

A mass conserving mixed stress formulation for incompressible flows

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One of the main difficulties in computational fluid dynamics lies in the proper treatment of the incompressibility condition. A weak treatment of this constraint can lead to a locking phenomena if the viscosity is small and results in bad velocity approximations. Recent developments (see [1]) show that $H(\text{div})$ -conforming finite elements for the approximation of the velocity provide major benefits such as exact mass conservation, pressure-independent (locking free) and polynomial robust (see [2]) error estimates. By introducing a new variable which approximates the gradient of an $H(\text{div})$ -conforming velocity we derive a new mixed stress formulation of the incompressible Stokes equations. For the analysis a new function space, the $H(\text{curldiv})$, is defined, in which we can show well posedness.

In the discrete setting two different approaches lead to a stable analysis. We present the construction of proper Finite elements, discuss solvability and verify our method with several numerical examples implemented in NGSolve (www.ngsolve.org) with the new NGS-Py interface. We conclude the talk by a further motivation of the new mass conserving mixed stress formulation: the numerical treatment of a linearized R13 model.

References.

- [1] Lehrenfeld, Christoph and Schöberl, Joachim: High order exactly divergence-free Hybrid Discontinuous Galerkin Methods for unsteady incompressible flows. *Computer Methods in Applied Mechanics and Engineering*, Vol. **307**, 339 – 361, 2016.
- [2] Lederer, P. and Schöberl, Joachim: Polynomial robust stability analysis for $H(\text{div})$ -conforming finite elements for the Stokes equations. *IMA Journal of Numerical Analysis*, drx051, 2017.