

Numerical Methods for Stochastic Perturbations of the Shallow Water Camassa-Holm Equation

Introduction:

The Camassa-Holm (CH) equation is a well-known model for shallow water waves, capturing important features such as nonlinear dispersive effects. Its deterministic form has been extensively studied, and various finite difference methods have been developed to ensure stability and consistency for general initial data [1]. However, in real-world applications, shallow water systems are often influenced by random perturbations [2], making it essential to develop robust numerical methods that account for stochasticity.

Main Goal:

This project focuses on developing and analyzing numerical methods for the stochastic version of the Camassa-Holm equation. The primary objectives are to:

1. *Develop Numerical Schemes:* Create finite difference schemes that are designed to handle the stochastic perturbations in the Camassa-Holm equation. This will involve adapting existing schemes [1] to incorporate noise terms that model stochastic influences on wave propagation [3].
2. *Prove Stability:* Prove the stability of these schemes, ensuring that the numerical solutions remain bounded and physically meaningful over time, even in the presence of stochastic forcing.
3. *Establish Consistency:* Ensure that the schemes are consistent with the stochastic perturbations of the CH equation by deriving energy estimates and ensuring that the numerical methods accurately approximate the underlying SPDE.

Possible Extensions:

As an extension, the project may explore the convergence properties of the developed numerical schemes. Building on the frameworks from [1,2], the student can investigate whether the schemes converge to weak solutions of the stochastic Camassa-Holm equation. This may include exploring compactness techniques and energy-based arguments to prove convergence in a stochastic setting [2].

References:

[1] G. M. Coclite, K. H. Karlsen, and N. H. Risebro. A convergent finite

difference scheme for the Camassa-Holm equation with general H^1 initial data. SIAM J. Numer. Anal., 46(3):1554–1579, 2008 .

[2] L. Galimberti, H. Holden, K. H. Karlsen, and P. H. Pang. Global existence of dissipative solutions to the Camassa–Holm equation with transport noise. J. Differential Equations, 387:1–103, 2024.

[3] U. S. Fjordholm, K. H. Karlsen, and P. H. Pang. Convergent finite difference schemes for stochastic transport equations. Submitted, 2023.