

Weak Convergence Methods for Stochastic Conservation Laws

Introduction:

Stochastic conservation laws appear in many applications, such as fluid dynamics, traffic flow, and shallow water models, where random perturbations play a crucial role. In the context of nonlinear partial differential equations (PDEs), traditional methods often rely on strong convergence, but weak convergence techniques have become indispensable in dealing with issues like oscillations and concentrations that arise from nonlinearities. In particular, weak convergence methods such as micro-local defect measures and compensated compactness have been highly successful in the deterministic setting. However, their extension to stochastic settings is less developed and presents significant challenges, particularly in managing oscillations and stochastic forcing.

Main Goal:

This project will investigate weak convergence methods for stochastic conservation laws, focusing on extending existing deterministic tools to the stochastic regime. Specifically, the goals of this project are:

1. **Introduce Micro-Local Defect Measures and Weak Convergence Methods:** The student will study the foundational methods of weak convergence, such as micro-local defect measures and compensated compactness, and explore their applications in deterministic PDEs, particularly conservation laws. This will include a detailed exploration of their role in managing oscillations and nonlinearity.
1. **Recent Extensions to Stochastic Conservation Laws:** The project will review and assess recent advances [1, 2] in extending these weak convergence techniques to stochastic conservation laws, including the stochastic averaging lemmas and the H-measures adapted to stochastic settings.
1. **Investigate Stochastic Conservation Laws with Transport Noise:** A core part of the project will be to investigate whether recent weak convergence results [1, 2] can be extended to stochastic conservation laws with transport noise terms [3]. This involves exploring how different types of stochastic perturbations affect weak convergence and developing appropriate methods for

dealing with the added complexity of noise.

This project requires a strong background in PDEs, functional analysis, Fourier analysis, and probability theory, particularly in the context of stochastic PDEs.

References:

- [1] K. H. Karlsen, M. Kunzinger, and D. Mitrovic. *A dynamic capillarity equation with stochastic forcing on manifolds: a singular limit problem*. Trans. Amer. Math. Soc., 377(1):85–166, 2024
- [2] M. Erceg, K. H. Karlsen, and D. Mitrovic. *Velocity averaging under minimal conditions for deterministic and stochastic kinetic equations with irregular drift*. Submitted, 2023.
- [3] P.-L. Lions, B. Perthame, and P. E. Souganidis. *Stochastic averaging lemmas for kinetic equations*. 2013.