Inference With Confidence

International FocuStat Workshop on Confidence Distributions and Related Themes

May 11-13 2015 **Teknologihuset, Oslo**

focusstat
FOCUS DRIVEN STATISTICAL
INFERENCE WITH COMPLEX DATA

UiO : University of Oslo

The Research Council of Norway

FocuStat, Focus Driven Statistical Inference With Complex Data, is a five-year project funded in part by the Research Council of Norway, operating from January 2014 to December 2018 at the Department of Mathematics, University of Oslo. The project group consists of Nils Lid Hjort (professor, project leader), Gudmund Hermansen and Kristoffer Hellton (PostDocs), Céline Cunen and Sam-Erik Walker (PhDs). Other PhD and Master's level students are also associated with the project, and we are collaborating with yet other colleagues, at the Department of Mathematics and elsewhere.

The themes of the project include and involve confidence distributions, model selection and model averaging, bridging the gaps between parametric, semiparametric and nonparametric modelling and inference, Bayesian nonparametrics, combination of information across diverse data sources, etc. A common thread is the notion of *focus*, the view that some aspects of experiments, data, and information are more important than others, and that such a science- and context-driven focus ought to contribute to the modelling and analysis of data, as well as to the performance evaluation of the relevant methods. This leads to focus driven model building and model selection, etc. The project is meant to develop relevant parts of general statistical methodology but will also involve actual applications to the analysis of real-world complex data. For further information, regarding publications, talks, 'who we are', news and events, etc., consult the project webpage

`www.mn.uio.no/math/english/research/projects/focustat/index.html`

and check (and contribute to) our Facebook page.

FocuStat has budget for arranging small-scale international workshops on designated themes, for each of 2015, 2016, 2017, 2018, as well as for lower-scale research kitchens. This year's **Inference With Confidence** workshop concerns confidence distribution and related themes. These include and involve construction methods for CDs, studying their behaviour and performance, second-order correction tools for enhancing accuracy, links to so-called objective Bayes and empirical Bayes methods, meta-analysis and more general methods for data fusion, application to real data stories, etc.

For May 2016 we are planning a **Focused model selection and model averaging** workshop, in particular involving versions of the FIC (the focused information criterion). More information concerning both this and other research activities will be found on our webpage in due time.

Inference With Confidence
Programme

The structure of the workshop is intended to encourage and facilitate active discussion, during, after and between talks. The time schedule indicated here is therefore not followed strictly.

Monday:

8:45 - 9:10	Good morning & tea and coffee
9:15 - 9:45	**Nils Lid Hjort**: Welcoming remarks; FocuStat; general workshop themes
10:00 - 10:45	**Tore Schweder:** Confidence is epistemic probability
	tea & coffee
11:15 - 12:00	**Jan Hannig:** Generalized fiducial inference: A review
	lunch
13:30 - 14:00	**Piero Veronese:** Fiducial and objective Bayesian inference for natural exponential families
	tea & coffee
14:15 - 15:00	**Dongchu Sun:** Objective Bayesian analysis of multivariate functional models

Tuesday:

9:00 - 9:50	**Min-ge Xie:** Confidence distribution as a unifying framework for Bayesian, fiducial and frequentist (BFF) inference: I. Core and recent advances
10:00 - 10:50	**Regina Liu:** II. Fusion learning done the right way
	tea & coffee
11:15 - 12:00	**Céline Cunen:** Optimal inference via confidence distributions for two-by-two tables: presentation and comparisons with Liu, Liu, Xie methods
	lunch
13:30 - 14:00	**Kristoffer Hellton:** PCA and the asymptotic distribution of high-dimensional sample eigenvectors
	tea & coffee
14:15 - 14:45	**Nils Lid Hjort:** Confidence distributions for break points
15:00 - 15:30	**Steffen Grønneberg:** On statistical inference and multiple use of the same data
18:00 -	Workshop Dinner & Confidence Party

Wednesday:

9:00 - 9:30	**Gudmund Hermansen:** Predictive confidence for time series
9:30 - 10:15	**Gunnar Taraldsen:** The road from conditional Monte Carlo to improper priors and fiducial inference
	tea & coffee
10:30 - 11:15	**Bo Lindqvist:** On the proper treatment of improper distributions
11:30 - 12:00	**Sam-Erik Walker:** Confidence distributions based on M-estimators
	lunch
13:30 - 14:15	**Arnoldo Frigessi:** Posterior uncertainty for rank data aggregation – and a priori plans for *Big Insight*
14:30 - 15:30	Summing up & rounding off

Titles & abstracts:

Céline Cunen (with Nils Lid Hjort):

Optimal inference for two-by-two tables via confidence distributions: presentation and comparisons with the Liu, Liu, Xie methods

Abstract: Meta-analysis of two-by-two tables is common in medical research, but most existing methods rely on normal approximations and are therefore unsuited for tables with rare events. The tables are usually modelled as pairs of binomial variables with a common odds ratio parameter (the focus parameter). In this talk we will present an optimal method for such an odds ratio parameter, and compare it to related methods by Liu, Liu and Xie (JASA, 2014). Both classes of methods are based on confidence distributions and claim to be exact. The methods will be compared both by displaying results for real datasets, by simulation studies and by calculating the confidence risk functions. According to certain optimality theorems our method should have uniformly higher confidence power than the methods of Liu, Liu and Xie (2014). We will also briefly discuss other models for two-by-two tables, notably models that allow heterogeneity among the odds ratio parameters, and we will propose an optimal method for inference on the ratio between odds ratios in this case.

Arnoldo Frigessi:

Posterior uncertainty for rank data aggregation – and a priori plans for *Big Insight*, a Norwegian centre for research-based innovation

Abstract: Analysis of rank data has received renewed interest, due to novel applications in the era of big data. The basic purpose is to estimate the latent ranks of n items on the basis of N samples, each of which is the ranking of the same n items by an independent assessor. A classical exponential model in a distance between ranks is the Mallows models. For some special distances, including Kendall's correlation, the partition function can be computed analytically, but for other distances it cannot. The footrule distance, which is the L_1 norm of the difference between the observed and latent ranks, and Spearman's distance (L_2 norm) are not tractable. We take a Bayesian approach, where the latent ranks are assumed to be random and have a prior distribution over the set of all permutations. We develop an MCMC algorithm for sampling the latent ranks and other design parameters, from the posterior distribution. For the partition function, we used a simple importance sampling scheme. The algorithm is tested on simulated data, on some simple preference experiments that we collected and on several benchmark data sets. This is joint work with Øystein Sørensen, Valeria Vitelli and Elja Arjas.

In the second part of my talk, I will present the new centre of excellence *Big Insight*, which starts now (April 2015), and will operate for the next eight years, with focus on personalised solutions and prediction of transient phenomena.

Steffen Grønneberg:

On statistical inference and multiple use of the same data

Abstract: In fields that are mainly nonexperimental, such as economics and finance, it is inescapable to compute test statistics and confidence regions that are not probabilistically independent from previously examined data. The Bayesian and Neyman-Pearson inference theories are known to be inadequate for such a practice. We show that these inadequacies also hold m.a.e. (modulo approximation error). We develop a general statistical theory, called the neoclassical inference theory, that is immune to this inadequacy m.a.e. The neoclassical inference theory appears to nest many statistical practices, whether they are labelled Bayesian or à la Neyman-Pearson. We derive a general, but simple adjustment to make standard errors account for the approximation error.

Jan Hannig:

Generalized fiducial inference: A review

Abstract: R.A. Fisher, the father of modern statistics, proposed the idea of fiducial inference in the 1930's. While his proposal led to some interesting methods for quantifying uncertainty, other prominent statisticians of the time did not accept Fisher's approach because it went against the ideas of statistical inference of the time. Beginning around the year 2000, the author and collaborators started to re-investigate the idea of fiducial inference and discovered that Fisher's approach, when properly generalized, would open doors to solve many important and difficult inference problems. They termed their generalization of Fisher's idea as generalized fiducial inference (GFI).

After more than a decade of investigations, the authors and collaborators have developed a unifying theory for GFI, and provided GFI solutions to many challenging practical problems in different fields of science and industry. Overall, they have demonstrated that GFI is a valid, useful, and promising approach for conducting statistical inference. The goal of this talk is to deliver a timely and concise introduction to GFI, to present some of the latest results, as well as to list some related open research problems. It is the authors' hope that their contributions to GFI will stimulate the growth and usage of this exciting approach for statistical inference.

Kristoffer Hellton:

PCA and the asymptotic distribution of high-dimensional sample eigenvectors

Abstract: The asymptotic distributions of sample eigenvalues and vectors in the high-dimensional setting have attracted substantial attention the last decade. An important application for these results is principal component analysis (PCA), where the eigenvectors are used to construct low-dimensional projections of the high-dimensional data. The inconsistency of the high-dimensional eigenvectors has discredited the general use of classical PCA, but the method is still extensively and successfully used in certain genetic applications. The asymptotic distribution of the sample eigenvectors and projections can, however, offer an explanation for this paradoxical situation and also give other insights into high-dimensional PCA.

Gudmund Hermansen (with Nils Lid Hjort):

Predictive confidence distributions for time series

Abstract: We apply recent advancements and developments in statistical inference methodology via confidence distributions to provide new and richer tools for assessing and exploring the quality of predictions. The main focus is on examining the possibilities for constructing robust prediction confidence distribution based inference in a variety of (short memory) time series models. Methodology for time series processes comprising trend and covariates is studied and we derive confidence distributions, both for future outcomes and for relevant 'predictive functions', like the probability that the next realisation will exceed a certain threshold. The methods are illustrated for several real data cases.

Nils Lid Hjort:

Opening remarks

Abstract: I will first sketch some of the intended research lines for the FocuStat project (2014–2018), pointing also to similar workshops for 2016, 2017, 2018 (but partly dedicated to other themes). Then I will attempt to describe some of the themes for the present *Inference With Confidence* workshop, including pointers to yet other confidence distribution related themes which also could have been presented.

Nils Lid Hjort (with Céline Cunen and Gudmund Hermansen):

Confidence distribution for a break point

Abstract: Tirant lo Blanch is a 'romance novel' written in Catalan in the 1460ies and published in 1490, and is considered an important precursor for what most literature historians hail as 'the world's first novel' (which appeared more than a hundred years later). The 1490 novel consists of 487 chapters, and we address the problem of assessing precisely when author II (M.J. de Galba) took over for author I (J. Martorelli). The statistical problem is to construct a confidence distribution for the break point τ, where observations $y_1, \ldots, y_\tau$ come from one distribution but $y_{\tau+1}, \ldots, y_n$ stem from a different distribution. My talk will be based on preliminary and hence ongoing work with Cunen and Hermansen. I will also point to other 'break point' applications.

Bo Lindqvist (with Gunnar Taraldsen):

On the proper treatment of improper distributions

Abstract: The axiomatic foundation of probability theory presented by Kolmogorov has been the basis of modern theory for probability and statistics. In certain applications it is however necessary or convenient to allow improper (unbounded) distributions, which is often done without a theoretical foundation. The talk presents the elements of a mathematical theory which includes improper distributions, and which is related to Renyi's theory of conditional probability spaces. An obvious motivation for the study is the extensive use of improper priors in Bayesian statistics, where the present theory gives an alternative to common ad hoc arguments which are not based on an underlying theory. In particular, the approach leads to simple explanations of apparent paradoxes known from the literature. The role of improper distributions in fiducial statistics and conditional sampling will also be briefly considered.

Tore Schweder:

Confidence is epistemic probability

Abstract: Probability has a Janus face with aleatory probability to one side and epistemic to the other. The lack of understanding fiducial probability/confidence as epistemic has hindered its acceptance and use (cf. Hampel, 'An outline of a unifying statistical theory', 2011). Confidence is not ruled by Kolmogorov's axioms.

Dongchu Sun:

Objective Bayesian analysis of multivariate functional models

Abstract: A general version of multivariate functional models using smoothing spline with correlated errors and correlated curves is introduced. A suitable symmetric smoothing parameter matrix is introduced, and practical priors are are developed for the unknown covariance matrix of the errors and the smoothing parameter matrix. An efficient algorithm for computing the multivariate smoothing spline is derived, which leads to an efficient Markov chain Monte Carlo method for Bayesian computation. Key to the computation is a natural decomposition of the estimated curves into components intrinsic to the problem that extend the notion of principal components. These intrinsic principal curves are useful both for computing and for interpreting the data. The methods are applied to the multivariate yield curves of Chinese Government Bonds and Bank Bonds.

Gunnar Taraldsen (with Bo Lindqvist):

The road from conditional Monte Carlo to improper priors and fiducial inference

Abstract: We started out in 1997 on a project to clarify certain aspects of a conditional Monte Carlo method suggested by Engen and Lillegård. The method gives conditional samples given a sufficient statistic, and can hence be used to construct optimal inference procedures. In some cases it can also be used to eliminate nuisance parameters by conditioning. The steps in the algorithm gives strong links to fiducial inference, and also to the foundations of statistics both in mathematical and philosophical terms. The talk will give an informal sketch of the initial method, and the many related foundational and practical issues.

Piero Veronese (with Eugenio Melilli):

Fiducial and objective Bayesian inference for natural exponential families

Abstract: We study the relationships between fiducial/confidence distributions and Bayesian posteriors when the model belongs to a natural exponential family with $k \geq 1$ parameters. In particular, if it exists, a prior generating a posterior which coincides with a fiducial distribution, is called *fiducial prior*.

For $k = 1$, following the original Fisher's idea, we propose an easy and direct way to construct a fiducial distribution for the parameter in both continuous and discrete exponential families. We prove that a fiducial prior exists if and only if the model is Normal (σ^2 known), Gamma (shape parameter known), Binomial, Negative-Binomial and Poisson. This prior coincides, or it is strictly related, with the Jeffreys prior. When the fiducial prior does not exist, the Jeffreys posterior and the fiducial distribution are shown to be asymptotically equivalent up to order $O(n^{-1})$. Thus a connection between fiducial and objective Bayesian inference is established, see Veronese and Melilli (2015).

For $k > 1$, the previous results are the starting point to find fiducial distributions for different parameterizations in several models. The idea is to factorize the distribution of a suitable sufficient statistic as a product of (possibly conditional) one-dimensional laws. Then the joint fiducial distribution for the parameters is defined as the product of the

one-dimensional fiducial laws obtained from the various elements of the factorization. It is interesting to note that the role of the Jeffreys prior in the univariate case is now played by the reference prior.

Some general results on equivalence between fiducial and reference posteriors are given for the conditionally reducible exponential families, which include the multinomial and the negative-multinomial models as relevant examples (see Consonni, Veronese and Guttiérrez-Peña, 2004, for a discussion of reference priors in conditionally reducible exponential families).

Sam-Erik Walker (with Nils Lid Hjort):

Confidence distributions based on M-estimators

Abstract: In this talk, we briefly discuss the possibility to base confidence distributions on various types of M-estimators (besides the MLE), including the special and important class of so-called strictly proper scoring rules. The motivation for this is mainly robustness, but also ease of use in the face of complex data. We discuss the subject matter mainly from a theoretical point of view, but we will also illustrate it using a few practical examples.

Min-ge Xie (part I) and **Regina Liu** (part II):

Confidence distribution (CD) as a unifying framework for Bayesian, fiducial and frequentist (BFF) inference: I. Core and recent advances. II. Fusion learning done the right way.

Abstract: CD, broadly speaking, refers to any sample-dependent distribution function that can represent confidence intervals of all levels for a parameter of interest. Conceptually, a CD is not much different from a point estimator or a confidence interval, but it estimates a parameter by using a sample-dependent distribution function defined on the parameter space. It provides "simple and interpretable summaries of what can reasonably be learned from data (and an assumed model)" (Cox 2013), and also meaningful answers to all questions related to statistical inference. A major theme emerging from recent developments on CD is "Any statistical approach, regardless of being Bayesian, frequentist or fiducial (BFF), can potentially be unified under the concept of confidence distributions, as long as it can be used to build confidence intervals of all levels, exactly or asymptotically."

We expand on the subject of CD in two parts:

(I) We describe and articulate a logic behind the recent developments of CD, and explore the close connections between CD and bootstrap, fiducial and Bayesian inferences. We demonstrate that CD can potentially serve as a unifying framework for statistical inferences in all areas, including estimation, hypotheses testing and prediction.

(II) We present practical examples to show that CD can be a powerful tool for solving statistical inference problems where methods with desirable properties have not been available or could not be easily obtained. Specific examples include: (1) Incorporating external information in analyses of clinical trials with binary outcomes; (2) Exact meta-analysis approach for discrete data and its application to 2×2 tables with rare events; (3) Efficient network meta-analysis: a confidence distribution approach; (4) Meta-analysis of heterogeneous studies using only summary statistics: efficiency and robustness; (5) Nonparametric combing inferences from multiple sources using CD, bootstrap and data depth. These many examples are but a small sample to show that CD can be an efficient approach for fusing learning. That is, CD can combine or fuse inferences from different sources or different inference methods to provide an efficient overall inference.

Participants:

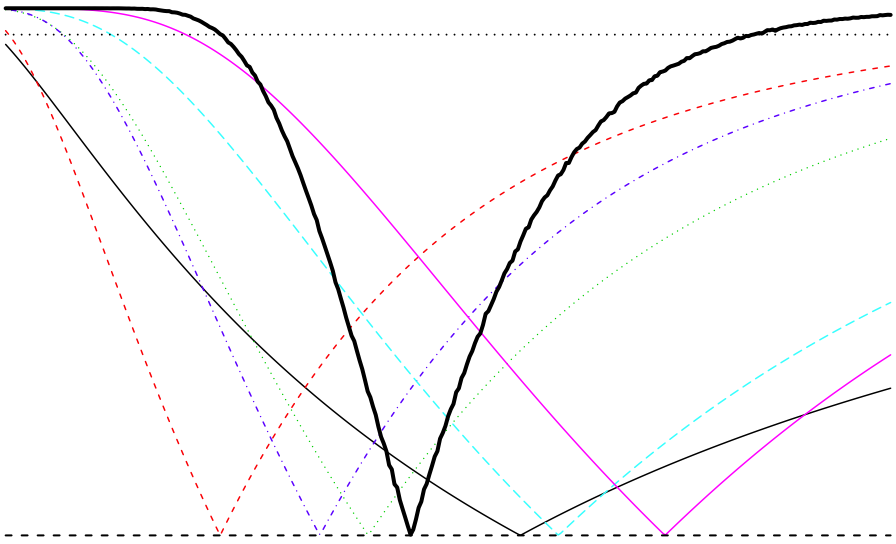
In addition to the 'core participants' listed here, a few more of the local Oslo statisticians (from the Department of Mathematics, the Norwegian Computing Centre, Big Insight, Institute of Basic Medical Science) will meet up for some of the talks.

From the FocuStat group: Céline Cunen, Gudmund Hermansen, Kristoffer Hellton, Nils Lid Hjort, Martin Jullum, Sam-Erik Walker

From the University of Oslo: Arnoldo Frigessi, Ingrid Glad, Tore Schweder

From the University of Trondheim: Bo Lindqvist, Gunnar Taraldsen

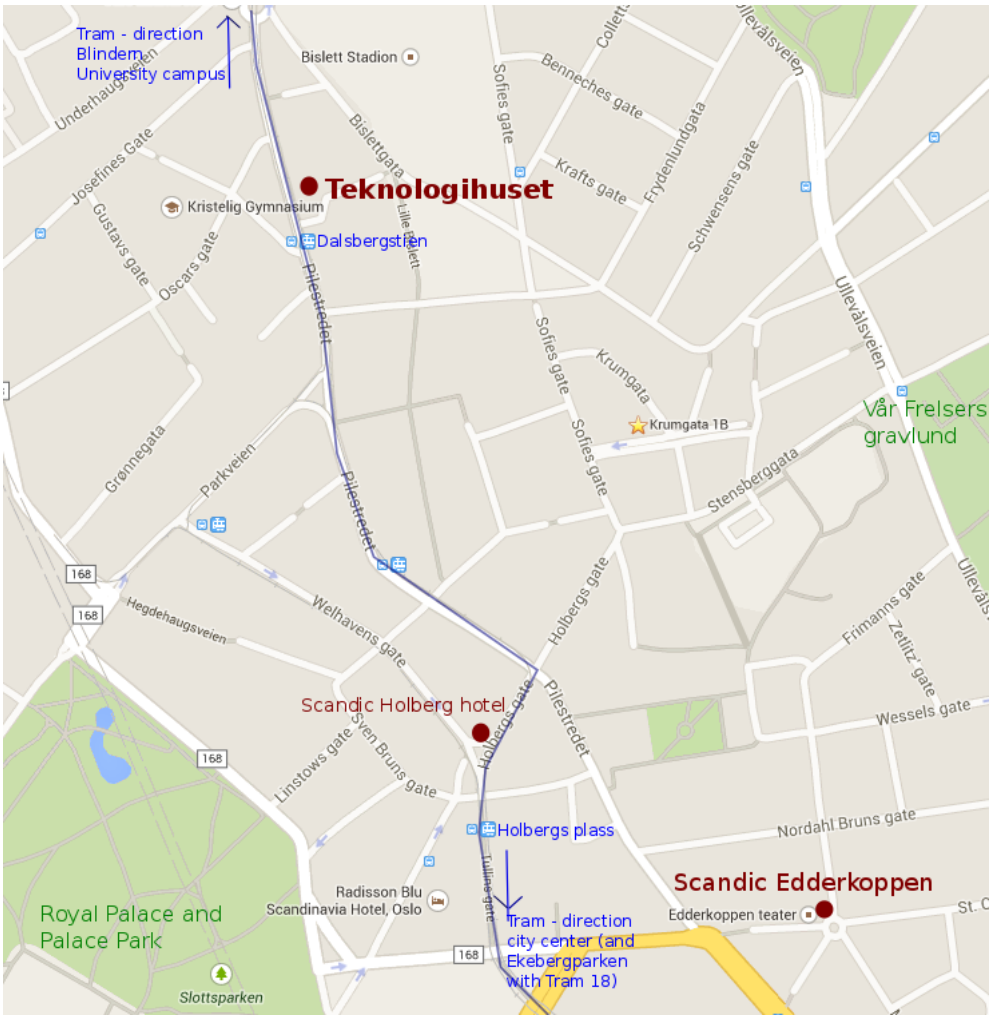
From elsewhere: Steffen Grønneberg, Jan Hannig, Regina Liu, Eugenio Melilli, Dongchu Sun, Piero Veronese, Min-ge Xie



Practical information:

Most of our workshop guests will stay at the hotel Scandic Edderkoppen, a 900 meters walk from Teknologihuset where the Workshop will take place (Pilestredet 56). If one prefers to use public transportation, one can take the trams 17 or 18 (direction Rikshospitalet) from the stop Holbergs plass to Dalsbergstien.

Both Teknologihuset and the hotel are close to the city centre and to several places of interest (if one has time for some sightseeing), with easy access by tram or the Apostle's horses. The Olympic stadion Bislett is a javelin's throw away from Teknologihuset (see the front cover), and the Royal Palace and Palace Park, as well as the Honorary cemetery 'Vår Frelsers gravlund', are peaceful places worth visiting. In the latter place one can find the graves of famous Norwegians like Henrik Ibsen, Edvard Munch, Bjørnstjerne Bjørnsen, Henrik Wergeland, etc. If one wishes to admire some art by Munch, the National Gallery is found only 250 meters from Scandic Edderkoppen. There one can also enjoy the exhibition *The Magic North. Finnish and Norwegian art around 1900*.



Map of area between Scandic Edderkoppen and Teknologihuset. Blue lines are Tram lines. *Google maps*

Confidence Dinner:

On Tuesday the 12th, the FocuStat group invites all participants to join us for a social, cultural and gastronomical evening.

We will take the tram together from Dalsbergstien at approximately 17:15 (tram 18 direction Ljabru to the stop Ekebergparken). If the weather is fine we will start (at 18:00) with a guided tour of Ekebergparken Sculpture Park, a relatively new attraction in Oslo with 31 artworks, both classic and modern. At 19:00 we will dine at the nearby Ekeberg restaurant, which offers a fine view of Oslo and is a prime example of Norwegian functionalistic architecture.

Ekeberg restaurant (*Photo by mellbye.com*)