

Well-Posedness of Fractional Bidomain Models for Cardiac Electrophysiology

Introduction:

The bidomain model plays an important role in simulating the propagation of electrical signals through cardiac tissue, which is fundamental for understanding cardiac function. It consists of a coupled system of reaction-diffusion equations and ordinary differential equations (ODEs) that describe the electrical dynamics in the intracellular and extracellular domains, as well as the ionic dynamics governing the behavior of cardiac cells. The fractional bidomain model introduces fractional diffusion operators to capture the multiscale and heterogeneous properties of cardiac tissue, providing a more realistic representation of cardiac electrophysiology [1,2].

Main Goal:

This project aims to prove the well-posedness of the fractional bidomain model under non-homogeneous Neumann-type and general mixed Robin boundary conditions. Building on the theoretical foundations of cardiac models [3,4], the student will analyze the existence, uniqueness, and stability of solutions for the coupled system of reaction-diffusion equations and ODEs. The overall goal is to investigate how fractional diffusion operators influence the model's behavior, particularly with respect to simulating cardiac arrhythmias.

References:

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