

joint with Anand Sawant.

- k a field (perfect) $S_{m,k}$ smooth scheme / k .
 - $\mathcal{A}_k(k) =$ sheaves of abelian groups on $(S_{m,k})_{Nis}$
 - $\mathcal{A}_{A'}^+(k) =$ strictly A' -invariant sheaves of ab groups on $(S_{m,k})_{Nis}$
- $$\stackrel{+}{M} \quad (=) \quad \forall X \in S_{m,k}, \forall i \geq 0 \quad H^i(X, n) \cong H^i(X \times A', n)$$

$$\forall X \in \text{Sm}_k, \quad H_i^{A'}(X) = (H_i(C_{X/A'}^{A'}(X))) \in \mathcal{A}b(k)$$

$H_0^{A'}(X)$ = the free strictly A' -univ. sheaf on X

$$n \geq 1 \quad H_0^{A'}(\mathbb{G}_m^{\wedge n}) = \underline{t}_n^{nw} \left(\begin{array}{c} \text{sections} \\ \text{over } F \end{array} \text{Tens}(F^{\times}, \eta) \right) / \begin{array}{l} [u] \in \text{deg } 1 \\ [uv] = [u] + [v] + \eta(u)[v] \\ [u] - [1-u] = 0 \\ \eta(u) = \eta(u) \\ h.\eta = \eta^2 + \eta\eta = 0 \end{array}$$

$$H_1^{A'}(\Sigma \mathbb{G}) = H_1^{A'}(\Pi) \cong \underline{t}_1^{nw}$$

$$n \geq 2 \quad H_{n-1}^{A'}(\Sigma^{\wedge n} \mathbb{G}^{\wedge n}) = H_{n-1}^{A'}(IA^{\wedge n}_{-1}).$$

Let $X \in \mathcal{S}_{\text{aff}}$

A cellular structure on X .
increasing sequence of op.

$$\Omega_0/\Omega_1 \cong \mathbb{A}^1(k) \quad \Omega_i/\Omega_{i+1} \cong \mathbb{A}^1(k) \cong T^{i+1}(k)$$

$$\Omega_0 \supset \Omega_1 \supset \dots \supset \Omega_n = X$$

where

each irreducible component are product of $\mathbb{A}^1$ & $\mathbb{A}^1$

Ex

G is split reductive group / k , G/p

P^* is cellular with $\Omega_i = P^* - P^{i-1}$ ($P^*/P^{n-1} \cong (\mathbb{A}^1/k)^{n-1}$)

Rh

as each $\Omega_i - \Omega_{i-1}$ is cohomologically trivial then the normal bundle is $X \times \mathbb{A}^1$

Let $\Omega \in \text{Alt}_n(k)$, the spectral sequence $E_r^{p,q} \Rightarrow H^{p+q}(X, \Omega)$

$$E_1^{p,q} = H^q(T^{p,0}(X), \Omega)$$

$$\Rightarrow H^i(X, \Omega) = H^i\left(\bigoplus_j E_{\infty}^{j,i-j}\right)$$

$$E_{\infty}^{x,0}(X, \Omega) = \underline{H_{\text{cl}}}(C_x^{\text{cl}}(X), \Omega) \quad \text{with} \quad C_x^{\text{cl}}(X) = \bigoplus_n \bigoplus_{\substack{p+q=n \\ p \geq 0}} H_p^{\text{cl}}(X)$$

Def $C_x^{\text{cl}}(X)$ is the cellular chain complex of X ,

$$\text{Alt}_{A'}(k) = \text{shifting } A\text{-invariant subspace}$$

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$$\forall X \in S_{m \times 2}, \forall i \geq 0$$

$$H^i(X, \mathbb{Q}) \cong H^i(X, \mathbb{R})$$

nehát $\forall C_x \in G_x(\alpha_{III}(1))$

$$[C_x^{N'}(X), c_x] \xleftarrow{D_{\mathcal{M}^+}(a, h)} [C_x^{u, l}(X), c_x] \xrightarrow{D_{\mathcal{M}^+}(u, l)}$$

Cor $X \mapsto C_X^{\text{all}}(X) \in \mathcal{D}(\text{al}_M(h))$
is a pseudo left adjoint to i

It follows that $C_x^{\text{cell}}(X)$ doesn't depend on the cellular structure on X
 two diff cellular structure define canonically homotopically equivalent
 chain complexes. $H_*^{\text{cell}}(X)$ is canonical, and is functorial
Rh, the functor i has always a pro-left adjoint. $C_x^{\text{cell}}(X) \in \text{pro-}\mathcal{D}(\text{Ab}_n(\mathbb{Z}))$
 $H_*^{\text{cell}}(X)$

- 1) ? is $C_x^{\text{cell}}(X)$ always contract?
- 2) $\forall X \in \text{Snc}_k$, $H_*^{\text{cell}}(X) = 0$ if $* > \dim(X)$
- 3) $H_*^{\text{cell}}(A^-(0)) = (\pi_0, 0, \dots, 0, \underbrace{K_n^{\text{hom}}}_{n-1}, 0, \dots, 0)$

as each $\Omega_i - \Omega_{i-1}$ is cohomologically trivial then the normal bundle to X_{i-1} is

$$5) \quad C_X^{\text{all}}(P^n) = \left[\begin{array}{ccccccc} \underline{k}_n^{nw} & \xrightarrow{\partial} & \underline{k}_{n-1}^{nw} & \rightarrow & 0 & \xrightarrow{\partial} & \underline{k}_2^{nw} \xrightarrow{\partial} \underline{k}_1^{nw} \xrightarrow{\partial} \mathbb{Z} \end{array} \right]$$

if n is odd $H_n^{\text{all}}(P^n) = \underline{k}_n^{nw}$
 — even $= [\eta] \underline{k}_n^{nw}$

$H_2^{\text{all}}(P^n) = [\eta] \underline{k}_2^{nw}$
 $H_1^{\text{all}}(P^n) = \underline{k}_1^n$
 X/X_{-1}

let us say that X smooth proper $\mathbb{A}^1$ -connected
 - if $\exists \mu_X: \underline{k}_n \xrightarrow{\sim} H_n^{\text{all}}(X)$ & $\forall x \in X(k), H_0(X) \rightarrow H_0(X/x_{-1}) \xrightarrow{\sim} \underline{k}_0^{nw}$
 - one says X is orientable if $\Lambda^*(T_X) \cong L^2$

Cor 1 if $X \in S_{n,k}$, A' con, proper d.c. a.

- 1) $(X \text{ admits a fundamental class}) \iff (X \text{ is orientable})$, & class of (pt)
- 2) if X is not orientable then $\exists H^{\text{odd}} \subset H^{\text{even}}(X) \rightarrow \underline{\text{or}} =$ an orbifold.

(3) Defn $H^{\text{or},*}(X) = K^*$ Can can be realized by a pair of points $\underline{\text{or}}$

Application X, Y are two high orientable, such prop, A' con - suffy 1)

$$\text{def } [X, Y]_{X/H} \rightarrow \text{Ab}(h) = \text{Hom}(K^{\text{or}}, K^{\text{or}})$$

$\mathcal{A}(k) =$ sheaves of abelian groups on $(\text{Spec } k)_{\text{fin}}$
 $\mathcal{A}'(k) =$ strictly $\mathbb{A}^1$ -invariant sheaves of ab. grp on $(\text{Spec } k)_{\text{fin}}$
 $\bigoplus_{i=0}^{\infty} H^i(X, \mathcal{A}) \cong \bigoplus_{i=0}^{\infty} H^i(X \times \mathbb{A}^1, \mathcal{A})$

Cor 1 is related to Poincaré duality -

Cor 2 if X is smooth proper $\mathbb{A}^1$ -con. dim n
 then $C_X^{\text{cell}}(X) \cong H_{\text{cell}}(C_X^{\text{cell}}(X), \underline{k}_n^{\text{mot}})$
 then Cor 1 & 2 are true for cellular smooth proper
 (for fixed X)

$X \times X \xrightarrow{\quad} X \xrightarrow{\quad} \text{pt}$
 $\text{Th}(v_X) =$
 $C_X^{\text{cell}}(\text{Th } v_X) \cong C_X^{\text{cell}}(X \times X \xrightarrow{\quad} X \xrightarrow{\quad} \text{pt})$
 $C_X^{\text{cell}}(X) \otimes_{\mathbb{A}^1} \underline{k}_n^{\text{mot}}[n] \cong \underline{k}_n^{\text{mot}}[n]$