannis Ivrissimtzis**
Durham University, UK

We study on polygonal meshes an information theoretic entropy function defined in [1] on abstract simplicial complices. The polygonal mesh entropy gives the number of bits expected in an optimal encoding of a sequence of mesh vertices following a probability distribution $\mathcal{P}$, assuming that vertices belonging to the same face are indistinguishable. The polygonal mesh entropy is closely related to the classic graph entropy, but the sets of indistinguishable vertices are readily given by the mesh faces, rather than the independent sets of a graph.

We show that the values of the entropy function are related to both the connectivity of the mesh, when $\mathcal{P}$ is uniform on the mesh vertices, and to the mesh geometry, when $\mathcal{P}$ depends on geometric properties of the vertices, such as normalised absolute values of discrete curvatures.

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Artifacts in Laplacian and order-randomized Laplacian mesh smoothing

Ying Yang, Holly Rushmeier and **Ioannis Ivrissimtzis**
Durham University, UK

Polygonal mesh artifacts have been studied in the context of subdivision surfaces, see [1]. Here, we show that artifacts similar to the *polar* and the *rotational* artifacts of subdivision surfaces can also appear when a mesh is smoothed with the discrete Laplacian operator, or algorithms based on it such as Taubin's smoothing algorithm.

We show that the creation of these smoothing artifacts is a resilient phenomenon and cannot be easily resolved by, for example, adapting the smoothing weights accord ing to the vertex degrees. The smoothing artifacts are also resilient under an order randomization of the smoothing process, that is, when the mesh vertices are updated instantly, one at a time, in an order that follows a random permutation of the vertex set. However, the latter order randomized variant of the Laplacian and Taubin's smoothing algorithms can overcome the problem of non convergence or slow convergence, which ap pears when general polygonal meshes whose underlying graph is bipartite, or has a large bipartite component, are smoothed.

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Parametric Polynomial Circle Approximation

Gašper Jaklič and Jernej Kozak
University of Ljubljana and University of Primorska, Slovenia

Uniform approximation of a circle arc (or a whole circle) by a parametric polynomial curve is considered. The approximant is obtained in a closed form. It depends on a parameter that should satisfy a particular equation, and it takes only a couple of tangent method steps to compute it. For low degree curves the parameter is provided exactly. The distance between a circle arc and its approximant asymptotically decreases faster than exponentially as a function of polynomial degree. The approximant can be applied for a fast evaluation of trigonometric functions.

A new family of Hermite subdivision schemes
reproducing cubic polynomials

Byeongseon Jeong and Jungho Yoon
Ewha W. University, Seoul, S. Korea.

The aim of this study is to provide a family of quasi interpolatory Hermite subdivision schemes with tension parameters. Acting on function values and their associated first derivatives, our schemes reproduce polynomials of degree up to three, yielding the fourth order accuracy. The smoothness analysis has been performed by using the factorization framework of subdivision operators. We see that the proposed scheme achieves C^4 smoothness for a certain range of tension parameters. Numerical examples are presented in order to demonstrate the performance of the proposed Hermite scheme.

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Hierarchical box splines and the
weak boundary approach for PDEs

Tadej Kanduč, Carlotta Giannelli, Francesca Pelosi, Hendrik Speleers
Università di Firenze, Italy

Isogeometric Analysis (IgA) is a recently developed framework alternative to Finite Element Analysis where the simple building blocks in geometry and solution space are replaced by more complex and geometrically oriented compounds. Box splines are an established tool to model complex geometry, and so represent a promising tool in the context of IgA. They form an intermediate approach between classical tensor product B splines and splines on triangulations. Local refinement can be achieved by considering hierarchically nested sequences of box spline spaces.

Box splines do not offer special elements to impose boundary conditions for the nu merical solution of partial differential equations (PDEs). In this talk we discuss a weak treatment of such boundary conditions. Near the domain boundary, a special domain strip is introduced to enforce the boundary conditions in a weak sense. The thickness of the strip is adaptively defined in order to avoid unnecessary computations. Numerical examples show the optimal convergence rate of box splines and their hierarchical variants for the solution of PDEs.

C^2-smooth isogeometric functions on two-patch geometries: Numerical examples and possible generalizations.

Mario Kapl and Vito Vitrih
Austrian Academy of Sciences, Austria, University of Primorska, Slovenia

This is a continuation of the talk C^2 *smooth isogeometric functions on two patch geometries: Basis construction and dimension study.* We use the therein presented C^2 smooth isogeometric functions for solving numerical problems, e.g. L^2 approximation, on different bilinearly parameterized two patch domains. The numerical results indicate op timal approximation power, which means in case of L^2 approximation convergence rates of order $\mathcal{O}(6)$ and $\mathcal{O}(7)$ with respect to L^2 norm for biquintic and bisixtic isogeometric functions, respectively.

In addition, we demonstrate the potential of these C^2 smooth functions for isogeo metric analysis by solving the triharmonic equation on different bilinearly parameterized two patch geometries. Finally, we describe possible extensions of the construction of C^2 smooth isogeometric functions to more general domains.

Solving Inequality Constrained Spline Optimization Problems

Joshua Holloway and **Scott Kersey**
Georgia Southern University, USA

We solve various variational spline curve problems subject to polygonal constraints, including best near interpolation, smoothing splines with obstacles, shape preserving splines, best spline by spline approximation, and polynomial degree reduction with polyg onal constraints. To solve these problems, we develop the active set method for quadratic programming. We provide necessary and sufficient conditions for global minima. We show how to efficiently implement the algorithm using rank one updates of QR factor izations, without the need for dual bases. We show that the algorithm will converge in finite steps (under certain conditions), which solves an open problem posed in the literature. We show that solutions to the problem of near interpolation under polygonal constraints are smoothing splines with weights determined from the multipliers in the active set method, which generalizes a result in the literature on near interpolation and smoothing splines, and allows us to choose optimal weights for smoothing splines. We generalize the problem of polynomial degree reduction with box constraints to polygonal and circular constraints. Furthermore, we supplement this with an iterative technique for better choosing data sites and knots so as to further minimize the bending energy of near interpolant spline curves, offering an easy solution to the problem of best spline interpolation with free data sites and free knots.

A comparison of the stability of interpolation formulae

J. M. Carnicer, **Y. Khiar** and J. M. Peña
Departamento de Matemática Aplicada, Universidad de Zaragoza, Spain

A pointwise condition number associated to a representation of an interpolation op erator is introduced. We show that the Lagrange formula is optimal with respect to this conditioning. For other representations of the interpolation operator, an upper bound for the conditioning is derived. A quantitative measure in terms of the Skeel condition num ber is used to compare the conditioning with the Lagrange representation. The relevant case of the Newton polynomial interpolation formula for equidistant nodes is analyzed.

From Quad Meshes to Quad Layout

Leif Kobbelt
Aachen university, Germany

The conversion of raw geometric data (that typically comes in the form of unstruc tured triangle meshes) to high quality quad meshes is an important and challenging task. The complexity of the task results from the fact that quad mesh topologies are subject to global consistency requirements which cannot be dealt with by local constructions. This is why recent quad meshing techniques formulate the mesh generation process as a global optimization problem. By adding hard and soft constraints to this optimization, many desired properties such as structural simplicity, principal direction alignment, as well as injectivity can be guaranteed by construction. An even more challenging problem is the computation of quad layouts, where a coarse segmentation of the input surface into essentially rectangular patches is sought which also satisfies global consistency and shape quality requirements. While being structurally related, both problems need to be addressed by fundamentally different approaches. In my talk I will present some of these approaches and demonstrate that they can generate high quality quad meshes and quad layouts with a high degree of automation but that they also allow the user to interactively control the results by setting boundary conditions accordingly.

Numerical Solutions of the Generalized Rosenau KdV-RLW Equation using radial basis functions

Bahar Korkmaz and Yilmaz Dereli
Anadolu University, Turkey

In this study, numerical solution of the initial boundary value problem of Generalized Rosenau KdV RLW equation is obtained by a meshless method based on collocation method with radial basis functions. We first use the Crank Nicolson and forward finite difference methods for discretization of the unknown function u and its time derivative, respectively. Then, we obtain the linear form of it by using the following Taylor's formula see [1]. The error norms and invariants are computed to determine the accuracy of the scheme. The stability analysis of the method is tested. It is shown that the scheme is unconditionally stable. Moreover, convergence rates of the solution is investigated. Finally, comparisons are made between the results of this method and some other earlier works [2] and [3].

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Conversion of Trimmed CAD Models to NURBS-compatible Subdivision Surfaces

Jingjing Shen, **Jiří Kosinka**, Malcolm A. Sabin, Neil A. Dodgson
University of Groningen, The Netherlands

Building on our previous work [1] on converting a single trimmed NURBS patch to a Catmull Clark subdivision surface, we extend the conversion process to CAD models composed of several trimmed NURBS patches. Our target representation is that of Cashman et al. [2], which allows us to preserve both uniform and non uniform boundary edges, to reduce approximation error and final control mesh density (compared to [1]), and to handle non convex corners via generalising Pixar sharp subdivision rules [3] to the non uniform setting.

A further advantage is that the general framework of [2] makes it possible to preserve some of the input NURBS patches or their parts exactly in the resulting subdivision surface.

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Regression analysis using a blending type spline construction

Tatiana Kravetc, Rune Dalmo, Børre Bang
UiT – The Arctic University of Norway

Regression analysis allows to track the dynamics of change in measured data and to investigate their properties. A sufficiently good model allows to predict the behavior of dependent variables with higher accuracy, and to propose a more precise data generation hypothesis.
By using polynomial approximation for big data sets with complex dependencies we get *piecewise* smooth functions. One way to obtain a smooth spline throughout the entire data set is to use local curves and to blend them using smooth basis functions. This construction allows computation of derivatives at any point on the spline. Properties such as tangent, velocity, acceleration, curvature and torsion can be computed, which gives us the opportunity to exploit these data in the subsequent analysis.
We can adjust the accuracy of the approximation on the different segments of the data set by choosing a suitable knot vector. This article describes a method for determining the number and location of the knot points, based on the change in Frenet frame.
We present a method of implementation using generalized expo rational B splines (GERBS) for regression problems (in two and three variables) and we evaluate the accuracy of the model using comparison of the residuals.

On the coupling of decimation operator with subdivision schemes for multiscale analysis

Zhiqing Kui, Jean Baccou, Jacques Liandrat
Aix Marseille univ., France

Subdivision schemes [1] are powerful tools for the fast generation of refined sequences ultimately representing curves or surfaces. Coupled with decimation operators, they generate multiresolution transforms that generalize the multiresolution analysis/wavelet framework [2]. The flexibility of subdivision schemes (a subdivision scheme can be non stationary, nonhomogeneous, position dependent, interpolating or not, non linear...) (e.g [3]) makes that, as a counterpart, the construction of suitable decimation operators is not direct and easy.

In the paper, we propose a generic approach for the construction of decimation oper ators consistent with a given subdivision. The conditions under which detail coefficients go to zero when the scale parameter increases are investigated as well as stability con ditions. Among others, the cases of homogeneous Lagrange interpolatory subdivision, spline subdivision, subdivision related to Daubechies scaling functions (and wavelets) and schemes switching from interpolating to non interpolating are revisited.

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Non-negative Data Interpolation using Spherical Splines

Victoria Baramidze and **Ming-Jun Lai**
University of Georgia, Athens, GA

This talk is based on an extension of the study from Reference [1] to the spherical setting. We present a spherical spline method to produce a smooth interpolatory spline which is non negative if the given data values are nonnegative. Under a minor condition on triangulation we establish the existence and uniqueness of the minimizer in two C^1 spline spaces: quintic, over a triangulation, and cubic, over a triangulated quadrangula tion in the spherical setting. We develop an algorithm to convert any triangulation to a one satisfying the minor condition based on acute triangulation. We then extend our study to compute the range restricted interpolatory splines. We shall study the approxi mating properties of the minimizer as well. Numerical examples are computed based on the standard projected gradient method and show the effectiveness of the nonnegative preserving/range restricted interpolatory spline method.

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Non polynomial B-splines

Arne Lakså
UIT The Artic University of Norway

Bézier and B spline curves can be expressed by using De Casteljau's corner cutting algorithm. This can also be formulated using factorization matrices. Each matrix reduce the coefficient vector with 1, and each line in each matrix represent a linear interpolation describing the corner cutting.

This opens for a further expansion, where the factors in the factorization which are linear functions in the polynomial case, is "deformed" by a perturbation function called a B function. The result is a non polynomial spline that was described in the general case in [1], but first introduced in the blending spline case in [2].

We will se that the typical properties of B splines are preserved or even improved when we add a B function to the formulas. We also compare with the theory of generalized B splines and Chebyshev systems.

Further, we describes the construction , evaluation and knot insertion of the B spline, and provide some practical examples.

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Learnt knot placement for curve and surface approximation

P. Laube, G. Umlauf, M. Franz
University of Applied Sciences, Constance, Germany

Knot placement for curve and surface approximation is an open problem. Selecting knot values to receive good approximation results is a challenging task. Proposed ap proaches range from parametric averaging to genetic algorithms. We propose the use of Support Vector Machines (SVMs) for finding suitable knot vectors in B spline curve approximation. The SVMs are trained to distinguish between locations along the curve that are well or not well suited as knots in the parametric domain. This score is based on different geometric features of a parameters corresponding point in the point cloud. A score weighted averaging technique is used to produce the final knot vector. We fur ther propose a method to use the score weighted averaging technique for t spline surface approximation.

On RBF-generated finite differences for PDEs on smooth surfaces

Erik Lehto
KTH Royal Institute of Technology, Sweden

A simple, accurate and efficient framework for solving partial differential equations (PDEs) on smooth surfaces is derived from (generalized) Hermite interpolation with radial basis functions (RBFs). The framework is based on projection from an extrinsic coordinate system, and only two ingredients are required for the discretization: i) node points on the surface, and ii) the surface normal at each node point[1]. By using locally defined interpolants to compute compact finite difference weights on these scattered nodes[2], the obtained system matrices are sparse and the corresponding linear systems can be solved efficiently using standard Krylov methods with simple pre conditioners. Convergence rates exceeding fourth order are easily achievable within this framework, as shown by numerical experiments.

A variety of applications, including reaction diffusion systems, harmonics of the Laplace Beltrami operator, and nonlinear convection, are presented for assorted surfaces.

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Adaptive Algorithms using Splines on Triangulations with Hanging Vertices

L. L. Schumaker, **S. Li**
Vanderbilt University, Nashville, USA

Adaptive approximation of functions is tested using polynomial splines on triangu lations with hanging vertices and indicates improved efficiency as compared to ordinary triangulations. Algorithms for generating data structures needed for triangulations with hanging vertices are also developed. Adaptive mesh generation algorithms using the fi nite element method(FEM) for solving a model problem involving a second order elliptic PDE are also discussed. Numerical examples using different *a posteriori* error indicators are given.

Surface modelling for vector graphics

Henrik Lieng, Jiří Kosinka, Flora Tasse, Jingjing Shen, Neil Dodgson
University of Cambridge, UK

Recent work on vector graphics aims to provide flexible, yet powerful, frameworks for defining and editing 'complex colour gradients'. By 'complex colour gradients', we mean colour gradients[1] that have been produced by a tool that has more degrees of freedom compared with the traditional linear gradient tool. Looking at current vector tools targeted towards colour gradients, we argue that there is room for improvement. For example, Adobe Illustrator's gradient mesh tool employs Ferguson patches to define 2D colour gradients. As a consequence, the user is forced to work with rectangular con trol meshes, heavily restricting the user's artistic freedom. In another Illustrator tool providing 2D colour gradients, based on skeletal strokes, the underlying framework em ploys simple linear gradients resulting in clear visible artefacts where input data requires smooth colour gradients.

In this talk, we present our recent work within this field and discuss remaining issues. Our principal argument is that subdivision surfaces provide clear advantages compared with current solutions. We propose a neat drawing framework where subdivision can be employed out of the box [1]. We then present a colour interpolation scheme, based on Catmull Clark subdivision, for gradient meshes of arbitrary manifold topology [2].

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A weighted binary average for subdivision schemes of point-normal pairs

Evgeny Lipovetsky
School of Computer Sciences, Tel Aviv Univ., Israel

Subdivision is a well known and established method for generating smooth curves/ surfaces from discrete data by repeated refinements. The typical input for such a process is a mesh of vertices. In this work we propose to refine 2D data consisting of vertices of a polygon and a normal at each vertex. Our core refinement procedure is based on a circle average, which is a new non linear weighted average of two points and their cor responding normals. The ability to locally approximate curves by the circle average is demonstrated. With this ability, the circle average is a candidate for modifying linear subdivision schemes refining points, to schemes refining point normal pairs. This is done by replacing the weighted binary arithmetic means in a linear subdivision scheme, ex pressed in terms of repeated binary averages, by circle averages with the same weights. Here we investigate the so modified Lane Riesenfeld algorithm and the 4 point scheme. For the case that the initial data consists of a control polygon only, a naive method for choosing initial normals is proposed. An example demonstrates the superiority of the above two modified schemes, with the naive choice of initial normals over the correspond ing linear schemes, when applied to a control polygon with edges of significantly different lengths.

This is a joint work with Nira Dyn.

Geodesic medial axes on 2-manifold meshes and its application to cortical morphometry

Yong-Jin Liu
Tsinghua University, China

In brain morphometry, recent advances have attracted considerable attention for con structing a graphical representation of cortical folding patterns, by computing medial axes on triangulated cortical surface M. The cortical surface can be segmented into labeled gyri by a set of sulci. By treating their boundaries as generators, the Voronoi diagrams based on geodesic metric on M provide a tool to compute geodesic medial axes of these 2D shapes on M. Several properties that make geodesic medial axis on M distinct from it Euclidean counterpart are also presented. Finally practical algorithms for constructing geodesic Voronoi diagram and medial axis on M are presented.

G^2-Continuous Patches On Surface Transitions

Florian Lorenz
Siemens AG Digital Factory, Motion Control, Erlangen, Germany

During the milling process with 5 axis machines undesirable artifacts mostly appear at the motion path passes surfaces transitions in the CAD model. These effects can influence the cartesian coordinates as well as the orientation of the tool. The reasons for that behavior vary from technical issues (e.g. diameter of the tool) to qualitative insufficient designs (e.g. discontinuities in the model). To increase the quality of the resulting machining program one can cut out a portion and use a G^2 continuous patch to connect the common edge in a sufficiently smooth way. We derive conditions for the patch to be G^1 and G^2 continuous along curves in terms of cubic tensor product B Splines.

The Choice of the Shape Parameter in a Purely Scattered Data Setting

Lin-Tian Luh
Providence University, Taiwan

Unlike the approach we presented before, here a purely scattered data approach is presented. The optimal choice of the shape parameters contained in the multiquadrics(MQ) and inverse multiquadrics(IMQ) is crucial in the meshless method RBF(radial basis functions). Since the birth of the MQ and IMQ, this question had obsessed experts in this field for decades. An interesting and useful approach, based on evenly spaced data points, was presented by the author on the journal ACHA in early 2016. Although it is quite satisfactory both in theory and practice, the requirement that the data points should be evenly spaced is a drawback and somewhat restricts its usefulness. In fact, a purely scattered data setting of finding the optimal shape parameters can be constructed and is also quite satisfactory.

In contrast to the general approach of constructing models by experimental results, we thoroughly discard models and choose the shape parameters by a function developed from theory. This function is called MN function, denoted by $MN(c)$, where c is just the shape parameter. In this talk the interpolated functions belong to a space denoted by B_σ which consists of functions whose Fourier transforms are compactly supported with support radius σ. The MN function is essentially the absolute error bound when the B_σ functions are interpolated by the MQ or IMQ. The optimal value of c is just the one making $MN(c)$ smallest. The structure of $MN(c)$ is very informative. It shows that the choice of the shape parameter greatly depends on the domain size, dimension of the Euclidean space, σ, the exponent in the MQ, and the fill distence which measures the spacing of the data points. A slight change of any of these factors may cause a huge change of the choice of c. The intricacy of the MN function maybe frustrating to those who try to build models for finding the optimal shape parameter. However it also saves people's time because it makes the effects of the above mentioned factors completely transparent.

Cycloidal spaces and spherical Bessel functions

J. M. Carnicer, **E. Mainar**, J. M. Peña
Universidad de Zaragoza, Spain

Cycloidal spaces are generated by the trigonometric polynomials of degree one and algebraic polynomials. The critical length of a cycloidal space is the supremum of the lengths of the intervals on which the Hermite interpolation problems are unisolvent. We show that the half of the critical length of a cycloidal space is the first positive zero of an spherical Bessel function of the first kind. This fact can be used to obtain efficiently critical lengths of cycloidal spaces. It also provides a solution to the problem of the asymptotic behaviour of the critical lengths of cycloidal spaces.

Some existence results for Chebyshev nets with singularities

Yannick Masson, Olivier Baverel, Alexandre Ern,
Laurent Hauswirth, Arthur Lebée, Laurent Monasse
CERMICS, Ecole des Ponts, UPE, Champs sur Marne, France

Chebyshev nets occur in the modeling of fabric and grid shells (covering structures of flexible rods), and it is of practical importance of being able to mesh surfaces with Chebyshev nets. Given a surface, the question arises whether it can be covered globally by a Chebyshev net. In general, the answer is negative: if the total Gaussian curvature of the surface is too large, no global Chebyshev net exists. We first present an existence result with an improved bound on the total Gaussian curvature [1].

In order to circumvent this restriction, we then propose the construction of Chebyshev nets starting from singularities, following ideas from [2]. The singularities can be obtained by splitting the surface into sectors on which we can construct Chebyshev nets. The final net is obtained by joining the Chebyshev net "patches" along the borders of the sectors. This allows to split the condition on the global total Gaussian curvature into conditions on each sector. Moreover, the junctions between sectors can be smoothed by adding a hole in the surface around the singularities, allowing the parametrization to be smoothly continued from neighbouring sectors. We prove that the introduction of singularities improves the existence bound on total Gaussian curvature, notably in the case of finite negative total Gaussian curvature. We will also discuss the choice of Cauchy boundary condition type (two primal curves, a dual curve with a vector field or a dual curve and a primal curve). The proof is constructive and numerical examples illustrate the practical capabilities of the method.

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On piecewise polynomial approximation for image representation and nonlinear subdivision schemes

Basarab Mateï and **Sylvain Meignen**
LJK Laboratory, Grenoble, France

A classical model used in image processing is to consider each row of an image as a piecewise polynomial. Because the latter is not bandlimited, classical linear representa tions are not efficient for its encoding. To design nonlinear representations adapted to this type of signal, one needs to determine the amplitude and location of the jumps sep arating the polynomial parts. For that purpose, different strategies have been proposed such as ENO SR [1][2] or those based on finite rate of innovation (FRI) approaches [3]. In spite these techniques are optimal when the signal to be represented is truly a piecewise cubic polynomial, they lack stability when the latter differs from the expected model. In practical situations such as image encoding, it is of great importance to have a reliable model for singularities estimation. After having recalled that singularities computation corresponds to the estimation of a stream of Diracs, we will review different techniques available for that purpose and how these can be beneficial for image encoding.

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Regularity of Vector and Hermite Subdivision Schemes

Jean-Louis Merrien
INSA de Rennes, IRMAR, France

In vector subdivision schemes, the data $\boldsymbol{f}_n : \mathbb{Z} \to \mathbb{R}^n$ is interpreted as an approxima tion to a limit function on the grid, and it is not so straightforward to prove more than the Hölder regularity of the limit function.

On the other hand, for Hermite subdivision schemes, the components of the vector valued function are the successive exact or approximated derivatives, so that the notion of convergence automatically includes regularity of the limit.

In this talk, we establish a link between both schemes and an equivalence between spectral condition and operator factorizations is given. This is a key to prove the conver gence hence the regularity of the limit function of a Hermite scheme. Then we study how such Hermite schemes with smooth limit functions can be extended into ones with higher regularity. As an example, we propose a new approach to cardinal splines as Hermite subdivisions schemes. We conclude by the construction of new Hermite schemes starting from a contractive vector scheme.

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Low-rank parameterization of planar domains

Dominik Mokriš, Bert Jüttler
Johannes Kepler University Linz, Austria

We study the classical problem of finding a spline parameterization of a planar do main enclosed by four given boundary spline curves. We consider this problem from a new viewpoint, namely we examine the rank of the resulting parameterization. More precisely, we would like to control the tensor rank of the coefficient tensor of the resulting parameterization.

We present a simple yet powerful method that generates a parameterization with a rank that does not exceed 5. This is compared with Coons interpolation [1], Laplacian smoothing [1] and a method based on solving biharmonic equation [3]. We show that these methods generally lead to parameterizations with a rank larger than 5. We also compare the parameterizations in several examples. The low rank of the parameterization is linked to the rank of the determinant of the Jacobian matrix and other geometric quantities. This hints at the possibility of controlling the rank throughout the isogeometric pipeline and establishes a connection to the results in [2].

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Hermite subdivision on manifolds

Caroline Moosmüller
TU Graz, Austria

In this talk we study Hermite subdivision schemes, which are refinement algorithms that operate on discrete point vector data and produce a curve and its derivatives in the limit. Most results on Hermite schemes concern data in vector spaces and rules which are linear. We are interested in Hermite schemes that operate on manifold valued data and are defined by the intrinsic geometry of the underlying manifold (in particular, by geodesics and parallel transport). We analyse such nonlinear subdivision rules with respect to Hölder and C^1 smoothness, approximation order, etc. Similar to previous work on subdivision in manifolds, we use the method of proximity to conclude convergence and smoothness properties of a manifold valued scheme from a "close by" linear one.

Symbols and exact regularity of symmetric pseudo-splines of any arity

Georg Muntingh
SINTEF, Norway

Using a generating function approach, we derive expressions for the symbols of the symmetric m ary pseudo spline subdivision schemes. We show that their masks have a positive (discrete time) Fourier transform, making it possible to compute the exact Hölder regularity algebraically as the spectral radius of a matrix. We apply this method to compute the regularity explicitly in some special cases, such as the symmetric binary and ternary pseudo spline schemes.

Tangent plane continuity of non-stationary subdivision schemes at extraordinary vertices

Marco Donatelli, **Paola Novara**, Lucia Romani
University of Insubria, Como, Italy

To study convergence and C^1 smoothness of non stationary subdivision schemes when the initial data define a regular polygonal mesh, several analysis tools have been proposed in the literature [1, 2]. The goal of this research is to extend the existing results to ana lyze the behavior of a non stationary subdivision scheme also when the initial polygonal mesh contains extraordinary vertices and/or extraordinary faces. Although in this case the analysis technique cannot simply rely on the eigenanalysis of the local subdivision matrix, as in the stationary case [3], we can still derive sufficient conditions for checking convergence and tangent plane continuity of any arbitrary non stationary subdivision scheme exploiting linear algebra tools. The usefulness of the derived conditions is il lustrated by a numerical example based on a non stationary version of Catmull Clark's subdivision scheme proposed in [4].

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Arbitrary holes in surfaces using blending type spline constructions

Aleksander Pedersen
UiT The Arctic University of Norway

Constructing parametric surfaces present several challenges, such as intended gaps and discontinuities. Generalized expo rational B Splines (GERBS) [1] is a blending type spline construction, where local functions associated with each knot are blended by the C^k smooth basis functions. Using features of the blending type spline construction, including local refinement and insertion of multiple knots, we can model parametric surfaces with arbitrary inner boundaries [2] which look geometrical discontinuous while still being mathematical smooth(C^k). In this paper we investigate how to construct free form inner boundaries on parametric surfaces using the above mentioned blending type spline construction. An example is modeling of physical properties such as, but not limited to, crack propagation, tearing and penetration of a surface.

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Tensor-product surface patches with Pythagorean-hodograph isoparametric curves

Rida T. Farouki, **Francesca Pelosi**,
Maria Lucia Sampoli, Alessandra Sestini
Università di Roma "Tor Vergata", Italy

The parameterization of a surface is a necessary artefact in specifying and analyzing its geometry. Although the isoparametric curves have no intrinsic geometrical signifi cance, they are nevertheless useful in practical applications, such as path planning for the machining or inspection of surfaces.

The Pythagorean hodograph (PH) curves have a distinct advantage over ordinary polynomial curves in this respect, since their arc lengths are simply polynomial functions of the curve parameter. The intent of the present is to investigate the feasibility of con structing surface patches with Pythagorean hodograph isoparametric curves. The sim plest non trivial instances, interpolating four prescribed patch boundary curves, involve degree $(5,4)$ tensor product surface patches $\mathbf{s}(u,v)$, whose $v =$ constant isoparametric curves are all spatial PH quintics. The construction can be reduced to solving a novel type of quadratic quaternion equation. A closed form solution for this type of equation is derived, and conditions for the existence of solutions are identified. The surfaces in corporate three residual scalar freedoms, that can be exploited to improve the interior shape of the patch. The implementation of the method is illustrated through a selection of computed examples.

On the optimal choice of local RBF approximants in the partition of unity interpolation

Roberto Cavoretto, Alessandra De Rossi, **Emma Perracchione**
University of Torino, Italy

The interpolation via Partition of Unity (PU) method consists in decomposing the domain into several *subdomains* or *patches* forming a covering of the original domain. In literature, except for particular and well known cases, [1], such subdomains are always supposed to be hyperspheres of a fixed size.

Here, considering radial basis functions as local approximants, the aim is to develop a method which enables us to select suitable sizes of the different PU subdomains and safe values for the shape parameter of the local basis functions. To such scope, we compute subsequent error estimates depending on these quantities. Then, we select for each PU subdomain the *optimal* couple of values (for its size and for the shape parameter) used to solve the local problem, [2].

Moreover, we also take into account the complexity of the algorithm; in fact, the computational issue consisting in finding all the points belonging to a given subdomain is performed with the use of a truly fast multivariate data structure.

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Design and Analysis of Manifolds with Irregular Points

Jörg Peters
University of Florida

This talk addresses the construction of geometric representations of manifolds with emphasis on smoothness and refinability also at irregular points in the layout of bivariate or trivariate piecewise polynomials (joint work see References [1 5]).

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Approximation of functions with curvilinear singularities by shearlet systems on bounded domains

Philipp Petersen
TU Berlin, Germany

Driven by an overwhelming amount of applications, the numerical approximation of partial differential equations was established as one of the core areas in applied math ematics. During the last decades a trend for the solution of PDEs emerged, focusing on employing systems from applied harmonic analysis for the adaptive solution of these equations. Most notably, wavelet systems have been applied and lead to provably optimal solvers for elliptic PDEs, [1].

Another popular system is that of shearlets, [3], which, in contrast to wavelets, ad mits optimal non linear approximation rates for functions that have singularities along smooth curves. Nonetheless, in order to apply these systems in adaptive discretization algorithms it is necessary to have a system on a bounded domain which forms a frame, is able to incorporate boundary conditions, and characterizes Sobolev spaces. However, up to date no suitable shearlet construction has been proposed that satisfies all these desiderata. In this talk we will introduce a novel shearlet system that meets all of the requirements mentioned above and enjoys optimal approximation rates for functions with discontinuities along curves. In addition, we will present first numerical examples. This talk is based on the works [2,4]

This is joint work with Philipp Grohs, Gitta Kutyniok, Jackie Ma and Mones Raslan.

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Interactive Modeling of Developable Surfaces

Helmut Pottmann
TU Wien and KAUST

Developable surfaces can be mapped into the plane without distortion and thus rep resent shapes obtainable with thin materials which do not stretch. They are important in a variety of applications including freeform architecture, sheet metal based industries, industrial design and curve creased origami. We discuss recent progress on interactive geometric design of these important surfaces and thereby address topics such as panel layout and curved support structures in freeform architecture, surfaces from developable strips and corresponding semi discrete representations, and a computational framework for developable NURBS surfaces and curve creased origami. We represent developable surfaces as splines and express the nonlinear conditions relating to developability and curved folds as quadratic equations. This allows us to utilize a constraint solver which may be described as energy guided projection onto the constraint variety, and which is fast enough for interactive modeling. A combined primal dual surface representation en ables us to robustly and quickly solve approximation problems. This is joint work with Chengcheng Tang, Pengbo Bo, Martin Kilian and Johannes Wallner.

A variational formulation of the sub-Riemannian model of the primary visual cortex

Dario Prandi
Université Paris Dauphine, France

In this talk we present a new biologically inspired mathematical model of images. This approach consists in an axiomatic formalization of the lift of images to the human primary visual cortex V1, modeled as a sub Riemannian manifold. This (now well ac cepted) mathematical structure represents the primary visual cortex as the bundle $PT\mathbb{R}^2$ of directions of the plane, where each point corresponds to a neuron with both spatial location and local orientation preferences. This structure is then endowed with a sub Riemannian metric, mimicking the long and short range connections between neurons. Our main contribution is to define a lift of images to this space, and more generally to solve image restoration tasks (e.g. denoising or inpainting), through a dissipation of energy by the cortical structure.

In sharp contrast with previous definitions of lifts (e.g. via Gabor wavelets), this method does not make use of an *a priori* fixed lift procedure. This allows us to integrate into this procedure the sub Riemannian structure of the cortex, which in turn, makes the lift sensitive to the curvature of objects composing the image, without resorting to any post processing diffusion over the cortical surface.

This is a joint work with J. M. Mirebeau, G. Peyré, and A. Sarti.

Classification of S-shaped and C-shaped Transition Curve

M. Yushalify Misro, **Ahmad Ramli**
Universiti Sains Malaysia, Penang, Malaysia

Transition curve or spiral easement is important in robot trajectory pathway, highway and road design. Meek and Walton [1] define S shaped and C shaped transition curve by classifying transition curve into five templates or five cases. Among those cases, S shaped is formed with circle to circle either with similar radii or different radii, where and C shaped construction can be divided into two cases which are a small circle lies inside a big circle and, from one circle to another circle. Few years later, Habib and Sakai [2] developed different definition to construct S shaped and C shaped transition curve and compared curves by Meek and Walton. Between both of these greatest duos, we will discuss this paper in term of manipulation of control point, degree of rotation where the angle will turn to form S shaped and C shaped by using cubic trigonometric Bezier. Behaviors of curvature profile, either monotonous decrement or monotonous increment and others differences are discussed. We will also discuss the similarities between two approaches.

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Approximation with Ambient B-Splines and intrinsic PDEs on manifolds

Ulrich Reif
Technical University of Darmstadt, Germany

The approximation of data given on some manifold is a difficult task since, in general, it is not clear how to construct finite dimensional function spaces with favorable proper ties on that manifold. A natural idea is to extend the given data to the ambient space of the manifold, for instance by requesting constant values along lines perpendicular to the manifold. In this way, a new function is defined, which can be approximated by standard tensor product splines in the ambient space. Restricting these splines to the manifold yields the desired approximation. We show that this method has optimal approximation power and illustrate its properties by practical examples. The method can also be used to define parametrizations of free form surfaces of arbitrary topology. Unlike other ap proaches, like geometric continuity or subdivision, the ambient B spline method yields any desired order of smoothness easily.

In the second part of the talk, we discuss the potential of the method for the solution of intrinsic partial differential equation on manifolds. Here, not a given function but functionals like the Laplace Beltrami operator must be extended to the ambient space of the manifold. Once this is done, the resulting higher dimensional PDE can be approxi mated by tensor product B splines, and again, the actual solution is found by restricting the solution to the manifold. Theorems on the existence and uniqueness of solutions are provided, while convergence results are not available, yet.

Spectral analysis of matrices arising
in isogeometric methods with GB-splines

Fabio Roman, Carla Manni, Hendrik Speleers
University of Turin, University of Rome “Tor Vergata”, Italy

Generalized splines are smooth piecewise functions with sections in spaces more gen eral than classical algebraic polynomials. Interesting examples are spaces comprising trigonometric or hyperbolic functions. Under suitable assumptions, generalized splines enjoy all the desirable properties of polynomial splines, including a representation in terms of basis functions (the so called GB splines) that are a natural extension of the polynomial B splines.

Tensor product GB splines are an interesting problem dependent alternative to tensor product polynomial B splines and NURBS in isogeometric analysis (IgA). Like any dis cretization method, the IgA paradigm (either in the Galerkin or in the collocation for mulation) requires to solve large linear systems. A deep understanding of the spectral properties of the related matrices is crucial for the design of fast solvers for these linear systems.

In this talk we focus on IgA Galerkin/collocation discretizations based on trigono metric or hyperbolic GB splines. In particular, we prove that the corresponding stiffness matrices possess an asymptotic eigenvalue distribution which can be compactly described by a function, the so called symbol, see [2]. These results extend those obtained for IgA discretization methods based on polynomial B splines, see [1], and strengthen the struc tural similarity between the polynomial and the generalized setting.

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Adaptivity with B-spline Elements

Malcolm Sabin
Numerical Geometry Ltd., UK

This paper takes a stage further the work of Kraft[1] and of Grinspun, Krysl and Schröder[2] who used subdivision formulations to show that finite element formulation can be expressed better in terms of the basis functions used to span the space, rather than in terms of the partitioning of the domain into elements. Adaptivity is achieved not by subpartitioning the domain, but by nesting of solution spaces.

This paper shows how, with B spline elements, their approach can be further simplified: a B spline element of any degree and in any number of dimensions can be refined independently of every other within the basis. This completely avoids the linear dependence problem, and can also give slightly more focussed adaptivity, adding extra freedom only, and exactly, where it is needed, thus reducing the solution times.

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Subdivision Surfaces C2 at the extraordinary points

Malcolm Sabin
Numerical Geometry Ltd. UK

The conditions for subdivision surfaces which are polynomial in the regular region to have continuity higher than C1 were identified by Reif[1]. The conditions are ugly and although schemes have been identified and implemented which satisfy them, those schemes have not proved satisfactory from other points of view.

This paper explores what can be created using schemes which are not polynomial in the regular regions, and the picture looks much rosier. The key ideas are

(i) use of quasi interpolation

(ii) local evaluation of coefficients in the irregular context.

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Proposal of Tangential Angle Parameterizaiton Curves

Takafumi Saito, Norimasa Yoshida
Tokyo University of Agriculture and Technology, JAPAN

We propose a novel representation of freeform planer curves named "Tangential Angle Parameterization Curves (TAP curves)", which enables easy control of both position (Hermite interpolation) and curvature profile. In TAP curves, curvature radius $\rho(\theta)$ is defined as a function of tangential angle θ. The position of a point on the curve is represented in the complex plane as

$$\mathbf{Q}(\theta) = \int_{\theta_0}^{\theta} \rho(\phi) e^{i\phi} \mathrm{d}\phi + \mathbf{Q_0}, \tag{1}$$

where $\mathbf{Q_0}$ and θ_0 are the position and the tangential angle at the start point. TAP curves have the following nice features.

1. Curvature profile can be controlled directly (excluding an inflection point).
2. Principle of superposition holds for curvature radius $\rho(\theta)$ and position $\mathbf{Q}(\theta)$.
3. Offset curves are simply represented by shifting the function $\rho(\theta)$.
4. If $\rho(\theta)$ is a polynomial, $\mathbf{Q}(\theta)$ becomes a closed form expression, so it can be calcu lated without numerical integration.

We propose Bézier TAP curves by representing $\rho(\theta)$ as Bernstein polynomials, where direct control of curvature profile is possible with "control curvature radii", and G^1, G^2 Hermite interpolation is realized by solving linear equations.

Multi-patch parameterization using template mapping

Bert Jüttler, **Svajunas Sajavičius**, Jaka Špeh
Johannes Kepler University Linz, Austria

We present an approach for multi patch parameterization of a multiply connected planar domain (target geometry). Our approach consists of two main steps   domain segmentation and multi patch parameterization. In the segmentation part we employ a mapping between the template and target geometry. The template is an easily parameter ized geometry with naturally defined segmentation lines. The application of generalized offsetting allows to simplify inner boundaries of the target geometry and to get a bijective hierarchical map [1, 2]. This map transfers the segmentation lines from the template to target geometry and segments it into quadrilateral subdomains. In the second step, we need to parameterize the areas between the offsets and the boundary curves of the do main (inner rings) and the rest of the domain. The parameterizations of the inner rings are provided by the construction of the generalized offsets. For the rest of the domain we use the Gordon Coons parameterization for each patch of the template and make compositions with the hierarchical map.

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On the isogeometric formulation of a symmetric Galerkin Boundary Element Method

A. Aimi, M. Diligenti, **M. L. Sampoli**, A. Sestini
University of Siena, Italy

Boundary Element Methods (BEMs) are schemes studied since the mid '80, for the numerical solution of those Boundary Valued Problems, which can be transformed to Boundary Integral Equations (BIEs). Their key points are the provided dimension reduction and the natural applicability to exterior problems. Even if BEMs can be formulated also by collocation, their Galerkin formulation is superior from a theoretical point of view because stability, consistency and convergence can be shown for a very general class of BIEs. The isogeometric approach [1] which benefits of spline flexibility has been recently combined with success also to BEMs, even if mainly in their collocation form. The study in [2] has however preliminarily shown the advantages in terms of accuracy and/or efficiency of the Isogeometric formulation of a Symmetric Galerkin BEM (IGA SGBEM) compared to the classical SGBEM formulation based on Lagrangian functions.

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Designing impossible NURBS objects via anamorphic shape modification

Javier Sánchez-Reyes, Jesús M. Chacón
Universidad de Castilla La Mancha, Spain

3D impossible objects are those that, seen from a specific viewpoint, result in a (seemingly) contradictory view. These views are particular instances of the so called optical anamorphosis, where an object looks similar to another one when observed from a certain viewpoint, but appears distorted unless if the observer moves away from the viewpoint. Typically, 3D impossible objects are generated by taking an initial model and then cutting or deforming it in some way. We propose a general tool to design impossible objects from initial NURBS models, by deforming them in an anamorphic fashion. This technique implies moving the NURBS control points in a radial direction through the viewpoint and simultaneously changing their weights. Thus, we preserve the silhouettes locally, but at the same time we can change the distance to the observer of different parts of the object and alter their precedence, which leads to the contradiction when seen by the observer. We also show how to employ textures to enhance this effect.

C^1 multipatch spaces for isogeometric analysis

Giancarlo Sangalli
Università di Pavia, Italy

One key feature of isogeometric analysis is that it allows smooth shape functions. Indeed, when isogeometric spaces are constructed from p degree splines (and extensions, such as NURBS), they enjoy up to C^{p-1} continuity within each patch. However, global continuity beyond C^0 on so called multi patch geometries poses some significant dif ficulties. The work I present, *which is a collaboration with Pablo Antolin, Annabelle Collin and Thomas Takacs,* consider multi patch domains that have a parametrization which is only C^0 at the patch interface. On such domains we study the h refinement of C^1 continuous isogeometric spaces. These spaces in general do not have optimal approximation properties. The reason is that the C^1 continuity condition easily over constrains the solution which is, in the worst cases, fully *locked* to linears at the patch interface. However, recently [1] has given numerical evidence that optimal convergence occurs for bilinear two patch geometries and C^1 splines of polynomial degree at least 3. This is the starting point of our study. We introduce the class of analysis suitable G^1 geometry parametrizations, which includes bilinears parametrizations. Following [2], we analyze the structure of C^1 isogeometric spaces over analysis suitable G^1 geometry parametrizations and, by theoretical results and numerical testing, discuss their aproxi mation properties.

References:

[1] M. Kapl, V. Vitrih, B. Jüttler, and K. Birner. Isogeometric analysis with geometrically continuous functions on two patch geometries. *submitted*, 2015.
[2] Annabelle Collin, Giancarlo Sangalli, and Thomas Takacs. Approximation properties of multi patch C^1 isogeometric spaces. *arXiv preprint arXiv:1509.07619*, 2015.

Kernels of convolution and subdivision operators

Tomas Sauer
University of Passau, Germany

It is well known from systems theory or the theory of difference equations [1] that the kernels of discrete convolution operators or filters consist of exponential polynomials. More precisely, if h is the finitely supported impulse response of a filter, then the zeros of the z *transform*

$$h^*(z) = \sum_{k \in \mathbb{Z}} h(k)\, z^{-k}$$

and their multiplicity completely determine it kernel.

The talk is concerned with the multivariate counterpiece of this classical result where an interesting interplay between shift and differentiation invariant polynomial spaces takes place. Moreover, this is strongly connected with the notion of multiplicity of com mon zeros of polynomials. Finally, we apply the results also to the kernels of subdivision operators and relate them to symmetric zeros of the symbol.

References:
[1] Jordan, C.: Calculus of finite differences, 3rd edn., Chelsea (1965).

Adaptive Curvature-Based Composite Grid Generation Techniques Applied to 3D Visualizations

Bonita V. Saunders
National Institute of Standards and Technology, Gaithersburg, MD

A composition of tensor product B spline mappings is used to generate grids for computing plots of complex mathematical functions that arise in many scientific appli cations. Curvature information derived from the function values is used to adapt the grids to improve the color map. The effectiveness of the technique is demonstrated in the development of interactive surface visualizations for the NIST Digital Library of Mathematical Functions (DLMF). We will show the latest implementations in WebGL, a JavaScript API for rendering 3D graphics in a web browser without the use of a plugin.

Bijective Parameterization with Free Boundaries

Scott Schaefer
Texas A&M University

This talk will discuss the issue of generating a bijective map between the 3D trian gulated surface and a region of the 2D plane that minimizes a distortion metric without constraining the boundary to a fixed shape. First I will discuss methods of measuring the distortion of the parameterization as well as how to modify such distortion metrics to guarantee they form a bijective map. The talk will pay special attention as to how to optimize such distortion metrics through an interior point method and a modified line search to guarantee such parameterizations form a bijective map. In addition, we will discuss methods for accelerating the optimization.

Polynomial Interpolation: from GC sets to simplicial complexes

Henry K. Schenck
University of Illinois, USA

Geometrically characterized (GC) sets were introduced by Chung Yao in their work on polynomial interpolation in R^d. We investigate behavior of GC sets in terms of the algebraic combinatorics of Bi Cohen Macaulay simplicial complexes.

Adaptive Numerical Quadrature for the
Isogeometric Discontinuous Galerkin Method

Agnes Seiler, Bert Jüttler
Johannes Kepler University, Linz, Austria

Isogeometric Analysis [1] uses single or multi patch NURBS domain parameterizations to derive discretizations of partial differential equations. Discontinuous Galerkin (dG) methods [2] are one of the main approaches to perform the coupling of the individual patches in the multi patch case. The dG based discretization of a partial differential equation is based on a modified variational form, where one introduces additional terms that measure the discontinuity of the values and normal derivatives across the interfaces between patches.

We consider multi patch domain parameterizations with non matching parameterizations of the interface and consider two problems that arise in the context of isogeometric dG methods: Firstly, one needs to identify pairs of associated points on the common interface of the two patches for correctly evaluating the additional terms in the variational form. We will use reparameterizations to perform this task. Secondly, suitable techniques for numerical integration are needed to evaluate the quantities that occur in the isogeometric dG discretization with the required level of accuracy. Here we explore two possible approaches, which are based on subdivision and adaptive refinement, respectively. The talk will discuss the trade off between computational efficiency and numerical accuracy for addressing both problems.

References:
[1] Hughes, T., Cottrell, J. and Bazilevs, Y. (2005), Isogeometric Analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement, Computer Methods in applied mechanics and engineering, vol 194, p. 4135 4195
[2] Langer, U., Mantzaflaris, A., Moore, S.E. and Toulopoulos, I. (2014), Multipatch Discontinuous Galerkin Isogeometric Analysis, Lecture Notes in Computational Science and Engineering, vol 107, p. 1 32

Interactive shape control of complete log-aesthetic surfaces

Minato Sekiguchi, Takafumi Saito
Tokyo University of Agriculture and Technology, JAPAN

In order to improve aesthetic quality of a curved surface, its curvature profile is important, but it is not easy to be controlled with a conventional CAD system. In this talk, we extend log aesthetic curves, which ensure nice curvature property, to surfaces. Especially, we propose generation and shape control methods of "complete log aesthetic surfaces", where iso parameter lines of both u and v are always log aesthetic curves. Complete log aesthetic surfaces are generated by rotational sweeping with scaling *i.e.* sweeping a log aesthetic curve segment as a profile curve along a logarithmic spiral as a guide curve. We try to control the surfaces by interactive control of shapes and positions of guide and profile curves. For controlling each curve, the plane it belongs to is specified, and then, both end points and the cross section of their tangent lines are manipulated, which enables G^1 Hermite interpolation. In case of using a logarithmic helix as a guide curve, a planer logarithmic spiral and shift amounts of its end points from the plane are specified.

An adaptive data reduction scheme based on hierarchical Hermite spline quasi-interpolation

C. Bracco, C. Giannelli, **A. Sestini**
Università di Firenze, Italy

Data reduction approaches are of interest whenever a certain shape is suitably approx imated by a reference function whose storage is expensive. By focusing on the bivariate case and assuming to deal with a spline reference function, here we consider an adaptive ascending strategy producing a final spline approximation which can be more cheaply stored. Such final spline is expressed in the truncated hierarchical B spline basis and is derived by approximating the reference function with an adaptive quasi interpolation approach. In particular, we rely on the hierarchical extension developed and analyzed in [1] of the Hermite quasi interpolation scheme originally introduced in [2].

References:

[1] C. Bracco, C. Giannelli, F. Mazzia, A. Sestini, Bivariate hierarchical Hermite spline quasi interpolation, BIT Numer. Math. (2016), to appear.

[2] F. Mazzia, A. Sestini, The BS Class of Hermite Spline Quasi interpolants on Nonuni form Knot Distributions, BIT Numer. Math. 49 (2009), 611 628.

Multiwavelets with exponential vanishing moments

Nada Sissouno
University of Passau, Germany

We present a class of filter banks whose construction is based on a family of level dependent Hermite subdivision schemes preserving elements of the spaces spanned by exponential and polynomial functions.
The key to our construction is the factorization property of the involved Hermite sub division operators in terms of convolution operators that annihilate the aforementioned spaces.
As a result, the corresponding multiwavelet bases, which give rise to a level dependent MRA, possess polynomial and exponential cancellation properties, useful in many kind of applications.
This is joint work with Mariantonia Cotronei.

Deconfliction and surface generation from bathymetry data using LR B-splines

Vibeke Skytt
SINTEF, Norway

The typical situation is an input consisting of a set of point clouds representing sea bottom acquired by different measurement techniques at different times, having different accuracy and varying patterns of points. These point clouds shall be combined into a seamless surface using the "best" of the initial points to represent the sea bottom with good accuracy and at the same time reduce the data size considerably. We discuss how to use a rough approximation of the combined point clouds as a reference surface to select a consistent set of points for further processing (deconfliction). The surface is then updated with the selected points to create an accurate approximation. LR B splines is the selected surface format due to its ability to represent local detail without globally increase the data size of the surface and suitability for adaptive refinement and approximation. The set of point clouds is potentially a case of "big data" and the final seamless surface will often be represented as several LR B spline surfaces. Suitable continuity, we use C1, between adjacent surfaces can again be achieved without globally increasing the data size by performing local knot insertion into the LR B spline surfaces.

<u>References</u>:

[1] T. Dokken, T. Lyche, and K. F. Pettersen. Polynomial splines over locally refined box partitions. Computer Aided Geometric Design 30(3): 331 356 (2013)

Approximation of Manifolds by Moving Least Squares (MMLS)

Barak Sober, David Levin
Tel Aviv University, Israel

In order to avoid the curse of dimensionality, frequently encountered in Big Data analysis, there was a vast development in the field of linear and non linear dimension reduction techniques in recent years. These techniques (sometimes referred to as man ifold learning) assume that the scattered input data is lying on a lower dimensional manifold, thus the high dimensionality problem can be overcome by learning the lower dimensionality behavior. However, in real life applications, data is often very noisy. In this work, we propose a method to approximate a d dimensional smooth submanifold $\mathcal{M}$ residing in $\mathbb{R}^n$ ($d << n$) based upon scattered data points (i.e., a data cloud). We assume that the data points are located "near" a lower dimensional smooth manifold and perform a non linear moving least squares projection. This method can also be used for approximating functions over the manifold. Under some mild assumptions, the resulting aproximation to the manifold $\mathcal{M}$ is shown to be infinitely smooth and of approximation order of $O(h^{m+1})$. Where h is the mesh size w.r.t. $\mathcal{M}$, and m is the degree of the local polynomial approximation. Furthermore, the algorithm presented here is linear in the large dimension n.

Divergence-free splines on Alfeld split of a simplex

Tatyana Sorokina
Towson University, USA

We consider the application of standard differentiation operators to spline spaces and spline vector fields defined on simplicial partitions in $\mathbb{R}^n$, following recent advances in the bivariate case, see [1]. In particular, we develop Bernstein Bézier techniques for working with divergence of spline vector fields. We apply the techniques to prove a version of a generalized Stokes' Theorem for continuous splines. Additionally, using the new Bernstein Bézier techniques, we obtain the dimension and a minimal determining set for divergence free splines on Alfeld split of a simplex in $\mathbb{R}^n$.

References:

[1] ALFELD P. and T. SOROKINA, Linear Differential Operators on Bivariate Spline Spaces and Spline Vector Fields, *BIT Numerical Mathematics*, to appear.

Effortless quasi-interpolation in hierarchical spline spaces

Hendrik Speleers, Carla Manni
University of Rome “Tor Vergata”, Italy

Hierarchical spline spaces provide a flexible framework for local refinement coupled with a remarkable intrinsic simplicity. They are defined in terms of a hierarchy of locally refined meshes, reflecting different levels of refinement. The so called *truncated hierar chical basis* is an interesting basis for the hierarchical spline space with an enhanced set of properties compared to the classical hierarchical basis: its elements form a convex partition of unity, they are locally supported and strongly stable [1].

In this talk we discuss a general approach to construct quasi interpolants in hier archical spline spaces expressed in terms of the truncated hierarchical basis [2]. The main ingredient is the property of *preservation of coefficients* of the truncated hierar chical basis representation. Thanks to this property, the construction of the hierarchical quasi interpolant is basically effortless. It is sufficient to consider a quasi interpolant in each space associated with a particular level in the hierarchy, which will be referred to as a one level quasi interpolant. Then, the coefficients of the proposed hierarchical quasi interpolant are nothing else than a proper subset of the coefficients of the one level quasi interpolants. No additional manipulations are required. Important properties like polynomial reproduction of the one level quasi interpolants are preserved in the hi erarchical construction. We also discuss the local approximation order of the hierarchical quasi interpolants in different norms, and we illustrate the effectiveness of the approach with some numerical examples.

References:
[1] C. Giannelli, B. Jüttler, H. Speleers. *Strongly stable bases for adaptively refined multilevel spline spaces*, Advances in Computational Mathematics 40, 459 490, 2014.
[2] H. Speleers, C. Manni. *Effortless quasi interpolation in hierarchical spa ces*, Nu merische Mathematik 132, 155 184, 2016.

Enumerating Quadrilateral Meshes

Petra Surynková
Johannes Kepler University Linz, Austria

Quadrilateral meshes are widely studied in the literature as they possess several advantages compared to triangular meshes, e.g., for applications in graphics. In this contribution we present a theoretical framework and a practical method for enumerating all topologically unique quadrangulations of a certain class. More precisely, we propose an algorithm that enumerates all possible quad layouts with respect to the number of internal vertices with the given valences. The incremental recursive construction produces lists of quad layouts with duplications. We weed out equivalent results by constructing a unique numbering of the vertices for each quad layout. Experimental results of enumerated meshes for different inputs are provided. Finally, we suggest a method to select a suitable quadrangulation of a given input shape by considering suitable quality criteria. This is joint work with Bert Jüttler.

References:

[1] Bommes, D., Lévy, B., Pietroni, N., Puppo, E., Silva, C., Tarini, M., and Zorin, D. (2012). State of the Art in Quad Meshing. *Eurographics STARS.*

[2] Botsch, M., Kobbelt, L., Pauly, M., Alliez, P., and Lévy, B. (2010). Polygon Mesh Processing. *A K Peters.*

Optimal approximation properties for analysis-suitable C^1 multi-patch isogeometric spaces

Thomas Takacs
Johannes Kepler University Linz, Austria

Given a physical domain $\Omega \subset \mathbb{R}^2$, we consider a geometry parametrization of Ω that is composed of multiple B spline patches forming a regular partition. On this multi patch representation we define a standard isogeometric function space. We present approximation error bounds for the C^1 smooth subspace of such an isogeometric space. We prove that if the space is an *analysis suitable C^1 multi patch isogeometric space* (as presented in [1]) the convergence rate is optimal under h refinement. Note that analysis suitable geometries cover but are not limited to piecewise bilinear parametrizations. The results presented here are based on a joint work with Annabelle Collin and Giancarlo Sangalli.

References:

[1] A. Collin, G. Sangalli and T. Takacs. Analysis suitable G^1 geometry parametrizations for isogeometric analysis. Submitted to *CAGD*, 2016. Pre print available as arXiv:1509.07619.

Designer friendly rebuild algorithm for freeform curves

Daisuke Taki, Misako Nishii, Takafumi Saito
Tokyo University of Agriculture and Technology, JAPAN

In design process of freeform curves using a CAD system, it is often required to regenerate a curve with specified degree and number of control points based on previously created curve sequence. This is implemented as "Rebuild" function in a CAD system. The purpose of this study is to improve the rebuild function so as to match with designer's kansei. Control point placement generated automatically by existing rebuild function does not satisfy designer's rule of thumb in curve design. For example, according to the rule of thumb, control points tend to be placed more densely in large curvature regions, while existing rebuild function tends to place the control points more densely near the end points. In this talk, we formulate the rule of thumb about control point density and curvature by focusing on the sensitivity of control points. Then, we propose a new rebuild algorithm which outputs a curve with better control point placement.

Industrial Shape Design at Airbus

R.M. Tookey
Airbus Operations Ltd, Filton, Bristol, UK

Long range civil aircraft are optimised to meet precise aerodynamic cruise perfor mance targets. The aircraft shape, therefore, has a direct impact on the operating cost of the aircraft per passenger seat per mile. At Airbus, aircraft shapes are surface modelled and then managed as the Master Geometry for all downstream engineering activities. The process goal is to deliver the right geometry at the right time for all departments. This means surface quality is driven by programme maturity to either satisfy modelling requirements or achieve manufacturing efficiencies. Since aircraft have a relatively low volume and long life compared to the automotive industry, Airbus has developed a shape design process that supports incremental delivery and continuous improvement. The paper outlines some of the challenges for the shape designer when deploying this process with industry standard NURBS surfaces.

On Bézier forms in Lie Groups

H. Ugail, A. Yılmaz Ceylan, M. C. Márquez
University of Bradford, UK

We study Bézier curves and surfaces in Lie groups particularly in the Heisenberg group $Heis^3$. We describe a modified de Casteljau's algorithm to compute Bézier curves and surfaces on compact Lie groups. In particular, we work on $Heis^3$ since $Heis^3$ is a 3 dimensional, locally compact and 2 step nilpotent Lie group. We try to find the parametric equations of Bézier curves and surfaces in terms of Bernstein polynomials, corresponding to $Heis^3$. Furthermore, we provide interesting examples showing the differences between Bézier curves and surfaces in R^3 and in the Heisenberg group.

On Some Analogies Between Bézier Theory and Mathematical Physics

Márton Vaitkus
Budapest University of Technology and Economics, Hungary

We study mathematical connections between the theory of Bézier curves and surfaces and theoretical physics. It is known that Bézier curves and surfaces arise as projections of toric vareties [1,2]. Over the complex numbers, said toric varieties are even dimensional manifolds, with a so called symplectic structure, which allows them to be interpreted as phase spaces of classical mechanical systems. Finding the quantum mechanical systems corresponding to the systems related to Bézier curves/surfaces, we can interpret in the language of quantum physics key concepts of Bézier theory, such as control points, Bern stein bases and polar forms. In particular, Bézier curves are found to be related to the concept of spin in quantum mechanics. The mathematical basis of this relation is found in Lie group representation theory. These connections and analogies suggest numerous interesting avenues for further research.

References:

[1] Krasauskas, Rimvydas. "Toric surface patches." Advances in Computational Mathe matics 17.1 2 (2002): 89 113.

[2] Sottile, F. "Toric ideals, real toric varieties, and the algebraic moment map." In Goldman, R. and Krasauskas, R. (eds). Topics in Algebraic Geometry and Geometric Modeling (2003): 225 240.

Geometric characteristics of rational Bézier quadric patches

A. Cantón, L. Fernández Jambrina,
E. Rosado María, **M.J. Vázquez-Gallo**
Universidad Politécnica de Madrid, Spain

In this talk we will present closed formulas for geometric elements of rational tri angular Bézier quadric patches in terms of just their weights and control points, using algebraic projective geometry. These results can be extended to quadric tensor product patches. In addition, we will derive closed, coordinate free, formulas for implicit equa tions of quadrics in terms of their weights and control points and we will give an affine classification of quadrics in Bézier form.

References:

[1] A. Cantón, L. Fernández Jambrina, E. Rosado María, and M. J. Vázquez Gallo, J. of Comp. and Appl. Math. 300, 400 419 (2016)

C^2-smooth isogeometric functions on two-patch geometries: Basis construction and dimension study.

Mario Kapl and **Vito Vitrih**
Austrian Academy of Sciences, Austria, University of Primorska, Slovenia

In the talk, we will consider the space of C^2 smooth isogeometric functions defined on bilinearly parameterized two patch domains $\Omega \subset \mathbb{R}^2$. The construction of these functions is closely related to the concept of geometric continuity of surfaces, more precisely, the C^2 smoothness of isogeometric functions is found to be equivalent to the G^2 smoothness of their graph surfaces (c.f. [1, 2]). The basic construction and the complete study of the dimension of the space of biquintic and bisixtic C^2 smooth isogeometric functions on such domains will be presented. The talk will be continued by the talk C^2 *smooth isogeometric functions on two patch geometries: Numerical examples and possible generalizations.*

References:
[1] D. Groisser, J. Peters, Matched G^k constructions always yield C^k conti nuous isoge ometric elements, Comput. Aided Geom. Design (2015).
[2] M. Kapl, V. Vitrih, B. Juttler, K. Birner, Isogeometric analysis with geometrically continuous functions on two patch geometries, Computers & Mathematics with Applica tions (2015).

Interpolation of convex scattered data in $\mathbb{R}^3$ using minimum norm networks

Krassimira Vlachkova
Sofia University, Bulgaria

We consider the interpolation of convex scattered data in $\mathbb{R}^3$ associated with a planar triangulation T. We propose a solution that constructs a minimum norm piecewise quadratic network defined and convex on the edges of T and then builds a Powell Sabin six split interpolant. We obtain necessary and sufficient geometric conditions for the convexity of the resulting interpolant.

Anisotropic Scaling Matrices with Minimum Determinant

Mira Bozzini, Milvia Rossini, **Elena Volontè**
Università degli studi Milano Bicocca, Italy

Scaling matrices are the key ingredient in multiscale analysis because they design how to handle the given data. It is known that the absolute value of the scaling matrix deter minant gives the number of disjoint cosets which is strictly connected with the number of filters needed to analyze a signal and then to computational complexity. Among the clas sical scaling matrices, we find the family of shearlet matrices that have many interesting properties. For example the capability to detect the directionality makes them attractive when dealing with anisotropic problems. Instead their drawback is their relatively large determinant. In this talk we propose for the three dimensional case, that is interesting in application, new scaling matrices with the same good properties of shearlet matrices, such as handling directionality and generate multiple multiresolution analysis but, on the same time, with minimum determinant.

Translation surfaces and isotropic nets on rational minimal surfaces

Jan Vršek, Miroslav Lávička
Univ. of West Bohemia, Plzeň, Czech Republic

In CAGD basic modelling surfaces, with the property being simple and widely used, are applied to construct complex models. Typical examples are ruled surfaces, rotational surfaces, swept surfaces and translation surfaces. We will deal with the translation surfaces that are shapes generated by translating one curve along another one. Hence, they are a simple solution of the task to interpolate a surface going through two prescribed curves. We will focus on the geometry of translation surfaces generated by two algebraic curves in space and study their properties, especially useful for geometric modelling purposes.

Next, we will investigate the minimal surfaces as special instances of translation surfaces. A vector $(a_1, a_2, a_3) \in \mathbb{C}^3$ is called isotropic if $a_1^2+a_2^2+a_3^2 = 0$, then the isotropic curve is a complex curve whose tangent directions are all isotropic. It is a classical result that each minimal surface may be obtained as a translation surface generated by some isotropic curve and its complex conjugate. Hence all the results about translation surfaces will be directly applied also to minimal surfaces. Finally, we will present a construction of rational isotropic curves with prescribed tangent which leads to the description of all rational minimal surfaces.

References:

[1] Sonia Pérez Díaz and Li Yong Shen. 2014. *Parametrization of translational surfaces.* In Proceedings of the 2014 Symposium on Symbolic Numeric Computation (SNC '14), Stephen M. Watt, Jan Verschelde, and Lihong Zhi (Eds.). ACM, New York, NY, USA, 128 129.

[2] Dirk Jan Struik. 1961. *Lectures on Classical Differential Geometry.* Addison Wesley series in mathematics. Addison Wesley Publishing Company

Computations and Applications of Medial Axis Transform of 3D Shapes

Wenping Wang
The University of Hong Kong

As a complete shape description, the medial axis of a geometric shape possesses a number of favorable properties it encodes symmetry, local thickness and structural com ponents of the shape it represents. Hence, the medial axis has been studied extensively in shape modeling and analysis since its introduction by Blum in 1960s. However, the practical application of the medial axis is hindered by its notorious instability and lack of compact representation; that is, a primitive medial axis without proper processing is often represented as a dense discrete mesh with many spurious branches. In this talk I shall represent some recent studies on computing stable and compact representations of the medial axes of 3D shapes. Techniques from mesh simplification will be extended to compute a medial axis without spurious branches and represented by a small number of mesh vertices, while meeting specified approximation accuracy. New applications of the medial axis will also be presented.

Edge-preserving variational methods for manifold-valued data

Andreas Weinmann
Helmholtz Center Munich, Germany

Nonlinear data spaces appear in various image processing and imaging tasks and ap plications. Examples are the euclidean motion group appearing in registration problems, the manifold of postive definite matrices appearing as data space in diffusion tensor imag ing, or spheres appearing, for instance, as direction fields. Since the aquired images and volumes are often noisy, schemes to perform edge preserving smoothing for data with values in such manifolds are necessary. For this task we present variational approaches based on free discontinuity problems and on total variation type regularization. These methods approximate the data while regularizing it in an edge preserving way.

Mesh-free semi-Lagrangian methods for advection on the sphere and other surfaces

Grady B. Wright and Varun Shankar
Boise State University, USA

Advection is a mechanism by which a quantity is transported from one location to another by the bulk motion of a fluid. In several applications advection occurs on a two dimensional surface embedded in three dimensional space, which makes the problems particularly challenging. Examples include the transport of various quantities, such as volcanic ash, in the upper atmosphere, which is often modeled as advection on the sur face of the sphere, and the transport of proteins and other molecules on cell membranes, which is of increasing interest in the biological and material sciences. We compare and contrast three mesh free semi Lagrangian methods for numerically simulating transport on surfaces. At the core of these methods is an interpolation scheme that is built on radial basis functions (RBFs). This allows the methods be formulated entirely in ex trinsic (Cartesian) coordinates, thus avoiding any singularities associated with intrinsic coordinate systems on the surfaces. The new methods, which include global RBFs, local RBF FD, and RBF partition of unity, also avoid having to use any stabilization terms such as hyperviscosity during time integration. We present results showing high orders of spatial convergence for transport on the sphere, and demonstrate the ability of our methods to handle transport on other surfaces.

Non-consistent cell-average multiresolution operators based on a new non-linear subdivision scheme

R. Donat, **D. F. Yáñez**
U. Católica de Valencia, Spain

We present and analyze a non linear subdivision scheme following the ideas pre sented by Marquina and Serna in [2]. It consists of a decomposition of the scheme $(Sf)_{2i+\lambda} = f_i + ((-1)^\lambda/8)(f_{i+1} - f_{i-1})$, with $\lambda = 0, 1$ obtaining the linear Chaikin's scheme plus a non linear part. In this work, we study the convergence and the stability of the scheme. Finally, we use the scheme as prediction operator in the Harten's mul tiresolution representation. This transform is based on two operators: decimation, D, and prediction, P, that satisfy the consistency property $DP = I$, where I is the identity operator. These operators do not satisfy this property. In order to apply them in a Harten's multiresolution scheme we use the algorithm designed by Aràndiga and Yáñez in [1]. We apply these schemes to compress digital images and compare our method with the operators obtained using classical techniques and non linear PPH function.

References:

[1] Aràndiga, F.; Yáñez, D. F.: Non consistent cell average multiresolution operators with application to image processing Applied Mathematics and computation 272 1 (2016), 208 222.

[2] Serna, S.; Marquina, A.: Power ENO methods: a fifth order accurate weighted power ENO method, J. Comput. Phys. 194 2 (2004) 632 658.

Non-linear approximation scheme for piecewise smooth data on uniform grids

Hyoseon Yang and Jungho Yoon
Department of Mathematics, Ewha Womans University, Seoul, Korea

The aim of this study is to construct a non linear scheme to approximate piecewise smooth data on uniform grids. Quasi interpolation is a very useful tool for data approx imation. However, it often suffers from ringing artifacts when approximating data near discontinuities. In order to overcome this drawback, this paper presents a non linear method which prevent spurious oscillations around discontinuities, while achieving high order accuracy in smooth regions. To this end, we first discuss a smoothness indicator which measures the local smoothness of the given data and then construct a local non linear approximation scheme to approximate data with singularities. Some numerical experiments are performed to demonstrate the ability of the proposed scheme.