

Discontinuous Galerkin methods for stochastic Degasperis-Procesi equations

Introduction:

The Degasperis-Procesi (DP) equation is a well-known integrable partial differential equation that models shallow water wave phenomena and supports both peakon and shockpeakon solutions. Recent developments have extended the deterministic DP equation to include stochastic perturbations, leading to the stochastic Degasperis-Procesi (SDP) equation. These stochastic versions provide a rich framework for understanding wave dynamics under random influences, such as noise, which can model real-world uncertainties in fluid systems. The multi-shockpeakon phenomenon in the SDP equation introduces additional complexities in terms of stability and solution behavior. In this context, discontinuous Galerkin (DG) methods have emerged as a powerful numerical tool to address these challenges, offering high accuracy in capturing discontinuous solutions while preserving important physical invariants like mass and energy.

Main Goal:

The goal of this project is to develop and analyze discontinuous Galerkin methods for the stochastic Degasperis-Procesi equation. The project will consist of the following key objectives:

1. **Algorithm Development:** Design a novel discontinuous Galerkin scheme suitable for solving the stochastic Degasperis-Procesi equation. The method should incorporate appropriate numerical fluxes to handle discontinuities, particularly shockpeakon solutions, and capture the stochastic dynamics of the system accurately.
1. **Stability Proofs:** Prove the stability of the proposed method, ensuring that the numerical scheme conserves critical quantities such as mass and energy over time, even in the presence of stochastic terms.
1. **Numerical Experiments:** Conduct numerical simulations to validate the proposed algorithms. The experiments will focus on testing the accuracy, stability, and conservation properties of the method. Special attention will be given to scenarios involving multi-shockpeakon interactions under stochastic perturbations.

References:

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