

Towards An Efficient and Practical High Order Finite Element Methods

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High order finite element methods have been analysed extensively for a wide variety of applications and are known to be capable of producing exponential rates of convergence, even for challenging problems with singularities, sharp boundary layers and high frequency oscillations. High order polynomial approximations are commonplace in many areas of scientific computing including computer graphics, computer aided-geometric design, and spectral methods for PDEs.

It is not unusual to see the spectral method used with approximation orders in the 100s (or even 1000s). Yet, despite theory giving the nod to the use of very high order finite element methods, the range of polynomial degree used in practical finite element computations is rarely larger than eighth order. Few commercial codes allow the use of high order finite elements. The rather modest polynomial degrees seen in high order finite element analysis are due to efficiency considerations rather than any theoretical barriers.

Bernstein-Bézier polynomials have a number of interesting properties that have led to their being the industry standard for visualisation and CAGD. We explore the use of Bernstein polynomials as a basis for high order finite element approximation.

The talk contains material developed in collaboration with Gaele Andriamaro, Oleg Davydov (Giessen), Guosheng Fu (Brown), Manuel Sanchez (Minnesota), L. Schumaker (Vanderbilt) and S. Jiang (Brown).