Workshop on Evolution Equations

This event is a small workshop on

CONSERVATION LAWS AND FLUID EQUATIONS

that will be held on ***Tuesday, 31st of October, 2023***. Speakers will present new ideas and insights in the theory and numerical analysis pertaining to

conservation laws, transonic flow, and magnetohydrodynamics (MHD) systems.

We hope this workshop can provide an opportunity for young researchers in PDEs to come together, exchange ideas, and potentially establish new collaborations.

Zoom link: https://uio.zoom.us/j/64213010305?pwd=eGJjSml5TXRVcnVobFpXYjdaZHp5UT09
Conference No.: 642 1301 0305
Password: 725843

Schedule

All the times refer to the *local Oslo time* in zone **GMT +2**

10:25-10:30 **Opening**.

10:30-11:10 **Speaker**: Kaibo Hu (*University of Edinburgh*).
Title: Helicity-conservative discretization of incompressible MHD systems
Abstract: The magnetohydrodynamics (MHD) systems have several important conservative properties, e.g., the magnetic Gauss law and the conservation of energy and (magnetic, cross, hybrid) helicity in the ideal limit. These conserved quantities encode various kinds of intrinsic symmetry of the equations. To achieve physical fidelity and numerical stability, it is desirable to preserve these conditions precisely in the numerical discretization (up to the machine precision).

In this talk, we first review the conservative properties of the continuous Navier-Stokes and MHD systems. Then we construct finite element methods that precisely preserve these properties in the framework of the Finite Element Exterior Calculus and Discrete Differential Forms. Discrete de Rham sequences play a vital role in the study. We investigate solvers that are robust with physical and discretization parameters.

11:15-11:55 **Speaker**: Rahul Barthwal (*University of Stuttgart*).
Title: On a class of sonic-supersonic boundary value problems
Abstract: We analyze some classes of sonic-supersonic boundary value problems arising in many transonic flows and initial value problems for compressible flows. First, we discuss the existence and regularity of a semi-hyperbolic patch arising from the two-dimensional Riemann problem for compressible Euler equations using the ideas of characteristic decomposition in the self-similar plane. Next, we discuss the existence and regularity of a smooth solution for a supersonic-sonic patch arising in a modified Frankl problem in the

study of three-dimensional axisymmetric steady isentropic relativistic transonic flows over a symmetric airfoil. Using the well-received characteristic decompositions of angle variables and a partial hodograph transformation we prove the existence and regularity of solutions in the partial hodograph plane first and by using an inverse transformation we construct a smooth solution in the physical plane and discuss the uniform regularity of solutions up to the associated sonic curve. Finally, we analyze a sonic-supersonic degenerate boundary value problem for the relativistic magnetohydrodynamics system and prove the existence and uniqueness of solutions using an iteration argument and the inversion technique employed in the previous works.

12:00-13:00 **Break**.

13:00-13:40 **Speaker**: Adrian Ruf (*University of Oslo*).
Title: Uncertainty quantification for conservation laws with discontinuous flux
Abstract: In this talk I will present and apply recent results pertaining stability for *conservation laws with discontinuous flux* and convergence rates of numerical methods approximating their solutions. These results will then be applied in the framework of uncertainty quantification for these types of equations.

In the first part of the talk I will give an overview of *conservation laws with discontinuous flux* which has been an active research area during the last several decades. Many selection criteria to single out a unique weak solution have been proposed in this context and several numerical schemes have been designed and analyzed in the literature. Surprisingly, the preexisting literature on *convergence rates* for such schemes is practically nonexistent. In this talk, focusing on so-called adapted entropy solutions, I will present the first-ever convergence rate results for finite volume and front tracking methods as well as a flux-stability result.

The second part of the talk is devoted to demonstrating applications of these stability and convergence rate estimates. First, I will present a general *uncertainty quantification framework* for conservation laws with discontinuous flux where the problem data (the initial datum, the flux, and the spatial dependency coefficient) are uncertain. A particular application constitutes two-phase reservoir simulations for reservoirs with spatially varying geological properties where the reservoir interfaces are only known up to certain statistical quantities.

13:45-14:25 **Speaker**: Magnus Ørke(*University of Oslo*).
Title: Particle paths for hyperbolic conservation laws
Abstract: We interpret nonlinear, scalar conservation laws

$$\partial_t u + \partial_x f(u) = 0, \qquad u_0 \in BV_{\text{loc}} \cap L^\infty(\mathbb{R})$$

as continuity equations, and couple the Kruzkhov uniqueness theory with the well-posedness of associated particle paths

$$\frac{d}{dt}x_t = \frac{f(u(x_t,t)) - f(c)}{u(x_t,t) - c}, \qquad x_0 \in \mathbb{R};$$

a system of ODEs which govern the mass transportation of the solution u. Thus, on the one hand we obtain a novel selection criterion for scalar conservation laws, and on the other hand we prove well-posedness and regularity with respect to the initial condition for a new class of ODEs. This is joint work with Ulrik Fjordholm and Ola Mæhlen (both at UiO).

14:30-14:35 **Concluding remarks**.