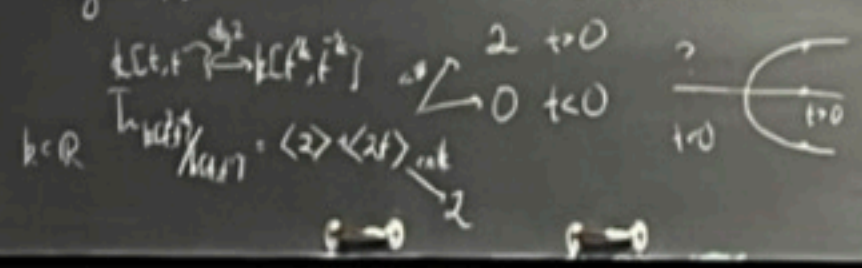




$\underline{E}_2$ X smooth proj $/k \rightsquigarrow \chi(X/k) \in GW(k)$

$\begin{matrix} \downarrow \text{is} \\ \mathbb{C} \rightarrow \mathbb{R} \end{matrix}$ $\begin{matrix} / \text{rk} & \text{sing} & / \text{nd} \\ \chi^{\text{top}}(X(k)) & \chi^{\text{top}}(X(\mathbb{C})) \end{matrix}$

$\deg \rightsquigarrow T_n$



Characteristic rings $\rightarrow \sum H^*, K_0, \Omega^*$

Quadratic $\rightarrow \tilde{CH}^*$ Chow-Witt sp $\rightarrow H^*(-, W)$ $W = GW/\mathbb{H}_1$

Main topic: Localization methods in quad. geom.

§2 Eigenvariet coh. & Borel-Moore homology

$G \subset GL_n$ $G \subset GL_n$ $E_m G \subset M_{n \times (nm)}^{\mathbb{Q}G}$ $\leftarrow$ matrices $\text{rk} = n$ $\leftarrow$ all Odom.

$X \in \text{Sch}_k^G \rightsquigarrow X \times^G EG := \text{colim}_m G \backslash X \times E_m G \subset \text{Spec}(k) \times \text{Proj}(\text{Sym}_k^* k[x_1, \dots, x_n])$

$X \times^G EG \downarrow BG = \text{Spec } k \times^G EG$

Def $E_G^{a,b}(X) = E^{a,b}(X \times^G EG)$ "quasicoherent coh."

$SH(k) \ni E \rightarrow Y \rightarrow E^{a,b}(Y)$ Borel-Moore homology Assume $X \in \text{Sch}_k^G$ $\rightarrow Y \in \text{Sch}_k^G$ $\deg Y = d$ $\left[\begin{matrix} E CH_G^*(X) \\ = HZ_G^{2d}(X) \end{matrix} \right]$

$$E_{a,b,G}^{BM}(X) = E_{G, X \in G}^{a,b}(Y \in EG)$$

$$E = H\mathbb{Z} \quad H\mathbb{Z}_{\text{tors}}(X) = CH_n^G(X)$$

Interpret $E = EM(W_*)$
 $E^{a,b}(X) = H^{a-b}(X, W)$

§3 Towards localization

$$T = G_m^S \quad X = \text{Sch}_k^T \quad E = \begin{pmatrix} H\mathbb{Z} \\ CH^* \end{pmatrix}$$

1. Atiyah-Bott localization (Eckmann-Grove)
 $CH^*(BT) = \mathbb{Z}[x_1, x_2, \dots]$ $\begin{matrix} \partial(x_i) \\ \downarrow \\ BT \end{matrix}$ $\begin{matrix} x_i \in T \\ \downarrow \\ x_i \in T_i \end{matrix}$
 $x_i = c_i(\partial(x_i)) = c_i^T(x_i)$

Thm $i: \bar{X}^T \hookrightarrow X$ Then $\exists P \in CH^{*+1}(BT)$ such that

$$i_*: CH_n^T(\bar{X}^T) \rightarrow CH_n^T(X) \quad CH_n^T(X) = CH_n(\bar{X}^T) \oplus CH_n^*(BT)$$

is an iso after modding $\cdot P$

Idea of proof $\bar{X}^T \hookrightarrow X \hookrightarrow X \oplus \bar{X}^T = U$ Then $H_*^T(U; \mathbb{Q}/P) = 0$

Sketch: let O be a homog space in T with $\dim O = 0$

$$O = T/T_0 \quad T_0 \in T \text{ paracompact} \Rightarrow \exists X, T = G_m \text{ with } T_0 \subset \text{lin } X$$

$$\begin{matrix} O \in T & \xrightarrow{\partial(x)} & \partial(x) \in \partial \\ \downarrow \pi & & \downarrow \\ BT & \xleftarrow{\partial(x)} & \partial(x) \end{matrix}$$

$\partial(x) = 0$ in $CH_T^*(O) \Rightarrow H_*^T(O; \mathbb{Q}/P) = 0$
 $\exists$ only finitely many orbit types in $X \Rightarrow P = \prod_i (1 - \partial(x_i))$

Cor Bott periodicity thm: Suppose X is smooth $\Rightarrow \bar{X}^T$ is smooth

$$\bar{X}^T = \coprod_j \bar{X}_j^T \hookrightarrow X \quad N_j = \text{Normal bundle } \bar{X}_j^T \text{ in } X \quad \text{Take } x \in CH_T^*(X)$$

let $y_j = i_j^* x$ Then $x = \sum_j i_{j*} \left(\frac{y_j}{e^T(N_j)} \right)$ afterwards $P, \pi, \dots$

A logarithmic HKR, Hodge and residue sequence for (log) HH
(with T. Linderoth, D. Park, DA Østhus)

§1 Introduction

$R \in \text{CRing}, A \in \text{Algebra}$
Thm. $\underline{\text{HH}}(A/R) \cong \underline{\text{HH}}_*(A/R)$ smooth $\underline{\text{HH}}_*(A/R) \cong \text{HH}_*(A/R)$
is a graded ring

$$\text{HH}(A/R) = A \oplus A$$

Following Quillen, the proof goes in 2 steps

- Show HKR holds for free algebras
- Derive the spectral sequence for AQ spectral sequence.

$$E_{2,p,q}^{\text{log}} = \pi_p(\wedge^q L_{A/R}) \Rightarrow \text{HH}_*(A/R)$$

$\nexists A/R$ modn. $L_{A/R} = \Sigma_{\mathbb{A}^1} \text{fol} \rightarrow \text{log algebra}$

We like HH (and variants) because of its relat. with K-theory

R_{log} unlike K-theory, HH does not satisfy localization
(Rognes, Hennrich, Madsen) you need to consider log structures

Ex. $\nexists A$ is DVR, then there is a cofiber seq. $R = A/\pi$

$$\text{HH}(A) \rightarrow \text{log HH}(A, A/\text{fol}) \rightarrow \text{HH}(R)[1]$$

$$K(A) \rightarrow K(F) \rightarrow \mathbb{Z}K(R)$$

§2 log maps

Def: A (divisor)