DIRICHLET SERIES AND OPERATOR THEORY: SCHEDULE AND BOOK OF ABSTRACTS

This two and a half day workshop is organized on June 16–18, 2021. Each session begins at quarter past the stated hour and last for 45 minutes. All times are with respect to local Norwegian time (CEST). The speaker names are clickable[1] and should take you to the title and abstract below.

	Wednesday	Thursday	Friday
09-10	Perfekt	Pushnitski	Saksman
10-11	Perfekt	Pushnitski	Saksman
11-12	Bayart	Miheisi	Ortega-Cerdà
12-13 13-14	*Lunch*	*Lunch*	*Lunch*
14-15	Olsen	Gerspach	
15-16	Schoolmann	Heap	

The physical component of the workshop takes place at the Blindern campus of the University of Oslo, while the digital component of takes place on Zoom.

Date: June 2, 2021.

[1]This claim is false for the printed version of this document.

Composition operators on the Hardy space of Dirichlet series

Speaker. Karl-Mikael Perfekt (University of Reading & NTNU).

Abstract. The presentation will begin with an abridged summary of the theory of composition operators on the Hardy spaces $H^p(\mathbb{D})$ of the unit disc $\mathbb{D}$. I will then introduce the Hardy spaces of Dirichlet series $\mathscr{H}^p$, and review the characterization of the composition operators on $\mathscr{H}^2$ (due to J. Gordon and H. Hedenmalm). The final part of the talk will focus on identifying the compact composition operators on $\mathscr{H}^2$. In particular, I will discuss a certain weighted mean counting function for Dirichlet series, which plays a similar role to the Nevanlinna counting function associated with the unit disc.

Introduction to multiplicative Toeplitz and Hankel operators

Speaker. Alexander Pushnitski (King's College London).

Abstract. In the first part, I will recall the definitions of the classical (additive) Toeplitz and Hankel operators and review some of their properties (boundedness, compactness, essential spectrum, etc). In the second part, I will discuss three representations for multiplicative Toeplitz and Hankel operators: as infinite matrices, as operators in the Hardy space of Dirichlet series and as operators in the Hardy space on the infinite multi-torus. I will also mention some open problems connected to these operators.

Some recent developments on bounded mean oscillation for Dirichlet series and related topics

Speaker. Eero Saksman (University of Helsinki).

Abstract. We will discuss a recent solution to a problem by Marzo and Seip (2011) concerning finding the Lebsgue spaces $L^p(\mathbb{T}^\infty)$ and $L^q(\mathbb{T}^\infty)$, where $\mathbb{T}^\infty$ is the infinite-dimensional torus), such that the Riesz projection on the infinite-dimensional torus $\mathbb{T}^\infty$ acts contractively between them. In addition, we will describe some results concerning BMO-spaces of Dirichlet series.

The talk is based on joint work with Sergei Konyagin, Herve Queffélec, and Kristian Seip.

On the topological structure of the set of composition operators on a Hilbert space of Dirichlet series

Speaker. Frédéric Bayart (University Clermont Auvergne).

Abstract. In this talk we will discuss the topological structure of the set of composition operators acting on the Hilbert space of Dirichlet series. In particular, we will focus on the two following questions: let C_φ and C_ψ be two composition operators. When do C_φ and C_ψ belong to the same component of the space of composition operators? When is $C_\varphi - C_\psi$ compact?

Pseudomoments of the Riemann zeta function

Speaker. Maxim Gerspach (KTH).

Abstract. The pseudomoments of the Riemann zeta function are the moments of the truncated Dirichlet series associated to zeta on the critical line. I will present some new bounds on these quantities and certain generalisations thereof, partly found in joint work with Y. Lamzouri. I will moreover give a heuristic explanation as to how these bounds arise, using ideas of a probabilistic flavour, in particular the concept of random multiplicative functions.

Random multiplicative functions and a model for the Riemann zeta function

Speaker. Winston Heap (Shandong University).

Abstract. We look at a weighted sum of random multiplicative functions and view this as a model for the Riemann zeta function. We investigate various aspects including its high moments, value distribution and maxima. Some of these are in accordance with the expected behaviour of the zeta function whereas some aspects remain undetermined and are potentially not. We also give some open questions regarding almost sure bounds for this sum. This is joint work with Marco Aymone and Jing Zhao.

Restriction theorems for multiplicative Hankel operators

Speaker. Nazar Miheisi (King's College London).

Abstract. A multiplicative Hankel matrix, also called a *Helson* matrix, is an infinite matrix of the form $\{\alpha(jk)\}_{j,k\geq 1}$. Their 'continuous' analogues are integral operators on $L^2(1,\infty)$ with integral kernels of the form $k(xy)$ — we refer to these as *integral Helson operators*. It is easily seen that, by an exponential change of variables, each integral Helson operator is unitarily equivalent to a classical Hankel operator and so this class is well understood.

In this talk we will consider a map from integral Helson operators to Helson matrices given by restricting the integral kernel onto integers (when this makes sense). We will discuss the boundedness of this operation with respect to the Schatten norms and operator norm. If time permits we will also describe an application to the problem of obtaining explicit spectral asymptotics for a family of compact modifcations of the multiplicative Hilbert matrix.

This is joint work with Alexander Pushnitski.

On an operator theoretic proof for the Prime Number Theorem

Speaker. Jan-Fredrik Olsen (Lund University).

Abstract. In 1931, Ikehara, under the supervision of Wiener, obtained a tauberian theorem on the Laplace transform of a certain class of functions that has the prime number theorem (PNT) as an almost immediate consequence. In this talk, we establish an operator theoretic analogue of Ikehara's theorem and use it to deduce the PNT, only using techniques that should be accessible to anyone with a basic knowledge of operator theory. We also discuss connections to the theory of function spaces of Dirichlet series.

Idempotent Fourier multipliers acting contractively on Hardy spaces

Speaker. Joaquim Ortega-Cerdà (University of Barcelona).

Abstract. I will present a joint work with Ole Fredrik Brevig and Kristian Seip. We describe the idempotent Fourier multipliers that act contractively on H^p spaces of the d-dimensional torus $\mathbb{T}^d$ for $d \geq 1$ and $1 \leq p \leq \infty$. When p is not an even integer, such multipliers are just restrictions of contractive idempotent multipliers on L^p spaces, which in turn can be described by suitably combining results of Rudin and Andô. When $p = 2(n+1)$, with n a positive integer, contractivity depends in an interesting geometric way on n, d, and the dimension of the set of frequencies associated with the multiplier. Our results allow us to construct a linear operator that is densely defined on $H^p(\mathbb{T}^\infty)$ for every $1 \leq p \leq \infty$ and that extends to a bounded operator if and only if $p = 2, 4, \ldots, 2(n+1)$.

Hardy spaces of general Dirichlet series and their maximal inequalities

Speaker. Ingo Schoolmann (University of Oldenburg).

Abstract. The by now established intimate link between ordinary Dirichlet series $\sum a_n n^{-s}$, holomorphic functions in infinitely many variables and functions from Hardy spaces on the infinitely dimensional torus lies at the very heart of the remarkable renaissance of the ordinary theory of Dirichlet series in the past 25 years. Their elaborated study eventually arises the question, whether a reasonable structure theory is possible on general Dirichlet series $\sum a_n e^{-\lambda_n s}$ naturally containing the 'ordinary' theory.

The jump to arbitrary frequencies $\lambda = (\lambda_n)$ reveals challenging difficulties and this talk gives an overview of the recent research efforts to overcome them. As one of the main results we will see that the transfer of important cornerstones of the ordinary theory to the framework of general Dirichlet series surprisingly produces the same class of frequencies, where the key argument is provided by the prominent Carleson-Hunt theorem and requires the framework of Hardy spaces of general Dirichlet series, which are modeled along certain compact groups called Dirichlet groups.