Automatic Variationally Stable Analysis for Finite Element Computations

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Abstract

We present a new type discontinuous Petrov-Galerkin (DPG) method for finite element (FE) approximations of boundary value problems of second order linear partial differential equations (PDEs) in the spirit of the DPG method introduced by Demkowicz and Gopalakrishnan [2, 3]. The new method, named Automatic Variationally Stable Finite Element (AVS-FE) method, uses a first order system weak representation of the governing PDEs, and distinguishes itself from the DPG method by using continuous function spaces for the trial functions, in an effort to reduce computational cost [1]. Discontinuous function spaces, however, are employed for the test functions and therefore the test functions can be solved locally at the element level by using the DPG philosophy in [2, 3]. Hence, they are optimal in the sense that they guarantee inherently stable FE approximations with best approximation properties in terms of the energy norm. The local contributions of test functions can be numerically solved on each element with high numerical accuracy and do not require the solution of global variational statements.

This is a joint work with Albert Romkes (South Dakota School of Mines & Technology, Rapid City, USA), Victor M. Calo (Curtin University, Perth, Australia), and Maciej Paszynski (AGH University of Science and Technology, Krakow, Poland).

References

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