

**A New Look at Calculus:
How an Old Idea Naturally Leads from Simple Algebra
to the Heart of Analysis**

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We discuss a novel approach to calculus and analysis that builds upon an old idea of René Descartes. We begin with simple algebra to solve the tangent problem for all algebraic curves without using any limits. By adding an elementary estimate we are naturally led to the idea of continuity and, more generally, of limits. In particular, one recognizes that the algebraic derivative can also be captured by a *non-algebraic* approximation process. This opens the door to handling transcendental functions such as $E_2(x) = 2^x$, where the need for new analysis concepts becomes clearly visible. Furthermore, by combining the approximation process with the algebraic derivative, we are naturally led to the traditional concept of differentiability in a *non-traditional* version that was introduced by Constantin Carathéodory over 60 years ago, and whose advantages should be recognized more widely. We hope that this approach will stimulate discussions about alternatives to the standard introduction to calculus.