

Compactness estimates for difference approximations of conservation laws

Introduction:

This project will explore the generalization of compactness estimates for difference schemes used in the numerical analysis of conservation laws, particularly those with discontinuous flux functions. Current work has focused on certain first-order schemes, but extending these methods to upwind and higher-order schemes is crucial to enhance accuracy in capturing shocks and discontinuities. This generalization will provide deeper insight into the behavior of numerical schemes in the context of conservation laws, and links to concepts in mathematical analysis such as compactness.

Main Goal:

The objective is to investigate the generalization of the compactness estimates of [1] for difference schemes to upwind and higher-order schemes. This will involve a detailed study of how these schemes behave under varying types of discontinuous flux and their corresponding convergence properties. The project will focus on developing new theoretical results, implementing the corresponding schemes, and testing them on various conservation law models to assess their performance.

References:

[1] K. H. Karlsen and J. D. Towers. Compactness estimates for difference schemes for conservation laws with discontinuous flux. *IMA J. Numer. Anal.*, 2024.