On subrecursive representability of irrational numbers

Lars Kristiansen

Department of Mathematics, University of Oslo

Irrational numbers are infinite objects. We cannot write them up. When we compute with irrationals, we need to represent them by some kind of program (procedure).

We may represent a real number by a (fast converging) Cauchy sequence. We may represent a real number by its Dedekind cut. We may represent a real number by a function that yields the digits of the base-10 expansion of the number one by one. There are many other options, . . . we may e.g. represent real numbers by continued fractions.

Does it matter how we represent real numbers?

If we assume full Turing computability, it does not matter. It does not matter in the sense that we end up with the same class of computable real numbers no matter which (reasonable) representation we chose.

If we do not assume full Turing computability, it does matter. The class of real numbers available depends on the representation if we work with any reasonable notion of computability that is not equivalent to full Turing computability, e.g., primitive recursive computability or polynomial-time computability. Moreover, we get a rather unexpected and surprising situation where we e.g. can expand more numbers in base 2 than in base 10.

I will discuss some theorems of Specker [4], Mostowski [3] and myself [1, 2]. The talk will not be very technical and should be easy to follow.

References

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4. Specker, E: *Nicht Konstruktiv Beweisbare Satze Der Analysis.* The Journal of Symbolic Logic **14**(3) (1949), 145–158.