

# C\*-algebras and geometry of groups and semigroups

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## **Adrien le Boudec (ENS Lyon)**

### **Uniformly recurrent subgroups and applications**

The talk will focus on the group theoretic notions of confined subgroups and uniformly recurrent subgroups. We will try to highlight the interest of these objects, and their connections with certain problems in group theory, and about group algebras and group  $C^*$ -algebras. Finally we will mention results about understanding confined subgroups and URSs for groups arising with a sufficiently rich and sufficiently non-free action by homeomorphisms on a topological space; and applications of these results to the study of the topological dynamics of these groups (joint work with Nicolás Matte Bon).

## **Chris Bruce (Queen Mary University of London University of Glasgow)**

### **Characterising algebraic number fields via Cartan pairs**

Nearly a decade ago, Li and Lück proved that given any two rings of algebraic integers, the associated ring  $C^*$ -algebras are always isomorphic. I will present a result showing that—in stark contrast with the result by Li and Lück—given any two such rings, there is a Cartan-preserving isomorphism between the associated ring  $C^*$ -algebras if and only if the rings are isomorphic. Thus, the ring  $C^*$ -algebra of a ring of algebraic integers together with its canonical Cartan subalgebra completely characterises the ring. As a consequence of this result, the semigroup  $C^*$ -algebra of the (full)  $ax+b$ -semigroup over a ring of algebraic integers together with its canonical Cartan subalgebra completely characterises the ring. This is joint work with Xin Li (University of Glasgow).

## **Tim de Laat (University of Münster)**

### **Gelfand pairs, spherical functions and exotic group $C^*$ -algebras**

For a non-amenable group  $G$ , there can be many group  $C^*$ -algebras that lie naturally between the universal and the reduced  $C^*$ -algebra of  $G$ . These are called exotic group  $C^*$ -algebras. Let  $G$  be a simple Lie group or an appropriate locally compact group acting on a tree. I will explain how the  $L^p$ -integrability properties of different spherical functions on  $G$  (relative to a maximal compact subgroup) can be used to distinguish between different exotic group  $C^*$ -algebras. This recovers results of Samei and Wiersma. Additionally, I will explain that under certain natural assumptions, the aforementioned exotic group  $C^*$ -algebras are the only ones coming from  $G$ -invariant ideals in the Fourier-Stieltjes algebra of  $G$ . This is based on joint work with Dennis Heinig and Timo Siebenand.

## **Mikael de la Salle (ENS Lyon)**

### **Weak amenability and $\tilde{A}_2$ -geometry**

After the celebrated work of Haagerup on free groups and many later developments, it is now well understood that the different approximation properties of group  $C^*$  (and von Neumann) algebras are very much connected to the geometry of the spaces on which the group acts. I will illustrate this through a new example : groups acting geometrically on  $\tilde{A}_2$  buildings fail to be weakly amenable. In particular, this provides a new geometric proof

of the known fact that lattices in  $SL_3(\mathbb{Q}_p)$  fail to be weakly amenable. This is based on a joint work with Jean Lécureux and Stefan Witzel.

## **Ulrik Enstad (University of Oslo)**

### **On the existence of bases in the orbit of unitary group representations**

In this talk we consider the following problem: Given an irreducible unitary representation  $\pi$  of a locally compact group  $G$  on a Hilbert space  $H$  and a lattice  $\Gamma$  in  $G$ , when can we find an element  $g$  in  $H$  such that  $\{\pi(x)g : x \in \Gamma\}$  is an orthonormal basis for  $H$ ? When  $G$  is the Heisenberg group, this problem has been extensively studied in the field of Gabor analysis, and is fully understood thanks to a result of Bekka.

Under certain assumptions on  $G$  and  $\pi$ , the above problem can be approached using the group von Neumann algebra and the group  $C^*$ -algebra of  $\Gamma$ . We present a new result for projective representations of abelian groups satisfying Kleppner's condition. If time permits, we will also discuss what is known when  $\Gamma$  is a discrete subset with no group structure: In particular, we discuss a density theorem for groups of polynomial growth due to Führ et al.

## **Gabriel Favre (Stockholm University)**

### **An algebraic characterization of the type I property for ample groupoids**

I will highlight some important parts of the non-commutative Stone duality between ample groupoids and boolean inverse semigroups. I will then explain how we exploited this duality to get an algebraic counterpart to van Wyk's topological criterion for the type I property for such groupoids, and give a brief application to discrete inverse semigroups. This is joint work with S. Raum.

## **Waltraud Lederle (UCLouvain)**

### **Conjugacy and dynamics in tree almost automorphism groups**

The almost automorphism group of a regular tree is one of the most important examples in the theory of totally disconnected, locally compact groups. In this talk, we will have a closer look at how its elements act on the tree boundary and how to determine whether two elements are conjugate or not.

This is joint work with Gil Goffer from the Weizmann Institute.

## **Kang Li (KU Leuven)**

### **Ghost projections and expanderish graphs**

Roughly speaking, a ghost operator is often an infinite matrix such that its matrix entries vanish at the infinity. This notion was introduced by Guoliang Yu in the study of the so-called coarse Baum-Connes conjecture. It is a very central topic in coarse geometry and operator algebras with applications to provide counterexamples to the coarse Baum-Connes conjecture, the existence of non-exact groups and the rigidity problem for Roe-type algebras. In this talk, we will visualize a class of ghost projections in terms of expanderish graphs.

## **Mario Klisse (TU Delft)**

### **A dynamical approach to Hecke operator algebras**

(Iwahori) Hecke algebras are deformations of the group algebra of Coxeter groups depending on a deformation parameter. They can be naturally represented on the  $\ell^2$ -space of the corresponding group and thus complete to  $C^*$ -algebras and von Neumann algebras. The aim of this talk is to introduce and discuss certain topological spaces associated with (Cayley graphs of) Coxeter systems which reflect combinatorial and order theoretic properties. These spaces turn out to nicely relate to various other important constructions (such as Gromov's hyperbolic compactification, the Higson compactification and Furstenberg boundaries of Coxeter groups) and are closely related to the Hecke operator algebras of the system. This allows to apply  $C^*$ -dynamical methods to the study of Hecke operator algebras. We will discuss some of the consequences of this connection.

## **Diego Martinez (UC3M)**

### **Schützenberger graphs and property A, or how to see exactness in the reduced $C^*$ -algebra**

In the talk we equip an inverse semigroup with a (not necessarily unique) proper and right invariant metric. For this, we will use the so-called Schützenberger graphs of the inverse semigroup, and study several geometric properties of them. Among these, of particular interest is Yu's property A, which, as we will see, lies in between the amenability of the associated universal groupoid and the exactness of the reduced inverse semigroup  $C^*$ -algebra. Along the way, we also introduce a desirable property for the metric to satisfy, and see how it plays a role in the study. This is joint work with Fernando Lledó.

## **Sam Mutter (Newcastle University)**

### **The K-theory of higher-rank graph algebras, coming from "k-cube groups"**

We define a  $k$ -cube group, which acts freely and transitively on the Cartesian product of  $k$  trees. The quotient of this action on the product of trees defines a  $k$ -dimensional cube complex which induces structure of a higher-rank graph, and hence can be associated a higher-rank graph algebra. These  $C^*$ -algebras are Kirchberg algebras, and as such are completely determined by their K-theory and the order of the identity class in  $K_0$ .

In this talk, we describe the structure of a  $k$ -cube group, and make deductions about K-theory of the corresponding higher-rank graph algebras. These are the first explicit computations of K-theory for an infinite family of  $k$ -rank graphs for  $k > 2$ .

This is joint work with Alina Vdovina and Cristiana Radu.

## **Piotr Nowak (IM PAN)**

### **On property (T) for $\text{Aut}(F_n)$**

The goal of this talk is to present the recent proof that  $\text{Aut}(F_n)$ , the automorphism group of the free group on  $n$  generators, has Kazhdan's property (T) for  $n \geq 5$ . This is joint work with Marek Kaluba and Taka Ozawa ( $n = 5$ ) and with Kaluba and Dawid Kielak ( $n \geq 6$ ). Our proof uses a characterization of property (T) via an algebraic notion of positivity in the group ring, due to Ozawa, and computer assistance in the form of semidefinite programming (i.e. convex optimization over positive definite matrices). As applications we confirm the explanation of the effectiveness of the Product Replacement Algorithm predicted by Lubotzky and Pak, as well as obtain new asymptotically optimal estimates of Kazhdan constant for  $\text{Aut}(F_n)$  and  $\text{SL}_n(\mathbb{Z})$ .

## **Eduard Ortega (NTNU)**

### **$C^*$ -unique groupoids**

The notion of  $C^*$ -unique Banach  $*$ -algebras was introduced in the 80's by Boidol, meaning that a Banach  $*$ -algebra accepts just one  $C^*$ -norm. This concept was then specialized to the case of the Banach group algebras  $L^1(G)$ , saying that a group is  $C^*$ -unique if  $L^1(G)$  is  $C^*$ -unique. An obvious necessary condition is amenability of the group, but turns out not to be sufficient, at least in the locally compact case. Applications of  $C^*$ -uniqueness can be found in measure theory, in the so-called Pompeiu Problem, but also in Gabor analysis, since it was proved by Gröchening and Leinert that  $C^*$ -uniqueness together with symmetry of the Banach algebra implies spectral invariance. In this talk I will talk about  $C^*$ -uniqueness of the Banach  $*$ -algebra  $L^1(G)$  for an étale groupoid  $G$ , and give conditions for when it is  $C^*$ -unique. Unless, the group case (topological) amenability it is no longer a necessary condition, but instead the weak containment property. This is a joint work with A. Austad.

## **Sanaz Pooya (IM PAN)**

### **Higher Kazhdan projections and Baum-Connes conjectures**

The Baum-Connes conjecture, if it holds for a certain group, provides topological tools to compute the K-theory of its reduced group  $C^*$ -algebra. This conjecture has been confirmed for large classes of groups, such as amenable groups, but also for some Kazhdan's property (T) groups. Property (T) and its strengthening are driving forces in the search for potential counterexamples to the conjecture. Having property (T) for a group is characterised by the existence of a certain projection in the universal group  $C^*$ -algebra of the group, known as the Kazhdan projection. It is this projection and its analogues in other completions of the group ring, which obstruct known methods of proof for the Baum-Connes conjecture. In this talk, I will introduce a generalisation of Kazhdan projections. Employing these projections we provide a link between surjectivity of the Baum-Connes map and the  $L^2$ -Betti numbers of the group. A similar relation can be obtained in the context of the coarse Baum-Connes conjecture. This is based on joint work with Kang Li and Piotr Nowak.

## **Eduardo Scarparo (NTNU)**

### **Boundary maps, germs and quasi-regular representations**

In recent years, there have been great advances in understanding when the reduced  $C^*$ -algebra of a group is simple and when it has the unique trace property. A central role in these discoveries has been played by Furstenberg's notion of boundary action (specially the so-called Furstenberg boundary).

We will present applications of the Furstenberg boundary for studying traces and  $C^*$ -simplicity of quasi-regular representations. We will be specially interested in quasi-regular representations coming from Thompson's groups.

We will also outline generalizations of these methods to covariant representations of minimal actions. This is joint work with Mehrdad Kalantar.

## **Tatiana Shulman (Chalmers/University of Gothenburg)**

### **On almost commuting matrices**

It is an old problem to investigate which relations for families of commuting matrices are stable under small perturbations, or in other words, which commutative  $C^*$ -algebras  $C(X)$  are stable. Extending the works of Davidson, Eilers-Loring-Pedersen, Lin, and Voiculescu on almost commuting matrices, we will obtain, for  $X$  of finite covering dimension, a complete characterization of when  $C(X)$  is stable. We also will discuss applications. Joint work with Dominic Enders.

## **Adam Skalski (IM PAN)**

### **On classifying Hecke algebras of right-angled Coxeter groups via K-theory**

Hecke algebras are deformations of group rings of Coxeter groups, arising naturally in the context of representation theory and group actions on buildings. In recent years there has been an increased interest in the study of their operator algebraic counterparts. I will introduce the relevant notions, describe the K-theory of Hecke  $C^*$ -algebras of right-angled Coxeter groups and discuss to what extent it can be used to distinguish the algebras in this class.

Based on joint work with Sven Raum.

## **Alina Vdovina (Newcastle University)**

### **Buildings, C\*-algebras and new higher-dimensional analogues of the Thompson groups**

We present explicit constructions of infinite families of CW-complexes of arbitrary dimension with buildings as the universal covers. These complexes give rise to new families of C\*-algebras, classifiable by their K-theory. The underlying building structure allows explicit computation of the K-theory. We will also present new higher-dimensional generalizations of the Thompson groups, which are usually difficult to distinguish, but the K-theory of C\*-algebras gives new invariants to recognize non-isomorphic groups.

We will also discuss new directions of generalizations to higher dimensions of the work of Vaughan Jones and his collaborators on connections of the Thomson's group and Theoretical Physics.