Shifts of finite type, Cuntz–Krieger algebras and their algebraic analogues, groupoids, and inverse semigroups

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Theorem 1

Let A and B be finite square $\{0,1\}$-matrices with no zero rows and no zero columns, and let R be an indecomposable reduced commutative ring with unit. The following are equivalent.

1. The one-sided shift spaces (X_A, σ_A) and (X_B, σ_B) are continuously orbit equivalent.
2. The groupoids G_A and G_B are isomorphic as topological groupoids.
3. The inverse semigroups S_A and S_B are isomorphic.
4. The Cuntz–Krieger algebras $\mathcal{O}_A$ and $\mathcal{O}_B$ are isomorphic by a diagonal-preserving isomorphism.
5. The Steinberg algebras RG_A and RG_B are isomorphic by a diagonal-preserving isomorphism.

Theorem 2

Let A and B be finite square $\{0,1\}$-matrices with no zero rows and no zero columns, and let R be an indecomposable commutative ring with unit. The following are equivalent.

1. The one-sided shift spaces (X_A, σ_A) and (X_B, σ_B) are eventually conjugate.
2. There is an isomorphism $\phi : G_A \to G_B$ such that $c_A = c_B \circ \phi$.
3. There is an isomorphism $\psi : S_A^e \to S_B^e$ such that $e_A = e_B \circ \psi$.
4. The Cuntz–Krieger algebras $\mathcal{O}_A$ and $\mathcal{O}_B$ are isomorphic by a diagonal-preserving isomorphism that intertwines the gauge actions λ_t^A and λ_t^B.
5. The Steinberg algebras RG_A and RG_B are isomorphic by a graded diagonal-preserving isomorphism.

Theorem 3

Let A and B be finite square $\{0,1\}$-matrices with no zero rows and no zero columns, and let R be an indecomposable commutative ring with unit. The following are equivalent.

1. The one-sided shift spaces (X_A, σ_A) and (X_B, σ_B) are conjugate.
2. There is an isomorphism $\phi : G_A \to G_B$ such that $\phi \circ \epsilon_A = \epsilon_B \circ \phi$.
3. There is an isomorphism $\psi : S_A \to S_B$ such that $\psi(S_A^r) = S_B^r$ and $\psi \circ r_A = r_B \circ \psi$.
4. The Cuntz–Krieger algebras $\mathcal{O}_A$ and $\mathcal{O}_B$ are isomorphic by a diagonal-preserving isomorphism that intertwines the positive maps τ_A and τ_B.
5. The Steinberg algebras RG_A and RG_B are isomorphic by a diagonal-preserving isomorphism that intertwines κ_A and κ_B.

Theorem 4

Let A and B be finite square $\{0, 1\}$-matrices with no zero rows and no zero columns, and let R be an indecomposable reduced commutative ring with unit. The following are equivalent.

1. The two-sided shift spaces $(\overline{X}_A, \overline{\sigma}_A)$ and $(\overline{X}_B, \overline{\sigma}_B)$ are flow-equivalent.
2. The groupoids $G_A \times \mathcal{R}$ and $G_B \times \mathcal{R}$ are isomorphic.
3. The inverse semigroups $\tilde{S}_A$ and $\tilde{S}_B$ are isomorphic.
4. The stabilised Cuntz–Krieger algebras $\mathcal{O}_A \otimes \mathcal{K}$ and $\mathcal{O}_B \otimes \mathcal{K}$ are isomorphic by a diagonal-preserving isomorphism.
5. The algebras $RG_A \otimes M_\infty(R)$ and $RG_B \otimes M_\infty(R)$ are isomorphic by a diagonal-preserving isomorphism.

Theorem 5

Let A and B be finite square $\{0,1\}$-matrices with no zero rows and no zero columns, and let R be an indecomposable commutative ring with unit. The following are equivalent.

1. The two-sided shift spaces $(\overline{X}_A, \overline{\sigma}_A)$ and $(\overline{X}_B, \overline{\sigma}_B)$ are conjugate.
2. There is an isomorphism $\phi : G_A \times \mathcal{R} \to G_B \times \mathcal{R}$ such that $\tilde{c}_A = \tilde{c}_B \circ \phi$.
3. There is an isomorphism $\psi : \tilde{S}^e_A \to \tilde{S}^e_B$ such that $\tilde{e}_A = \tilde{e}_B \circ \psi$.
4. The stabilised Cuntz–Krieger algebras $\mathcal{O}_A \otimes \mathcal{K}$ and $\mathcal{O}_B \otimes \mathcal{K}$ are isomorphic by a diagonal-preserving isomorphism that intertwines the actions $\lambda^A_t \otimes \mathrm{id}_{\mathcal{K}}$ and $\lambda^B_t \otimes \mathrm{id}_{\mathcal{K}}$.
5. The algebras $RG_A \otimes M_\infty(R)$ and $RG_B \otimes M_\infty(R)$ are isomorphic by a graded diagonal-preserving isomorphism.