

Thm (Borel-Ho-Serre-Vogt-W)

Let X be sm, proj, "cellular" scheme
over a perfect field K . Let $F: X \rightarrow X$

Then

$$d \log Z_{X,T}^A = \sum_{r=0}^n -(-1)^r \frac{d}{dt} \log P_r(T)$$

$$P_r(T) = \det(1 - T f | C_r^{\text{cell}}(X))$$

where $C_r^{\text{cell}}(X)$ denotes the
cellular complex of
Morel-Sawant

Morel-Sawant Cellular Homology

$X \in \text{Sm}_K$ is cellular if

$$\emptyset = \Omega_0 \subsetneq \Omega_1 \subsetneq \dots \subsetneq \Omega_n = X$$

$\Omega_i - \Omega_{i-1}$ smooth, affine, cohomologically trivial, and

eg opens A^1 ,
Spec L , KSL sep

"strict" products $(A^1)^{\times n}$
and products

ex G split reductive group

G/B flag variety

$\mathbb{P}^n$

Quadrics in $\mathbb{P}^3$

$\{P^1 \times P^1, P^1\}$
Res P^1

Def (Morel-Sawant)
Ex $C_r^{\text{cell}}(X)$
Thm (Morel-Sawant)
with X, Y
chain http

$C_r^{\text{cell}}(X)$
TAKE
SM-100
 $C_r^{\text{cell}}(X)$
is sm
 $X \in \text{Sm}_K$
 $d \log Z_{X,T}^A$

where $C^{\text{cell}}(X)$ denotes the cellular complex of Morel-Sawant

Def (Morel-Sawant) $C^{\text{cell}} := (H^{\text{cell}}(\mathcal{A}_k/\mathcal{A}_k) \mathcal{A})$

Ex. $C^{\text{cell}}(\mathbb{P}^1) = \underline{k}^{\text{an}}[1]$

Thm (Morel-Sawant) Let $F: X \rightarrow Y$ be in Sm_k with X, Y cellular. Then there is a canonical chain htpy class $C^{\text{cell}}(F): C^{\text{cell}}(X) \rightarrow C^{\text{cell}}(Y)$

Proof: Since cellularity from $H^{\text{cell}}(k) \geq H^{\text{cell}}(k)$ is a topology on Sm_k of cellular spaces. $H^{\text{cell}}(k) = \text{study of } \mathbb{A}^1\text{-homotopy}$

$C^{\text{cell}}: H^{\text{cell}}(k) \rightarrow [X/\mathcal{A}_k(k)]$ Spivak model

Take Spanier-Whitehead categories

$C^{\text{SW-cell}}: H^{\text{cell}}_{\text{SW}}(k) := H^{\text{cell}}(k)[\mathbb{P}^1] \rightarrow D^{\text{SW}}(\mathcal{A}_k(k)) = D(\mathcal{A}_k(k)) [C^{\text{cell}}(\mathbb{P}^1)]$ is symmetric monoidal

X Sm, pres cellular, dualizable in $H^{\text{cell}}_{\text{SW}}$

$$\begin{aligned} H^{\text{cell}}_{\text{SW}}(X) &= \sum_{n \geq 0} \tau(F^n) T^n = \sum_{n \geq 0} C^{\text{cell}}(\tau(F^n)) T^n = \sum_{n \geq 0} \tau(C^{\text{cell}}(F^n)) T^n \\ &= \sum_{n \geq 0} \sum_{i \geq 0} \tau(C^{\text{cell}}(F^n)) T^n = \sum_{i \geq 0} \tau(C^{\text{cell}}(F^i)) T^i \end{aligned}$$

ex G split reductive group

G/B flag variety

$\mathbb{P}^n$

Quotients in $\mathbb{P}^3$

$\begin{cases} \mathbb{P}^1 \times \mathbb{P}^1 \\ \mathbb{P}^2/F_1 \end{cases}$

dim:

Fin sep

Then $\text{Tr}(F) = \sum_{x \in X^F} \text{Tr}_{K(x)/K} \langle \det(1-dF) \rangle$ $2106 T^2 + 28,800 T^3 + 371,538 T^4$
 $+ (4822739 + (u)) T^5$

$\Rightarrow \text{Tr}(\text{Frob}_{X/\mathbb{F}_q}^m) = \sum_{i|m} \alpha(i) \text{Tr}_{\mathbb{F}_{q^m}/\mathbb{F}_q} \langle i \rangle$
 $\alpha(i) = \# \text{ points of } X$
 with residue field $\mathbb{F}_{q^i}$

rank q
 sign $\begin{cases} 0 & q \text{ even} \\ 1 & q \text{ odd} \end{cases}$
 $q_1 = \langle 1 \rangle + \langle -1 \rangle + \langle 1 \rangle + \langle -1 \rangle$

$d \log \hat{P}_{X/P'}(T) = \frac{d}{dT} \log \frac{1}{(1-T)(1-q_1^2 T)} + \langle -1 \rangle \frac{d}{dT} \log \frac{1}{(1-q_1 T)}$ $\text{rank}(K) = 5 > 5$

$d \log_{\text{Res}_{\mathbb{F}_q/\mathbb{F}_q}}^{A'} P'(T) = \frac{d}{dT} \log \frac{1}{(1-T)(1-q_1^2 T)} + \langle -u \rangle \frac{d}{dT} \log \frac{1}{1-q_1 T} + \langle -1 \rangle \frac{d}{dT} \log \frac{1}{1+q_1 T}$
 $u \in \mathbb{F}_q^* / (\mathbb{F}_q^*)^2$ non-trivial $x = 2u \frac{1}{q_1} \chi(\mathbb{F}_q) \frac{1}{q_1} = 5^2$

$d \log_{P^n}^{A'}(T) = \frac{d}{dT} \log \prod_{i \text{ even}} \frac{1}{(1-q_i^2 T)} + \langle -1 \rangle \frac{d}{dT} \log \prod_{i \text{ odd}} \frac{1}{(1-q_i T)}$ $(\text{rank}_{K/P'} P')/(\mathbb{F}_q) = 5^2$

$d \log_{G(1,3)}^{A'}(T) = \frac{d}{dT} \log \frac{1}{(1-T)(1-q_1 T)(1-q_1^4 T)} + \langle -1 \rangle \frac{d}{dT} \log \frac{1}{(1-q_1 T)(1-q_1^2 T)}$