Do Ceasefires Work?
A Bayesian autoregressive hidden Markov model to explore how ceasefire shape the dynamics of violence in civil war

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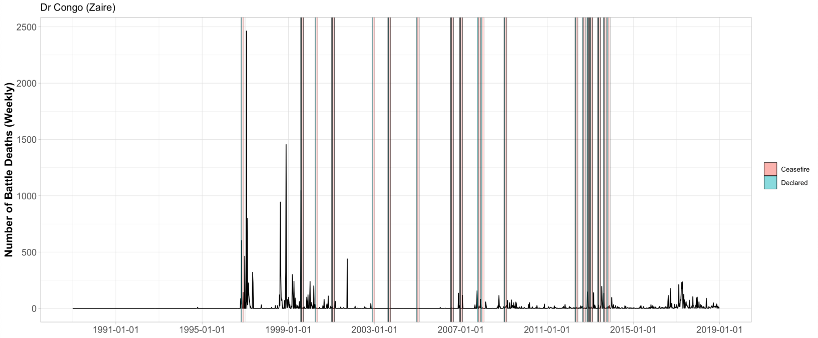
Weekly Number of Battle Related Deaths

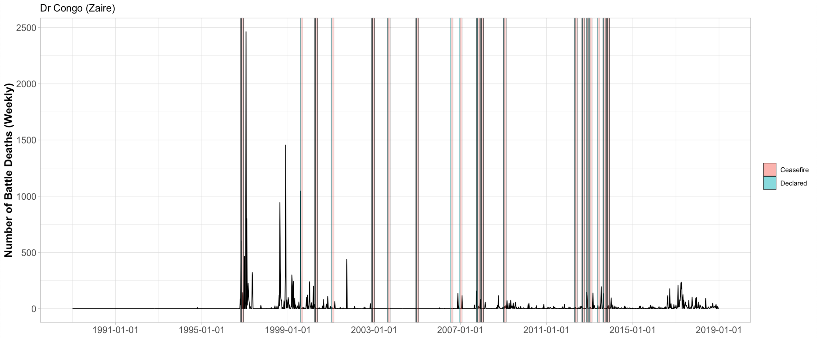


The response variable of interest is the weekly number of battle deaths[1] due to intrastate conflict and/or internationalised intrastate conflicts in a given country.

Ceasefires are *arrangements that include a statement by at least one conflict party to stop violence temporarily or permanently from a specific point in time.*

[1] https://ucdp.uu.se/

The Purpose of Ceasefires

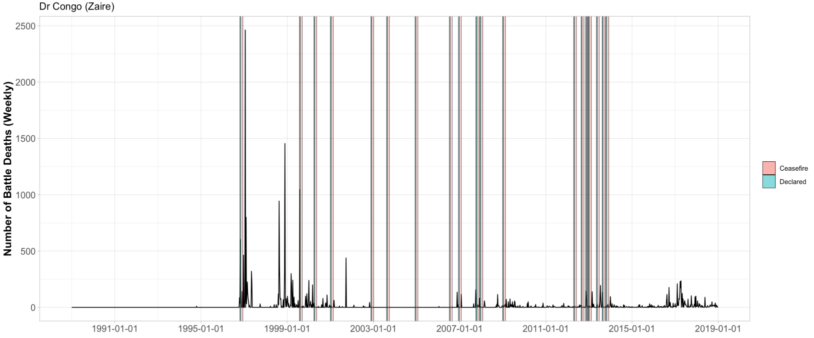


The underlying motivations for ceasefires are very different, they can last from a few hours to many days and some have no impact while others result in peace.

There has been no systematic investigation of their effectiveness.

The general purpose of a ceasefire is to shift a conflict to a less violent state, this should have a measurable effect on the battle deaths and the dynamics of violence.

Data and the Statistical Model



Here, we want to use a Bayesian Hidden Markov (HMM) modelling framework to try to answer some of these questions.

However, we need a references model (response function) capable of capturing the dynamics of such data, e.g. zero inflation, overdispersion, heteroscedastic, volatility clustering, etc.

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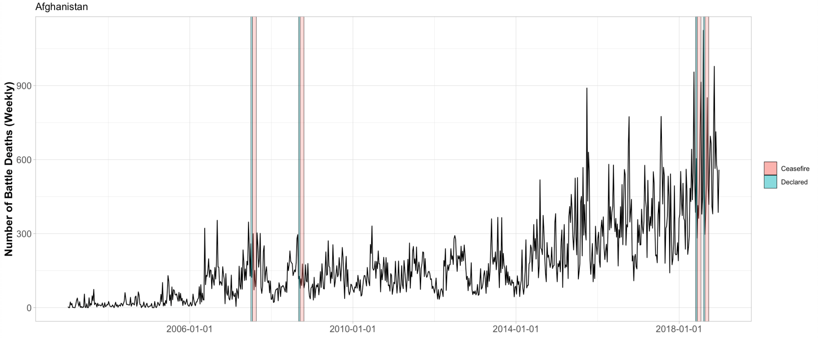

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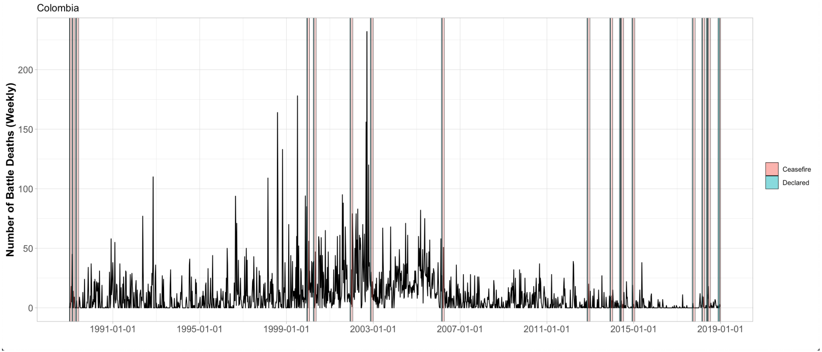

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The Statistical Model - Part 1

Let y_t be the number of battle deaths at time t and consider the model

$$y_t \mid \theta_t \sim \text{Poisson}(\theta_t)$$
$$\theta_t \mid y_{t-1}, a, b, c \sim \text{Gamma}(c(a + by_{t-1}), c),$$

where $a > 0$, $c > 0$ and $|b| < 1$ (to ensure weak stationarity; see Xu et al. (2012)).

Here, we view θ_t as the underlying stochastic intensity of the conflict.

From the above, it is straight forward to show that

$$y_t \mid y_{t-1}, a, b, c \sim \text{NB}(y_t \mid r_t, p)$$

is negative binomial for $t > 1$ with $r_t = c(a + by_{t-1})$ and $p = 1/(c + 1)$.

This means that

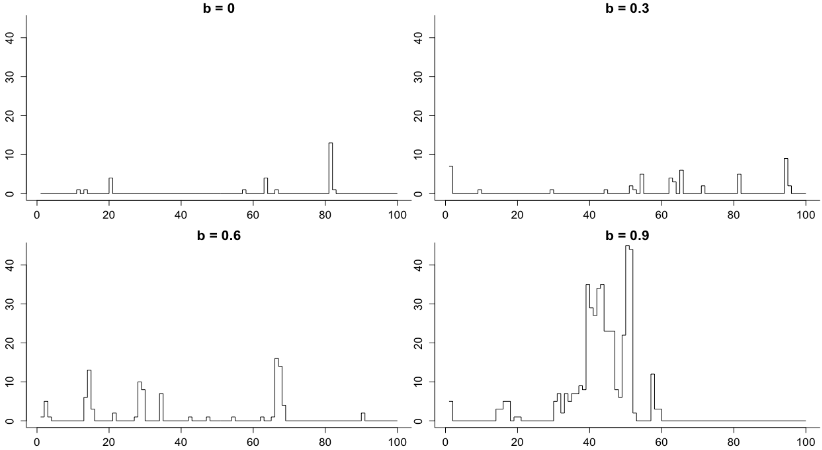
$$\text{E}\left[y_t \mid y_{t-1}\right] = a + by_{t-1} \quad \text{and} \quad \text{Var}(y_t \mid y_{t-1}) = (1 + 1/c)(a + by_{t-1}),$$

Note that the above is a baseline model, and more complexity can easily be added by including covariates via suitable functions in a, b or c.

We interpret the b parameter, the linear dependency in conditional expectation

$$\mathrm{E}\left[y_t \mid y_{t-1}\right] = a + b y_{t-1}$$

as a potential description of the dynamics of violence in a conflict.



The Statistical Model - Part 2

Furthermore, form the properties of the negative binomial distribution, we have that

$$y_t = \epsilon_t + \sum_{i=0}^{y_{t-1}} \delta_i, \tag{1}$$

where ϵ_t and δ_i are independent and

$$\epsilon_t \sim \mathrm{NB}(ca, p) \quad \text{and} \quad \delta_i \sim \mathrm{NB}(cb, p).$$

This is actually a known model, it is a NB-DINARCH(1), i.e. **N**egative **B**inomial **D**ispersion **I**nteger **A**utoregressive **C**onditional **H**eteroskedasticity); see Xu et al. (2012).

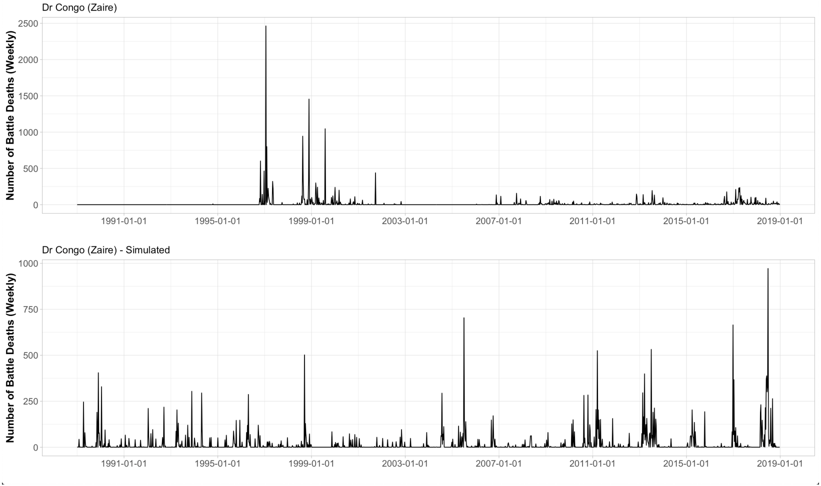
Such models tend to be motivated via a conditional expectation representation, similar to $\mathrm{E}\left[Y_t \mid y_{t-1}\right] = \beta_0 + \beta_1 y_{t-1} + \cdots$.

Or, through the use of a so-called (generalised) thinning operator, e.g. as in (1).

The NB-DINARCH(1) is the response function in our hidden Markov model.

Does it Work? - A Simple Simulation Study.

Is this a reasonable model with a sufficient amount of complexity for such data?



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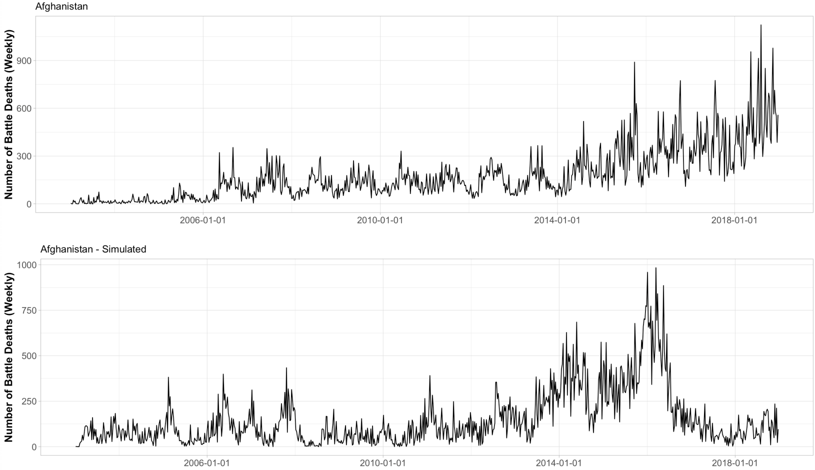

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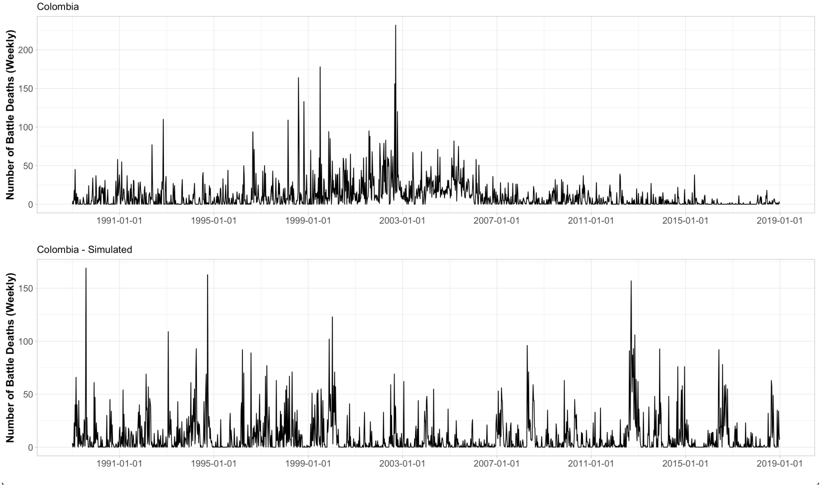

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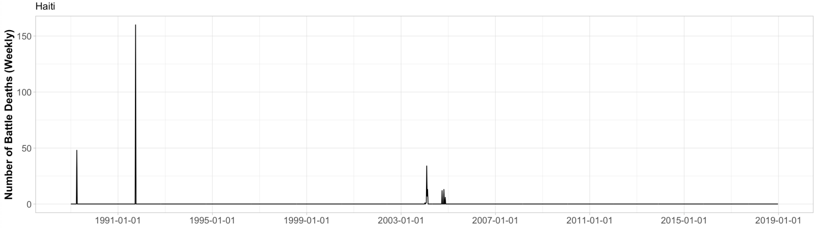
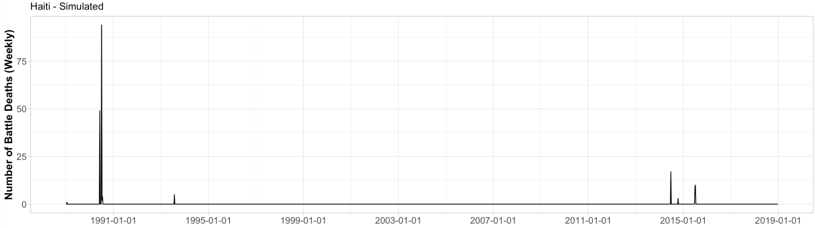




Ceasefire Data

Despite the prevalence of ceasefires, no agreed upon definition of ceasefire exists either in the academic nor the policy domain.

Here, we use a broad definition that covers a variety of arrangements (including: truces, cessation of hostilities, armistices, and preliminary and definitive ceasefire agreements).

However, we will distinguish between ceasefires that have been agreed on but have not yet entered into effect and ceasefires that are in effect.

- Ceasefire: *The week containing the start date of the ceasefire and the following four weeks (post-ceasefire).* We expect that conflicts are more likely to enter into a non-violent state following a ceasefire.
- Declared: *The two weeks prior to the start of a ceasefire (pre-ceasefire)*: Conflicts are more likely to enter into state of violence escalation in the period preceding a ceasefire.

In the model, the information on ceasefire or declared, is included as covariates.

The Hidden Markov Model - Part 1

For each country $i = 1, \dots, N$, let $y_{i,k}$ be the observed battle deaths for week $t_{i,k}$, where $k \in \{1, \dots, n_i\}$ and n_i is the number of weeks for country i.

Now, we assume a model where $y_{i,k}$ is dependent on a latent state $s_{i,k} \in \{1, 2, 3\}$, corresponding to the state space

$$\{\text{'peace'}, \text{'stable conflict'}, \text{'escalating conflict'}\}.$$

The state sequence, $\{s_{i,1}, \dots, s_{i,n_i}\}$ is assumed to be a Markov chain, where the state for $t_{i,k}$ only depends on the state of the process and covariates at time $t_{i,k-1}$.

Furthermore, the conditional distribution of $S_{i,k} \mid S_{i,k-1}$ is determined by

$$P := \begin{pmatrix} (1 + e^{q_1} + e^{q_2})^{-1} & 0 & 0 \\ 0 & (1 + e^{q_3} + e^{q_4})^{-1} & 0 \\ 0 & 0 & (1 + e^{q_5} + e^{q_6})^{-1} \end{pmatrix} \begin{pmatrix} 1 & e^{q_1} & e^{q_2} \\ e^{q_3} & 1 & e^{q_4} \\ e^{q_5} & e^{q_6} & 1 \end{pmatrix},$$

where q_1, q_2, q_3, q_4, q_5 and q_6 determine the rates of respective state transitions.

And, the transition probability parameters will depend on covariates:

$$\begin{pmatrix} q_1 & q_2 & q_3 & q_4 & q_5 & q_6 \end{pmatrix} := (z_k^{(i)})'\zeta,$$

where $z_k^{(i)}$ is a vector of covariates (including information about ceasefires) for country i at week $t_{i,k}$, and ζ is a coefficient matrix.

The Hidden Markov Model - Part 2

The HMM will use a variation of the NB-DINARCH(1) as response function.

Now, the full model is

$$Y_{i,k} \mid Y_{i,(k-4):(k-1)}, S_{i,k} \sim \text{Negative-Binomial}(r_{i,k}, p),$$

where $p = c/(1+c)$,

$$r_{i,k} = a + b \cdot \frac{1}{4} \sum_{l=1}^{4} Y_{i,k-l},$$

$$a = a_1 \mathbf{1}\{S_{i,k} = 1\} + a_2 \mathbf{1}\{S_{i,k} = 2\} + a_3 \mathbf{1}\{S_{i,k} = 3\},$$

and

$$b = \mathbf{1}\{S_{i,k} \neq 1\} \cdot e^{(\beta_1 \mathbf{1}\{S_{i,k}=2\} + \beta_2 \mathbf{1}\{S_{i,k}=3\})^{\mathrm{t}} x_k^{(i)}},$$

where a_1, a_2, a_3 and c are positive parameters, $x_k^{(i)}$ is a vector of covariates.

Note that we have seen empirically that averaging over the past four observations gives a more stable model.

To help distinguish between states 2 and 3 and improve state space identification, we impose the following constraints:

$$\beta_{11} \leq \beta_{21} \quad \text{and} \quad a_1 \leq a_2 \leq a_3$$

The Hidden Markov Model - Part 3

Note that if $s_{i,k}$ is state 1 (i.e. in peace), then

$$Y_{i,k} \sim \text{Negative-Binomial}(a_1, p),$$

and we may interpret a_1 and c as describing a model for rare conflict related deaths (e.g., isolated terrorist attacks).

Moreover, this forces the rate parameter b to be identified with respect to periods of conflict (i.e., when the system is in state 2 or state 3).

Since

$$E[Y_{i,k} \mid Y_{i,(k-4):(k-1)}, S_{i,k}] = \frac{a}{c} + \frac{b}{c} \cdot \frac{1}{4} \sum_{l=1}^{4} Y_{i,k-l}.$$

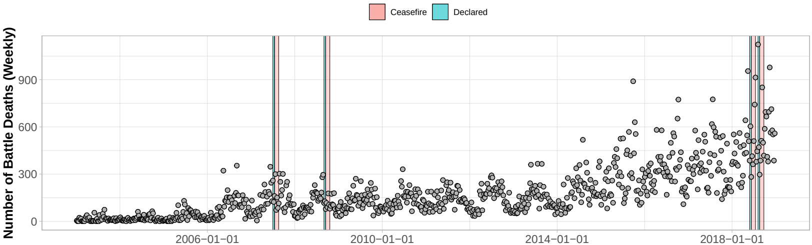
the number of conflict deaths during state 1 has a mean of a_1/c, whereas in the conflict states (2 or 3) the mean structure is autoregressive.

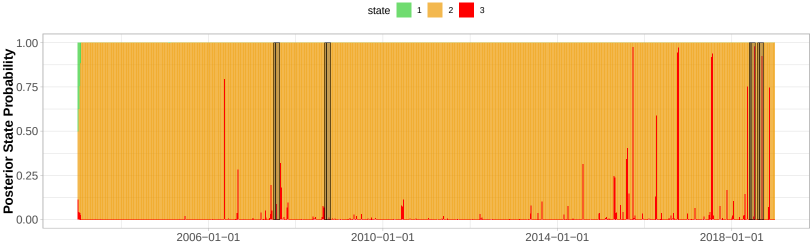
Our Bayesian approach uses mainly vague priors.

And, inferences is based on a custom Metropolis-within-Gibbs MCMC algorithm.

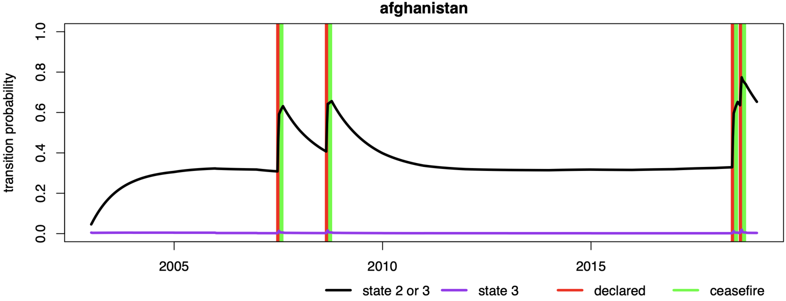
Results - Part 1

From the posterior model, we can now estimate the underlying state space.





And, plot the evolution of the probabilities of transition (on a week-by-week resolution).



Results - Part 3

All results are relative to an "average country" represented by using the average of the covariates.

- Periods of violent conflict are rare events, with an estimated probability for transition from state 1 to states 2 or 3, in one week, of 0.00063 and 6×10^{-7}.
- Ceasefires are associated with ongoing violence, the transition probability from state 1 to state 2 (stable conflict) increases by a factor of 52 for a country with a declared ceasefire, and by a factor of 18 for a country with a ceasefire in effect.
- Ceasefire increase the probability of transition into a less violent state, the probability of transition into state 1, from state 2, is increased by a factor of 3 when a ceasefire is in effect.
- We see a statistically significant effect for the declared variable coefficients appear for transitions $1 \to 2$, $2 \to 3$, and $3 \to 1$; all with a sign that indicates sustained or heightened levels of violence.
- Furthermore, the estimated "autoregressive coefficient" b/c increases from 0.87 to 1.68 in state 3, for weeks in which a ceasefire has been declared (and is also high/explosive under certain covariate conditions).

  Indicating immediate short-term escalation of violence before a ceasefire is effective.

  However, these time periods are very short lived with transition probabilities for $3 \to 1$, $3 \to 2$, and $3 \to 3$ of 0.09, 0.66, and 0.25, respectively.