Cohesive Dynamics and Fracture

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Results reported in:

Cohesive Dynamics and Fracture. arXiv:1411.4609v3 [math.AP] 11 Dec 2014. R. Lipton

Dynamic Brittle Fracture as a Small Horizon Limit of Peridynamics, *Journal of Elasticity* Jan, 2014, DOI 10.1007/s10659-013-9463-0, (Open access). R. Lipton

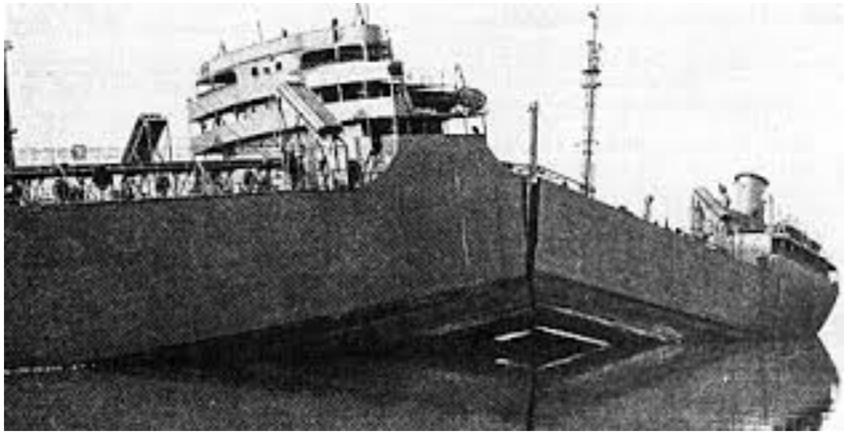
Some of these results presented in the survey article:

Peridynamics Fracture and Nonlocal Continuum Models, *SIAM News* April 2014, (With Qiang Du).

Dynamic fracture of Brittle Solids

Dynamic fracture of brittle solids is an example of collective interaction across disparate length and time scales.

Apply sufficient force to a sample of brittle material, atomistic-scale bonds will eventually snap, leading to fracture of the macroscopic specimen.



Outline of presentation

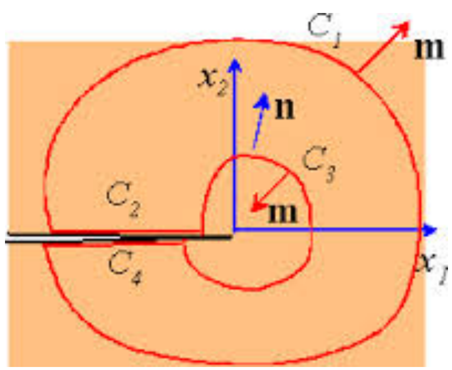
- Brief overview of continuum fracture models.
- Nonlocal cohesive model in peridynamic formulation
- Definition of process zone in terms of cohesive dynamics.
- Explicit estimates for size of process zone from cohesive dynamics.
- Vanishing nonlocal interaction and the localization of the process zone and the brittle fracture limit.
- Observations on the dynamics in the small horizon limit

Classic theory of Dynamic Fracture Mechanics

The theory of dynamic fracture is based on the notion of a deformable continuum containing a crack.

The crack is mathematically modeled as a branch cut that begins to move when an infinitesimal extension of the crack releases more energy than needed to create a fracture surface.

Fracture mechanics, together with experiment, has been enormously successful in characterizing and measuring the resistance of materials to crack growth and thereby enabling engineering design.



Challenges – quantitative modeling of the state of a solid body containing multiple freely propagating cracks.

Given a damaged Shear panel: how much more load can it sustain before failure ?

Continuum fracture modeling: top down approach

Classic top down approach. Find the state of deformation in the cracking body by:

Starting with a PDE model away from the crack

+

Provide a description of the physics in the process zone in the vicinity of the crack

+

Provide an equation for the time evolution of the crack

On modeling multiple cracks: top down approach

Application of cohesive zone elements:
Xu and A. Needleman, *Numerical simulations of fast crack growth in brittle solids*, J. Mech. Phys. Solids, 42 (1994), 1397– 1434.
A. Hillerborg, M. Modeer, and P.E. Petersson, *Analysis of crack formation and growth by means of fracture mechanics and finite elements*, Cem. Concr. Res., 6 (1976), 731–781.

Extended finite element XFEM, Generalized finite element methods GFEM, as Partition of unity methods to eliminate effects of mesh dependence on cohesive zone modeling of freely propagating cracks.

T. Belytschko and T. Black, *Elastic crack growth in finite elements with minimal remeshing*, Int. J. Numer. Meth. Eng., 45 (1999), 601–620.
C.A. Duarte and J.T. Oden. *An hp adaptive method using clouds*. Computer Methods in Applied Mechanics and Engineering, 139(1-4):237–262, 1996.
J.M. Melenk and I. Babuska. *The partition of unity finite element method: Basic theory and applications.* Computer Methods in Applied Mechanics and Engineering, 39:289–314, 1996.

Phase field methods

Coupling porous media flow and fracture evolution in Geomechanics:

M.F. Wheeler, T. Wick and W. Wollner. An augmented-Lagrangian method for the phase-field approach for pressurized fractures. Comput. Methods Appl. Mech. Eng. **271**, 69–85 (2014).

Mikelic, M.F. Wheeler, T. Wick*. A quasistatic phase field approach to pressurized fractures. Nonlinearity 28(5), 1371-1399 (2015)*

Quasistatic:

Francfort, G., Marigo, J.-J.: *Revisiting brittle fracture as an energy minimization problem*. J. Mech. Phys. Solids **46**, 1319–1342 (1998)

B. Bourdin, G. Francfort, J.-J. Marigo, Numerical experiments in revisited brittle fracture, J. Mech. Phys. Solids 48 (2000) 797–826.

Dynamic:

Bourdin, B., Larsen, C., Richardson, C.: *A time-discrete model for dynamic fracture based on crack regularization*. Int. J. Fract. **168**, 133–143 (2011)

Borden, M., Verhoosel, C., Scott, M., Hughes, T., Landis, C.: A phase-field description of dynamic brittle fracture. Comput. Methods Appl. Mech. Eng. **217–220**, 77–95 (2012)

Lattice models: bottom up approaches

Insight into crack tip instabilities and branching by modeling discreteness of fracture at the smallest length scales (breaking of atomic bonds).

M. Marder and S. Gross,
Origin of crack tip instabilities, J. Mech. Phys. Solids, 43 (1995),

M. Marder,
Supersonic rupture of rubber,
J. Mech. Phys. Solids, 54 (2006), 491–532.

M.J. Buehler, F.F. Abraham, and H. Gao
Hyperelasticity governs dynamic fracture at a critical length scale,
Nature, 426 (2003), 141–146.

Nonlocal models: bottom up approaches

Analysis of size effects in quasi-brittle materials

Fracture and Size effect in Concrete and Other Quasibrittle materials
Z. Bazant and J. Planas. CRC Prsss, Boca Raton 1998.

Modeling discreteness of fracture at atomistic length scales through nonlocal models & upscaling to classic fracture mechanics.

Nonlocal Energies for Quasi-static models:

A.Braides & M.S. Gelli, *Limits of Discrete Systems with Long-Range Interactions,* Journal of Convex Analysis, 2002 9:363–399.

Alicandro Focardi and Gelli, *Finite-difference Approximtion of Energies in Fracture Mechanics, Annali della Scuola Normale Superiore di Pisa, 2000 29:671-709.*

L. Truskinovsky*, Fracture as a phase transition*, In: ``Contemporary research in mechanics and mathematics of materials.'' R. Batra, M. Beatty(eds.), CIMNE, Barcelona, 1996, 322-332.

The Challenge – quantitative modeling of multiple freely propagating cracks: residual strength

Given a damaged
Shear panel:
how much
more load can
it sustain before
failure ?

Bottom up meso-approach:
Can we get quantitative predictions from a self consistent – well posed dynamic, mesoscopic continuum theory, informed by atomistic Simulations – AND, at the same time, be consistent with Macroscopic parameters, such as Shear modulus, & Energy release rate?

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Cohesive dynamics constitutive modeling

Unstable meso-scopic dynamics

Nonlocal Cohesive Evolution with Dynamic Instability for Quasi-brittle fracture

Quasi brittle to brittle fracture
As non-locality goes to zero

Convergence to a PDE Based Model for Dynamic Brittle Fracture Characterized by μ and G

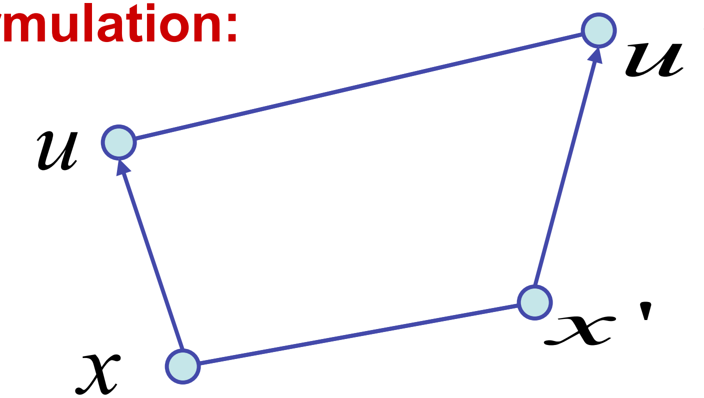
Cohesive dynamics as an Up-scaling of atomistic simulations

For nonlocal linear spring and Lennard-Jones MD models:
P. Seleson, M. Parks,
M. Gunzburger, R. Lehoucq
SIAM, MMS (2009)

Cohesive-dynamics in the Peridynamic Formulation: Background:

We adopt general nonlinear-nonlocal formulation:

S.A. Silling, *Reformulation of elasticity theory for discontinuities and long-range forces*, J. Mech. Phys. Solids, 48 (2000), 175–209. ``**Peridynamic Formulation**.''



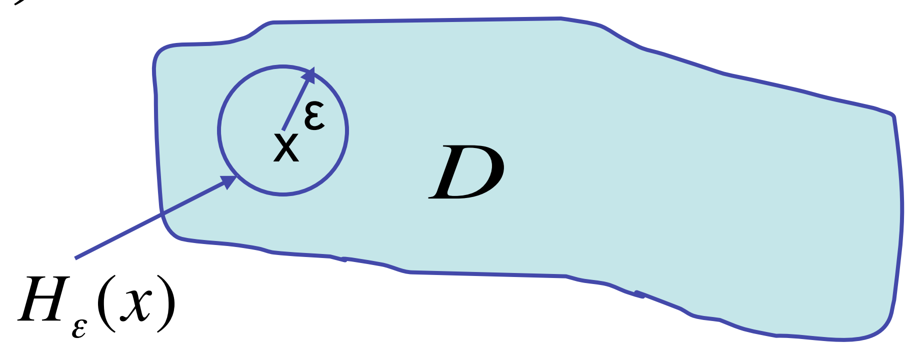
Small displacement theory ``u'' is displacement, x denotes position. Shear strain ``S''

$$S = ((u(x') - u(x))/\,|\,x' - x\,|\,) * e$$
$$e = (x' - x)/\,|\,x' - x\,|$$

ε is length of nonlocal interaction in units taken relative to sample size. Limit of vanishing nonlocality corresponds to ε→0

Force depends on shear strain S

$$\rho\ddot{u} = \int_{H_\varepsilon(x)} k^\varepsilon(S, x' - x)\,dx' + b$$



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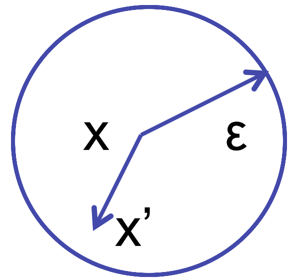
Choice of Potential Energy with Unstable Bonds



Peridynamic potential is a function of the strain along the direction `` x'-x ''

$$S = ((u(x') - u(x))/|x' - x|\,)*e \qquad e = (x' - x)/|x' - x|$$

$$W^{\varepsilon}(S, x' - x) = \frac{1}{\varepsilon} f\left(|x' - x| * |S|^2\right)$$

Non local law: Force between x and x' depends on the strain

$$force = k^{\varepsilon}(S, x' - x) = \partial_S W^{\varepsilon}(S, x' - x)$$

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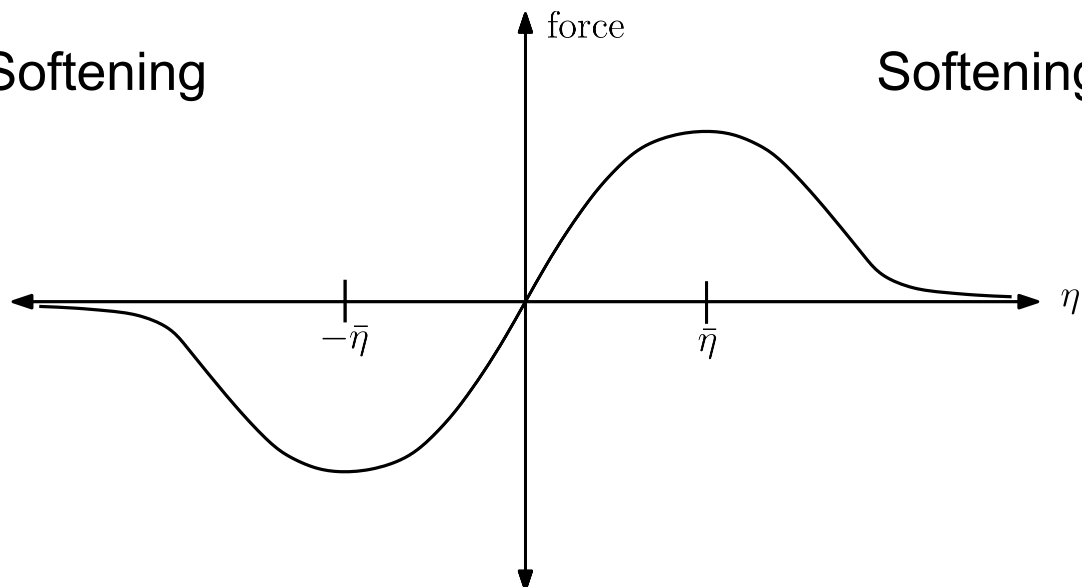
Introduce a cohesive dynamics via an unstable ``force vs. displacement'' law (L., J. Elast. 2014, ArXiv 2014)

$$force = \partial_S W^{\varepsilon}(S, x' - x)$$

Softening

Softening

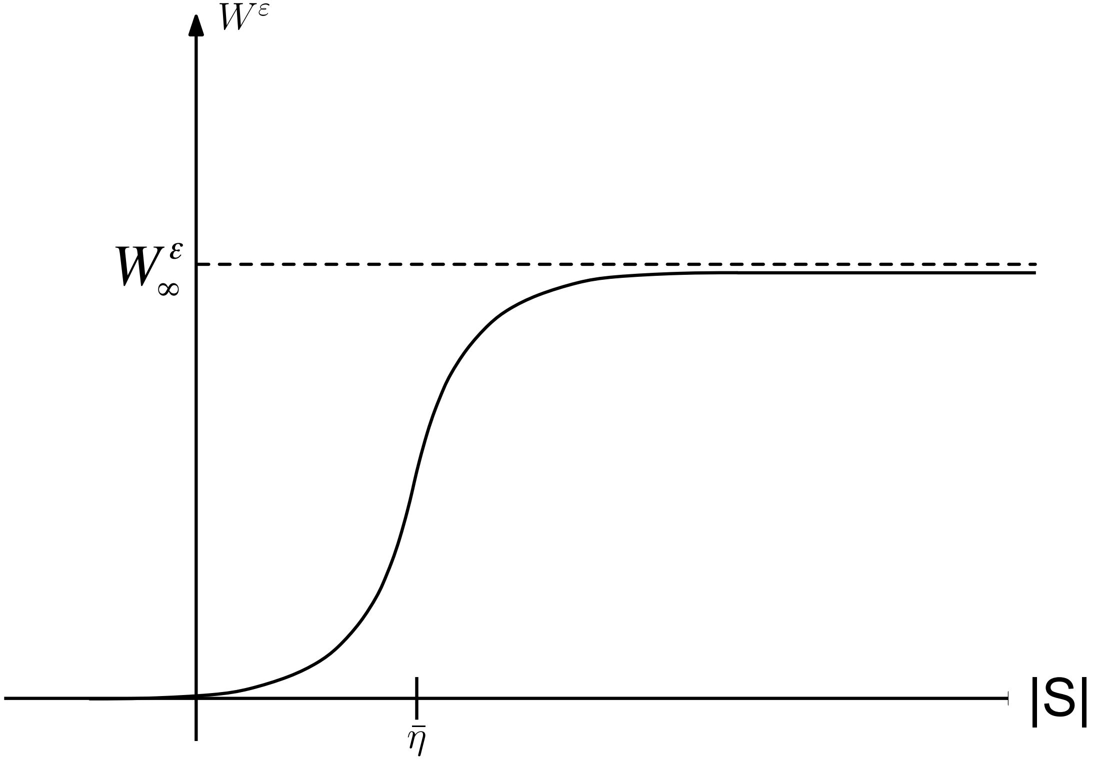
S



$\bar{\eta} = \bar{r} / \sqrt{|x' - x|}$ $\bar{r}$ Is strain concentration

Softening corresponding to square root concentration of strain

Convex – concave potential



$$|S| \mapsto W^{\varepsilon}(S, x' - x)$$

Cohesive Energy, Kinetic, Energy, Action Integral and Dynamics

Formulation of Cohesive potential energy:

Energy density

$$W^{\varepsilon}(S,x)=\frac{1}{Vol(H_{\varepsilon})}\int_{H_{\varepsilon}(x)} W^{\varepsilon}(S,x'-x)dx'$$

Potential Energy

$$PD^{\varepsilon}(u)=\int_{D} W^{\varepsilon}(S,x)\,dx$$

Kinetic Energy

$$K(\partial_t u)=\frac{1}{2}\int_{D}\rho\,|\,\partial_t u\,|^2\,dx$$

Externally applied energy

$$U(u(t))=\int_{D} b(t,x)u(t,x)\,dx$$

Initial conditions

$$u(0,x)=u_0(x) \qquad \partial_t u(0,x)=v_0(x)$$

Initial data doesn't depend on ε

Cohesive Energy, Kinetic, Energy, Action Integral and Dynamics at Mesoscale

Lagrangian $$L^{\varepsilon}(u(t),\partial_t u(t),t) = K(\partial_t u) - PD^{\varepsilon}(u) + U(u(t))$$

Action integral $$I(u) = \int_0^T L^{\varepsilon}(u(t),\partial_t u(t),t)\,dt$$

Least action principle delivers the Euler Lagrange equation for the dynamics described by

$$\rho \ddot{u}^{\varepsilon}(t,x) = -2 \int_{H_{\varepsilon}(x)} (\partial_S W^{\varepsilon}(S^{\varepsilon}, x'-x)dx' + b(x,t)$$

Formulation

For initial data $u(0,x)=u_0(x)$ and $u_t(0,x)=v_0(x)$ belonging to $L^2(D)$.

$$\rho \ddot{u}^{\varepsilon}(t,x) = -2 \int_{H_{\varepsilon}(x)} (\partial_S W^{\varepsilon}(S^{\varepsilon}, x'-x)dx' + b(x,t)$$

This problem is well posed and has a solution $u^{\varepsilon}(t,x)$
That belongs to $C^2([0,T];L^2(D))$ Lipton. J. Elasticity 2014

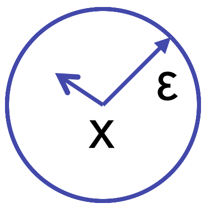
The force is a function of strain

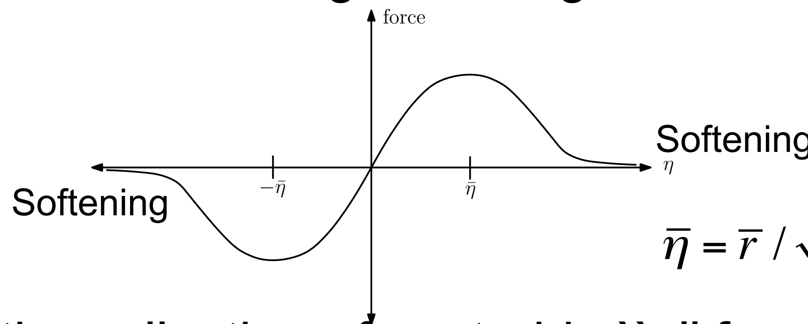
$$S = ((u(x') - u(x))/|x'-x| \,) * e$$

``a generalized directional derivative''
so both continuous and discontinuous deformation
$u^{\varepsilon}(t,x)$ can participate in the dynamics.

Process zone:

Collection of neighborhoods containing softening behavior





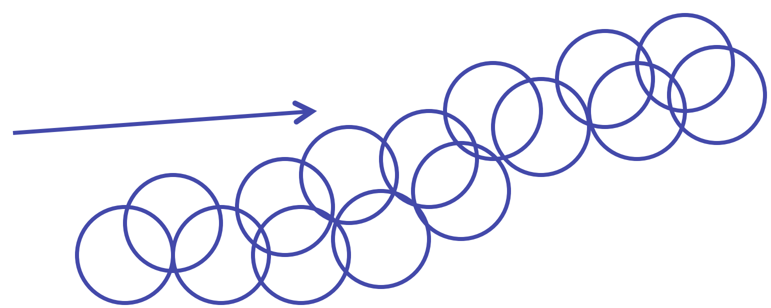
$\overline{\eta} = \overline{r} / \sqrt{|x' - x|}$

Process zone defined to be the collection of centroids ``x'' for which the proportion of bonds P with strain greater than critical value $\overline{\eta}$ is greater than α, $0 < \alpha < 1$, i.e., $P(\{y \in H_\varepsilon(x) : |S^\varepsilon| > \overline{\eta}\}) > \alpha$

Classic Barenblatt & Dugdale Cohesive zone models feature a process zone collapsed onto a prescribed crack surface

However for the *cohesive bond model* the dynamics selects which points lie in process zone

Process zone



Lipton 2013

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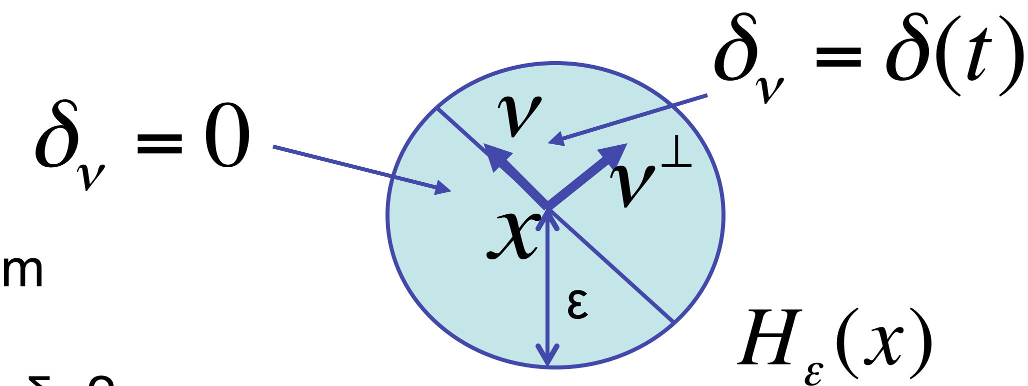
Fracture nucleation condition from instability inside the process zone:

Linear stability of jump perturbation across the neighborhood at x



Given a smooth equilibrium solution u(x) is it stable under a jump perturbation δ_v ?

Calculation gives the condition for linear stability

$$\rho\delta_{tt} = A_\nu \delta \qquad A_\nu = \frac{1}{2}\int_{H_\varepsilon(x)} (-\partial_S^2 W^\varepsilon(S, x' - x))dx'$$

Unstable	$A_\nu > 0$	$-\partial_S^2 W^\varepsilon(S) > 0 \ \lvert S \rvert > \bar{r} / \sqrt{\lvert x' - x \rvert}$	perturbation grows
Neutrally stable	$A_\nu < 0$	$-\partial_S^2 W^\varepsilon(S) < 0 \ \lvert S \rvert < \bar{r} / \sqrt{\lvert x' - x \rvert}$	perturbation stable

Lipton 2013, 2014

Dependence of the process zone with length scale of nonlocal interaction ε

<u>Dependence of the process zone with length scale of nonlocal interaction</u>

At each time ``t'' consider the process zone $C^{\alpha}_{\varepsilon,t}$ given by the union of centroids of neighborhoods for which

$$P(\{y \in H_{\varepsilon}(x) : |S^{\varepsilon}| > \overline{\eta}\}) > \alpha$$

Then the volume of the process zone is bounded above by.

$$\frac{\varepsilon}{\alpha \overline{r}(f'(0))} \times C(t)$$

L. 2014

where

$$C(t) = e^{t}(PD^{\varepsilon}(u_0) + \tfrac{\rho}{2}\|v_0\| + \tfrac{2}{\rho}\int_0^t \|b(\tau)\| d\tau)$$

This is obtained directly from dynamics using energy estimates & no assumptions

$$\rho \ddot{u}^{\varepsilon}(t,x) = -2 \int_{H_{\varepsilon}(x)} (\partial_S W^{\varepsilon}(S^{\varepsilon}, x'-x) dx' + b(x,t)$$

Now examine collapse of the process zone with vanishing nonlocality ``ε'' & convergence to evolution with energy associated with brittle fracture

For initial data $u(0,x)=u_0$ (x) and $u_t(0,x)=v_0(x)$ belonging to $L^2(D)$

$$\rho\ddot{u}^{\varepsilon} = -2\int_{H_{\varepsilon}(x)} (\partial_S W^{\varepsilon}(S^{\varepsilon}, x'-x)dx' + b$$

We start with nonlocal dynamics associated with horizon length scale ε. Then pass to the limit of vanishing nonlocality ε → 0 and follow the dynamics to recover dynamics associated an evolution with uniformly bounded Griffith energy associated with brittle fracture. (L. 2014)

Here ε is the ratio of non local interaction length to sample size.

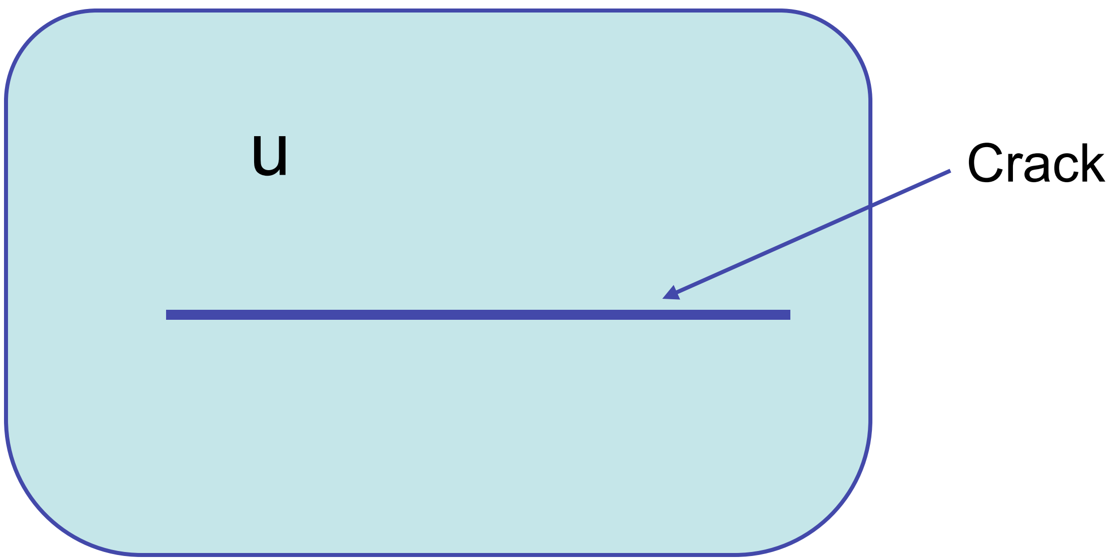
Motivation: Relation between Peridynamic Potential and Potentials from Linear Elastic Fracture Mechanics

An illustrative calculation for motivation:
Consider the displacement inside a body with a flat crack of length L.
Calculate Peridynamic energy and send ε to zero to see what the limit energy is.

$$PD^{\varepsilon}(u) = \int_{D} \int_{H_{\varepsilon}(x)} W^{\varepsilon}(S, x' - x)\, dx'\, dx \to ??$$

Let u denote the displacement.

Jump in u along normal to crack = δ



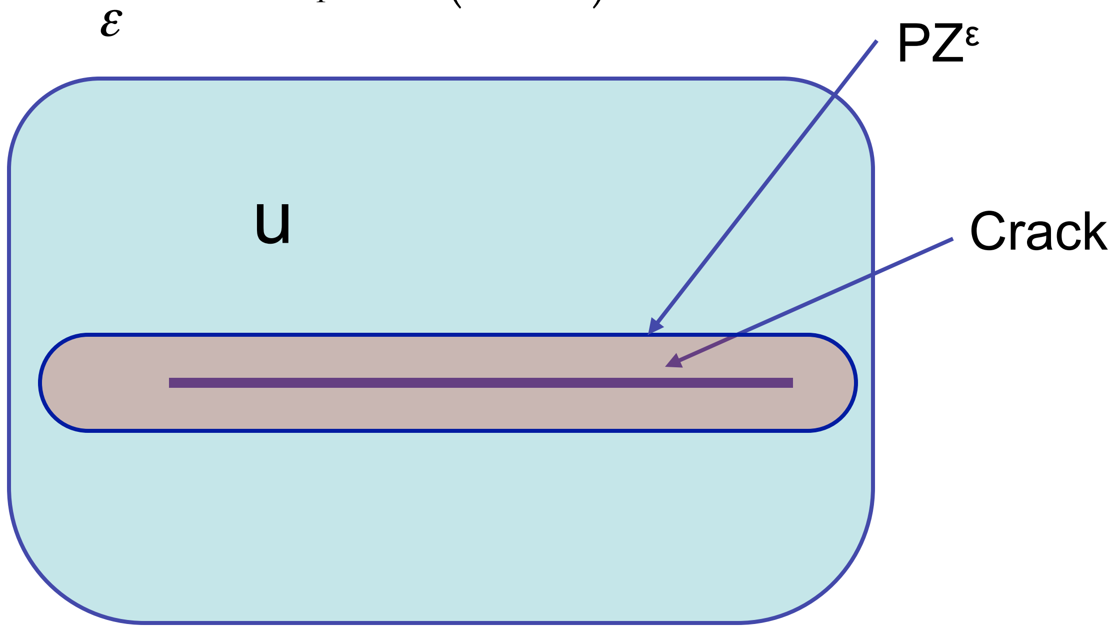
Relation between Peridynamic Potential and Potentials from Linear Elastic Fracture Mechanics

Peridynamic energy for displacement

$$PD^{\varepsilon}(u)=\int_{D}\int_{H_{\varepsilon}(x)} W^{\varepsilon}(S,x'-x)\,dx'\,dx$$

$$=\int_{D\setminus PZ^{\varepsilon}}\int_{H_{1}(0)} \tfrac{1}{\varepsilon|\xi|} f(\varepsilon|\xi|S^{2})|\xi|\,d\xi\,dx+\int_{PZ^{\varepsilon}}\int_{H_{1}(0)} \tfrac{1}{\varepsilon|\xi|} f(\varepsilon|\xi|S^{2})|\xi|\,d\xi\,dx$$

Change of variables $\xi=\dfrac{x'-x}{\varepsilon}$ $\quad H_{1}(0)=\left(|\xi|\leq 1\right)$



Relation between Peridynamic Potential and Potentials from Linear Elastic Fracture Mechanics

Send ε to zero: For x outside PZ^{ε}

$$S^2 = [((u(x+\xi)-u(x))/|\xi|)*e]^2 \rightarrow \left(E_{ij}(u(x))e_i e_j\right)^2$$

$$E_{ij}(u(x)) = (\partial_i u_j(x) + \partial_j u_i(x))$$

$$\tfrac{1}{\varepsilon|\xi|} f(\varepsilon|\xi|S^2) \rightarrow f'(0)(E_{ij}(u(x))e_i e_j)^2$$

$$\int_{H_1(0)} \tfrac{1}{\varepsilon|\xi|} f(\varepsilon|\xi|S^2)|\xi|\,d\xi \rightarrow \int_{H_1(0)} (E_{ij}(u(x))e_i e_j)^2|\xi|\,d\xi$$

$$\int_{H_1(0)} (E_{ij}(u(x))e_i e_j)^2|\xi|\,d\xi = C_{ijkl}E_{ij}(u(x))E_{kl}(u(x))$$

$$\int_{D\setminus PZ^{\varepsilon}} \int_{H_1(0)} \tfrac{1}{\varepsilon|\xi|} f(\varepsilon|\xi|S^2)|\xi|\,d\xi\,dx \rightarrow \int_{D\setminus PZ^{\varepsilon}} C_{ijkl}E_{ij}(u(x))E_{kl}(u(x))\,dx$$

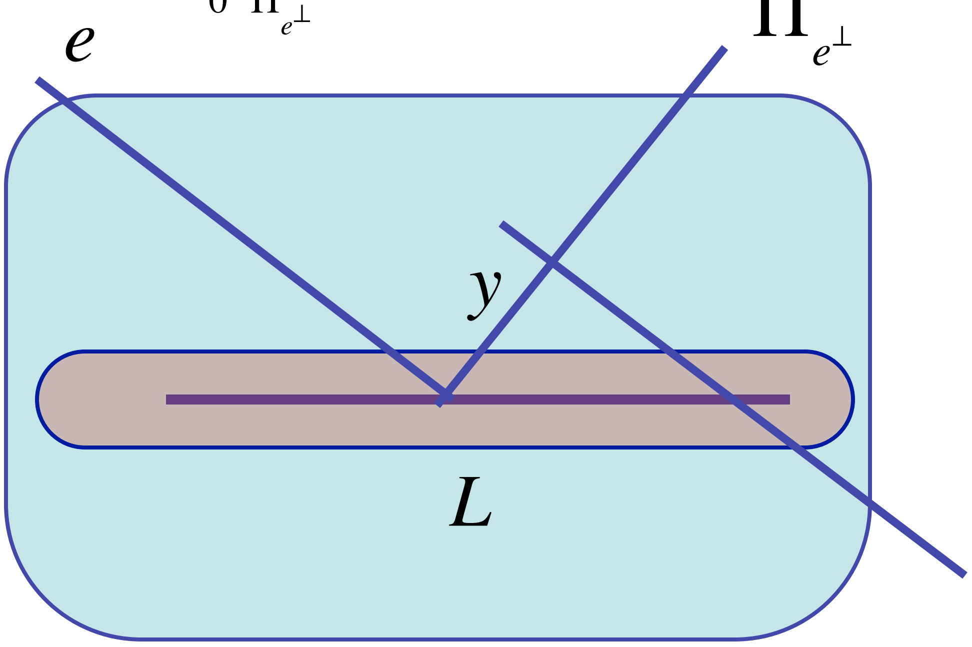
Relation between Peridynamic Potential and Potentials from Linear Elastic Fracture Mechanics

Send ε to zero: Calculate energy inside PZ$^{\varepsilon}$

$$\int_{PZ^{\varepsilon}}\int_{H_1(0)} \tfrac{1}{\varepsilon|\xi|} f(\varepsilon|\xi|S^2)|\xi|\,d\xi\,dx = \int_{H_1(0)}\int_{PZ^{\varepsilon}} \tfrac{1}{\varepsilon|\xi|} f(\varepsilon|\xi|S^2)|\xi|\,d\xi$$

$$\int_{H_1(0)}\int_{PZ^{\varepsilon}} \tfrac{1}{\varepsilon|\xi|} f(\varepsilon|\xi|S^2)|\xi|\,d\xi = \frac{4f_\infty}{\omega_2}\int_0^1 r^2\,dr \int_0^{\pi}\int_{\Pi_{e^\perp}} N(e,y)\,dy\,d\varphi + O(\varepsilon)$$

Crofton's formula: $$L = \frac{1}{2}\int_0^{\pi}\int_{\Pi_{e^\perp}} N(e,y)\,dy\,d\varphi$$



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Relation between Peridynamic Potential and Potentials from Linear Elastic Fracture Mechanics

For this displacement we recover

Griffith's fracture energy as ε → 0.

$$PD^{\varepsilon}(u) = \int_{D}\int_{H_{\varepsilon}(x)} W^{\varepsilon}(S, x'-x)\,dx'\,dx \rightarrow \int_{D} CE(u(x))E(u(x))\,dx + G_c L$$

Elastic Energy

Surface Energy

Cohesive evolution with bounded initial data

Initial displacement $u_0(x)$ **Initial velocity** $v_0(x)$

$$\sup_{x\in D} |u_0| < \infty \qquad \sup_{x\in D} |v_0| < \infty$$

Initial data with bounded peridynamic enegry

$$PD^{\varepsilon}(u_0) = \int_D \int_{H_\varepsilon(x)} W^{\varepsilon}(S_0, x'-x)\,dx'\,dx \leq \infty$$

Energy inequality: In anticipation of small horizon limit

The constants in the description of LEFM are related to nonlocal potential and influence function by:

$$\lambda = \mu = (1/4) f'(0) \int_0^1 r^2 J(r) dr \qquad G = (4/\pi) f_\infty \int_0^1 r^2 J(r) dr$$

Note that only f’(0) and f_∞ determine the elastic moduli parameters μ, λ and G

$$W^\varepsilon(S, x'-x) = \frac{1}{\varepsilon} f\left(|x'-x| \times |S|^2 \right)$$

Fundamental inequality:

$$LEFM(u_0) = \int_D 2\mu |E(u_0)|^2 + \lambda |div(u_0)|^2 dx + G(H^1(J_{u_0})) \geq$$

$$\geq PD^\varepsilon(u_0) = \int_D \int_{H_\varepsilon(x)} W^\varepsilon(S_0, x'-x) dx' dx$$

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Brittle fracture limit of cohesive evolutions

Compactness Theorem: (for small horizon limit of dynamics).
Let $u^{\varepsilon}(t,x)$ be a family of nonlocal cohesive evolutions associated with the same initial data. Then up to subsequences they converge to a limit evolution $u^0(t,x)$ that has bounded LEFM energy for [0,T]

and

$$\int_D 2\mu\,|E(u^0)|^2 + \lambda\,|div(u^0)|^2 dx + G(H^1(J_{u^0})) \leq C \quad t \in [0,T]$$

$$\lim_{\varepsilon \to 0}\{\sup_{0<t<T}\left|\int_D |u^{\varepsilon}(t,x) - u^0(t,x)|^2\, dx\right|\} = 0$$

$u^0(t,\bullet)$ in SBD(D)

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Energy Inequality

Total energy at time=t | Kinetic energy | Potential energy | Work done on the body

$$EPD^{\varepsilon}(t,u^{\varepsilon}) = \frac{\rho}{2}\| u_t^{\varepsilon}(t)\|^2_{L^2(D)} + PD^{\varepsilon}(u^{\varepsilon}(t)) - \int_D b(t)u^{\varepsilon}(t)\,dx$$

$$EPD^{\varepsilon}(0,u_0) = \frac{\rho}{2}\| v_0 \|^2_{L^2(D)} + PD^{\varepsilon}(u_0) - \int_D b(0)u_0\,dx$$

Energy Equality

$$EPD^{\varepsilon}(t,u^{\varepsilon}) = EPD^{\varepsilon}(0,u_0) - \int_0^t \int_D b_{\tau}(\tau)u^{\varepsilon}(\tau)\,dx\,d\tau$$

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Energy inequality for the limit evolution

$$LEFM(u_0) = \mu \int_D |\nabla u_0|^2 dx + G(H^1(S_{u_0}))$$

$$LEFM(u^0(t)) = \mu \int_D |\nabla u^0(t)|^2 dx + G(H^1(S_{u^0(t)}))$$

Total energy at time=t | Kinetic energy | Potential energy | Work done on the body

$$GF(t, u^0(t)) = \frac{\rho}{2} \| u_t^0(t) \|_{L^2(D)}^2 + LEFM(u^0(t)) - \int_D b(t) u^0(t)\, dx$$

$$GF(0, u_0) = \frac{\rho}{2} \| v_0 \|_{L^2(D)}^2 + LEFM(u_0) - \int_D b(0) u_0\, dx$$

Limit Flow Energy Inequality

$$GF(t, u^0(t)) \le GF(0, u_0) - \int_0^t \int_D b_\tau(\tau) u^0(\tau)\, dx\, d\tau$$

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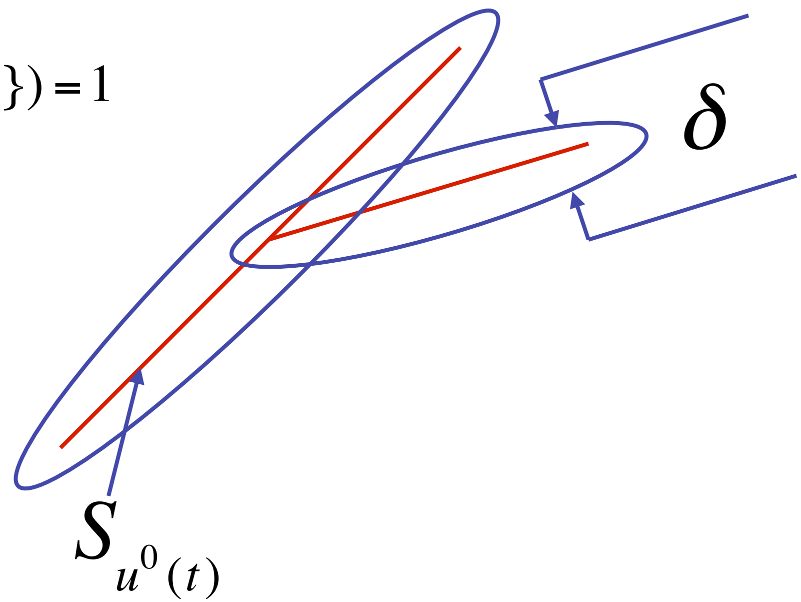
Now make a technical hypothesis

1. Assume jump set of u^0, $J_{u^0(t)}$ is closed
2. Fix δ & for ε sufficiently small assume all neighborhoods are stable outside a domain of ``width'' δ containing $S_{u^0(t)}$

$$P(\{y \in H_\varepsilon(x) : |S^\varepsilon| < \overline{\eta}\}) = 1$$



Recover classic wave equation for the limiting ``brittle'' evolution for points not on crack set

Distinguished limit of vanishing nonlocality

Theorem. For $\lambda = \mu = (1/4)f'(0)\int_0^1 r^2 J(r)dr$

The limit flow satisfies the wave equation

$$\rho \ddot{u}^0 = div(\sigma) + b \qquad \sigma = \lambda I Tr(Eu^0) + 2\mu E u^0$$

For points (x,t) not on the crack set $S_{u^0(t)}$

As ε-> 0 the cohesive evolution $u^\varepsilon(x,t)$ approaches PDE based fracture given by the deformation - jump set pair

$u^0(t,x) \quad S_{u^0(t)}$

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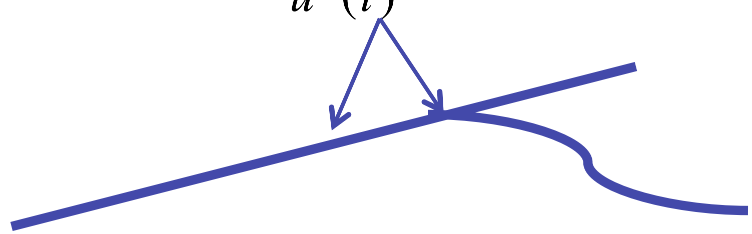

Some of the mathematical tools necessary for the analysis

- Gamma-convergence tools from image processing
- Specifically M. Gobbino's solution (2000) of conjecture by De Giorgi for the Mumford Shah Functional (1996).

M. Gobbino, *Finite difference approximation of the Mumford–Shah functional*, Commun. Pure Appl. Math., 51 (1998), 197–228.

Conclusions

- The nonlocal cohesive model in peridynamic formulation is mathematically well posed. It is a free process zone model providing nucleation and propagation of fracture surface driven by mesoscopic instability.

  This feature is distinct from classic cohesive zone models where cohesive zones are restricted to prescribed surfaces and the equations of elasticity are enforced off these surfaces.

- Cohesive dynamics provides a-priori estimates for size of process zone in terms of model parameters. Useful for calibrating the model to the material sample.

- These nonlocal models recover a brittle fracture limit with bounded Griffith fracture energy in the limit of vanishing nonlocality.

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Anti-plane shear:Journal of Elasticity 2014,
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