
Recent developments on the $\bar{\partial}$ -equation

In memory of Nils Øvrelid

The book of abstracts



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TITLES AND ABSTRACT

Mats Andersson.

Title: Segre and Chern forms associated with singular metrics

Abstract: Let $E \rightarrow X$ be a holomorphic vector bundle. We consider a quite wide class of Hermitian metrics on E with analytic singularities such that, in a reasonable way, one can associate (non-smooth) Segre forms and Chern forms. In particular, they coincide with the usual forms where the metric is smooth and they represent the expected Bott-Chern cohomology classes. We also show that the multiplicities (Lelong numbers) of the forms are integers if the singularities are integral analytic. This is a joint work in progress with Richard Lärkäng.

Bo Berndtsson.

Title: Supercurrents and convex geometry

Abstract: The formalism of superforms and -currents is a way to use methods from Kähler geometry in real geometry. It has been applied to tropical geometry by Lagerberg, to mixed volumes by Larsson and to Riemannian geometry by myself. In this talk I will outline further applications, focusing on the theory of valuations on convex bodies.

Judith Brinkschulte.

Title: L^2 -estimates with applications to stable sets of holomorphic foliations

Abstract: I will discuss estimates for the $\bar{\partial}$ -operator in weighted L^2 -spaces on Kähler manifolds with applications to the study of stable sets of holomorphic foliations. In particular, I will present a proof of Brunella's conjecture about exceptional minimal sets of codimension one holomorphic foliations with ample normal bundle.

Fusheng Deng.

Title: Rigidity and multiplicity of singularities associated to ample vector bundles

Abstract: We show that the irreducible desingularization of an isolated singularity given by the Grauert blow down of a negative vector bundle is unique up to isomorphism, and give a formula for the multiplicity of an isolated singularity given by the Grauert blow down of an exceptional divisor in a complex manifold. The second result can be applied to give exact formula or estimate for the multiplicity of an isolated singularity given by the Grauert blow down of a negative vector bundle. In the special case that the base is a compact Riemann surface and the line bundle is given by the divisor of a single point, this is closely related to the classical results about Weierstrass points of compact Riemann surfaces. This lecture is based on joint work with Yinji Li, Qunhuan Liu, and Xiangyu Zhou.

Franz Forstnerič.

Title: Oka tubes in holomorphic line bundles.

Abstract: In this joint work with Yuta Kusakabe, we show that unit disc bundles in certain ample Hermitian holomorphic line bundles on a compact complex manifold of dimension ≥ 1 are Oka manifolds, while their complementary exterior tubes are Kobayashi hyperbolic. This phenomenon contributes to the heuristic principle that Oka properties are related to metric positivity of complex manifolds.

Håkan Samuelsson Kalm.

Title: On the polynomially convex embedding dimension

Abstract: If M is an arbitrary smooth compact real n -dimensional manifold, what is the smallest integer N such that M can be smoothly embedded into $\mathbb{C}^N$ as a polynomially convex set? This natural question was asked by Izzo and Stout in 2018. A related question is how many smooth functions on M it takes to generate the uniform algebra of all continuous functions on M . An upper bound follows by classical works by Forstnerič and Rosay. This bound was recently improved by Gupta and Shafikov. I will present a joint work in progress with Arosio and Wold to answer the above questions when $n \leq 11$.

Ingo Lieb.

Title: Integral formulae in several complex variables – the beginning.

Abstract: The years 1968–1970 saw the development of a new class of integral formulae in complex analysis – the Ramírez/Henkin integrals. I will describe this development and the subsequent applications to the $\bar{\partial}$ -equation. So it will be a talk not about the present (or the future) of the Cauchy-Riemann equations, but about their past. The past, nevertheless, still determines the present.

Takeo Ohsawa.

Title: Remarks on neighborhoods of Stein submanifolds

Abstract: The following was proved in [D-Oh].

Theorem 1. *There exists a locally hyperconvex Stein domain in $\mathbb{CP}^5$ which is not hyperconvex.*

Here we call a complex manifold M *hyperconvex* if M admits a bounded strictly plurisubharmonic exhaustion function. A domain $\Omega \subset M$ is called *locally hyperconvex* if each boundary point of Ω has a neighborhood U such that $U \cap \Omega$ is hyperconvex.

The proof is based on Siu's theorem [S] which asserts that every closed Stein submanifold of a complex manifold admits a Stein neighborhood basis. A locally hyperconvex neighborhood of a locally closed Stein submanifold of $\mathbb{CP}^5$ is constructed by using the existence of a holomorphic retraction from a neighborhood, which is a consequence of Siu's theorem and a theorem of Docquier-Grauert [D-G].

The following will be proved here by a more direct method.

Theorem 2. *There exists a compact complex submanifold M in $\mathbb{CP}^5$ of dimension 2 and a Stein domain $D \subset M$ with a Levi-flat boundary such that D admits a Stein neighborhood in $\mathbb{CP}^5$ with Hölder continuous boundary.*

A review on the works preceding [S] will be given, too.

REFERENCES

- [D-Oh] Diederich, K. and Ohsawa, T., *On pseudoconvex domains in $\mathbb{P}^n$* , Tokyo J. Math. **21** (1998), no. 2, 353-358.
- [D-G] Docquier, F. and Grauert, H., *Levisches Problem und Rungescher Satz für Teilgebiete Steinscher Mannigfaltigkeiten*, Math. Ann. **140** (1960), 94–123.
- [S] Siu, Y.- T., *Every Stein subvariety admits a Stein neighborhood*, Invent. Math. **38**(1976/77), no.1, 89-100.

Ragnar Sigurdsson.

Title: Polynomials with exponents in compact convex sets and associated weighted extremal functions.

Abstract: The lecture is a report on a joint work in six papers with Benedikt Steinar Magnússon and our PhD students Álfheiður Edda Sigurðardóttir and Bergur Snorrason. I will discuss some fundamental concepts referred to in the title, then approximations and regularity, generalizations of the Siciak-Zakharyuta theorem, generalizations of Siciak's product formula, approximation of holomorphic functions with polynomials with exponents restricted to convex cones, and generalizations of the Bernstein-Walsh-Siciak extension theorem.

Sophia Vassiliadou.

Title: Differential operators of low order on some singular surfaces

Abstract: I will discuss some results describing differential operators of order at most 2 and 3 on some singular surfaces. This is joint work with Rachel Diethorn, Jack Jeffries, Claudia Miller, Nick Packauskas, Josh Pollitz and Hamid Rahmati.

Dmitri Zaitsev.

Title: Global regularity in the $\bar{\partial}$ -Neumann problem and finite type conditions.

Abstract: The celebrated work of Catlin on the global regularity of the $\bar{\partial}$ -Neumann operator for pseudoconvex domains of finite type links local algebraic and analytic geometric invariants through potential theory with estimates for the $\bar{\partial}$ -equation. Yet despite their importance, there seems to be a major lack of understanding of Catlin's techniques, resulting in a notable absence of an alternative proof, exposition or simplification.

The goal of my talk will be to present an alternative proof based on a new notion of a "tower multi-type". The finiteness condition of the tower multi-type is an intrinsic geometric condition that is more general than the finiteness of the regular type, which in turn is more general than the finite type.

Under the latter condition, we obtain a generalized stratification of the boundary into countably many level sets of the tower multi-type with certain convexity properties.

The existence of such stratification implies Catlin's potential-theoretic "Property (P)", which, in turn, is known to imply global regularity via compactness estimate. Notable applications of global regularity include Condition R by Bell and Ligocka and its applications to boundary smoothness of proper holomorphic maps generalizing a celebrated theorem by Fefferman.

Jan Wiegerinck.

Title: Pluripolar hulls and fine holomorphy.

Abstract: The pluripolar hull of a pluripolar set $E \subset \mathbb{C}^n$ is the set

$$E^* = \{z \in \mathbb{C}^n : \text{for all } h \in \text{PSH}(\mathbb{C}^n) \ h|_E = -\infty \Rightarrow h(z) = -\infty\}.$$

Here $\text{PSH}(\mathbb{C}^n)$ stands for the set of plurisubharmonic functions on $\mathbb{C}^n$.

If E is a sufficiently large part of an analytic variety $V \subset \mathbb{C}^n$, then clearly $V \subset E^*$. In case $E \subset \mathbb{C}^2$ is the graph of a holomorphic function f , it has been conjectured that E^* equals the graph of the maximal fine analytic extension of f .

We will discuss these notions and indicate how large pluripolar hulls in examples by Zwonek and by Poletsky and the author occur because of fine holomorphic extension, giving some support for the conjecture.

Jujie Wu.

Title: Density in weighted Bergman spaces and Bergman completeness of Hartogs domains.

Abstract: We study the density of functions which are holomorphic in a neighbourhood of the closure $\overline{\Omega}$ of a bounded non-smooth pseudoconvex domain Ω , in the Bergman space $H^2(\Omega, \varphi)$ with a plurisubharmonic weight φ . As an application, we show that the Hartogs domain

$$\Omega_\alpha := \{(z, w) \in D \times \mathbb{C} : |w| < \delta^{\alpha_D}(z)\}, \alpha > 0,$$

where $D \subset \subset \mathbb{C}$ and δ_D denotes the boundary distance, is Bergman complete if and only if every boundary point of D is non-isolated. This work is joint with Bo-Yong Chen and John Erik Fornæss. If time permits, I will give a counterexample which shows that the Mergelyan property does not hold for the pseudoconvex domain D , where D is the graph of a holomorphic motion of the unit disc and D has Stein neighborhood basis.