

Feynman Kac formula for differential operators with path-dependent coefficients

Jocelyne Bion-Nadal

*CNRS - CMAP, Ecole Polytechnique, 91128 Palaiseau Cedex, France
jocelyne.bion-nadal@cmaph.polytechnique.fr*

The Feynman Kac formula establishes a link between a solution of a parabolic second order PDE and probability measures solutions to a martingale problem:

Under regularity conditions there is a unique solution v to the PDE

$$\partial_u v(t, x) + L^{a,b} v(t, x) = 0, \quad v(T, \cdot) = h \text{ with}$$

$$L^{a,b} v(t, x) = \frac{1}{2} \text{Tr}(a(t, x)) D_x^2(v)(t, x) + b(t, x)^* D_x v(t, x).$$

From the Feynman Kac formula $v(t, x) = E_{Q_{t,x}^{a,b}}(h(X_T))$

(X_u) is the canonical process, and $Q_{t,x}^{a,b}$ is the unique solution to the martingale problem for the second order elliptic differential operator $L^{a,b}$ starting from x at time t . The martingale problem for the differential operator $L^{a,b}$ has been introduced and studied by Stroock and Varadhan (1969).

We generalize this result to the case where the coefficients a and b of the differential operator $L^{a,b}$ are path dependent.

The functions a and b are progressive functions defined on $\mathbb{R}_+ \times \Omega$ where Ω is the set of càdlàg paths. This means that for given t , $a(t, \omega)$ and $b(t, \omega)$ can depend on the whole trajectory of ω up to time t .

We construct a progressive function φ solution to the path dependent PDE:

$$\begin{aligned} D_t \varphi(t, \omega) + \frac{1}{2} \text{Tr}(a(t, \omega)) D^2 \varphi(t, \omega) + b(t, \omega)^* D \varphi(t, \omega) &= 0 \\ \varphi(T, \omega) &= h(\omega) \end{aligned}$$

The first step is to define notions of regularity for progressive functions.