

Well-Posedness and Time-Discretization of Stochastic Bidomain Models for Cardiac Electrophysiology

Introduction:

Stochastic models in cardiac electrophysiology introduce randomness to better capture the unpredictable behavior of electrical propagation in cardiac tissue, such as arrhythmogenesis. The bidomain model, which describes the electrical activity of the heart, involves a system of stochastic reaction-diffusion equations (SPDEs) coupled with stochastic differential equations (SDEs) to represent the stochastic nature of electrical signals [1]. This project aims to explore these stochastic extensions to better model uncertainty in cardiac activity.

Main Goal:

This project will focus on proving the well-posedness of weak solutions for the stochastic bidomain model, consisting of SPDEs and SDEs that describe electrical and ionic dynamics in the heart. The student will analyze the convergence of time-discretization methods, particularly implicit Euler schemes [3], ensuring that the numerical methods remain stable and accurate in capturing the stochastic dynamics. Additionally, as a potential extension, the project may explore the incorporation of fractional diffusion to better model multiscale effects in heterogeneous cardiac tissue.

References:

[1] M. Bendahmane and K. H. Karlsen. Stochastically forced cardiac bidomain model. *Stoch. Processes Appl.*, 129:5312—5363, 2019.

[2] M. Bendahmane, K. H. Karlsen, and F. Mroue. Stochastic electromechanical bidomain model. *Nonlinearity*, 37 075023, 2024.

[3] [1] N. Glatt-Holtz, R. Temam, and C. Wang. Time discrete approximation of weak solutions to stochastic equations of geophysical fluid dynamics and applications. *Chin. Ann. Math. Ser. B*, 38(2):425—472, 2017.