Do Japanese and Italian women live longer than women in Scandinavia?

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List of countries by life expectancy

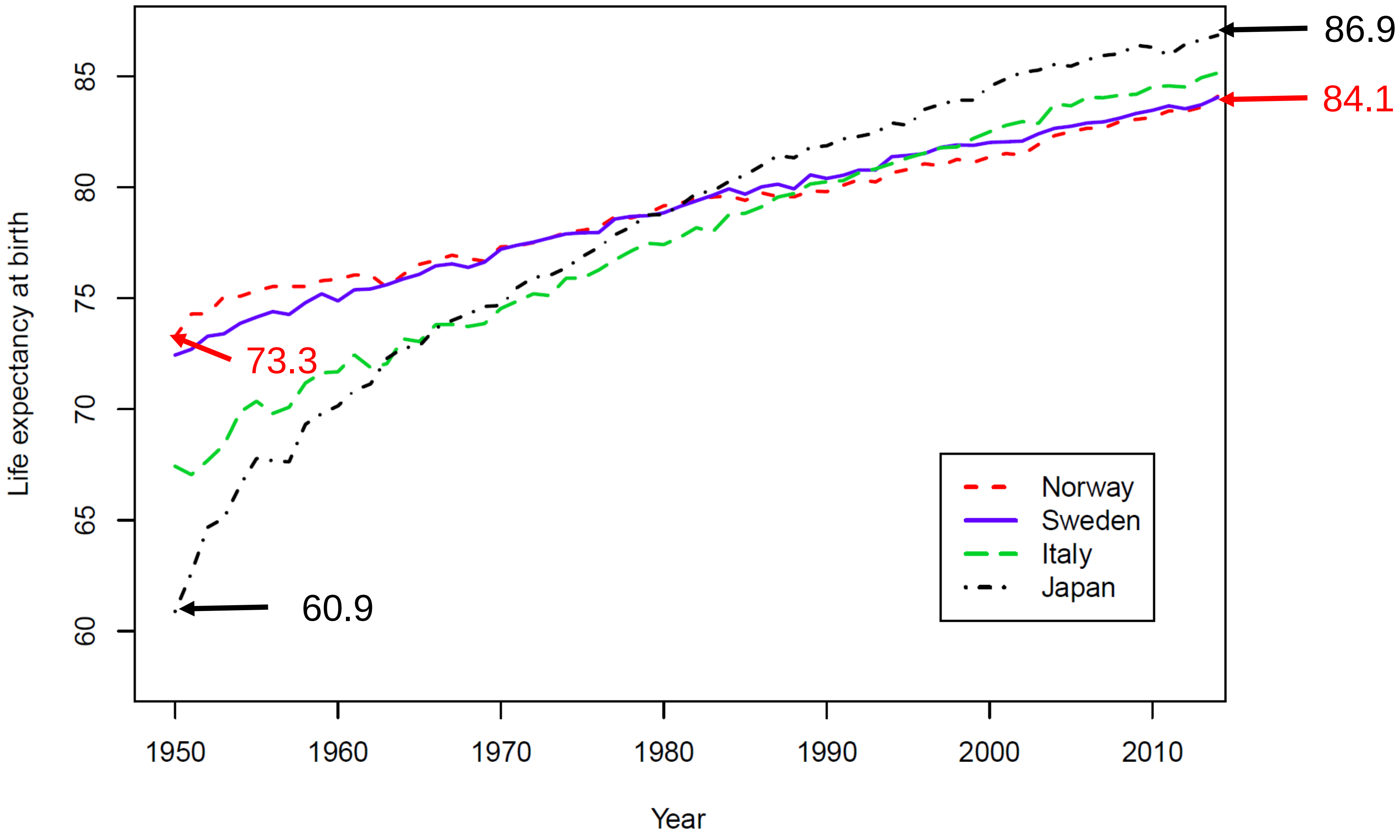
WIKIPEDIA
The Free Encyclopedia

List by the United Nations, for 2010–2015

Life expectancy at birth (years), UN World Population Prospects 2015

Rank	State/Territory	Overall	Male	Female
1	Japan	83.74	80.91	86.58
2	Italy	83.31	80.00	86.49
10	Sweden	81.93	80.10	83.71
16	Norway	81.32	79.22	83.38

Life expectancy at birth for women 1950-2014



Data from The Human Mortality Database (mortality.org)

Scandinavian women had a longest life expectancy at birth in 1950, but they have later been overtaken by women from Japan and Italy

So why the question in the title of the talk?

The reason why one may ask the question, is that life expectancy at birth is a more complicated concept than one may think at first glance

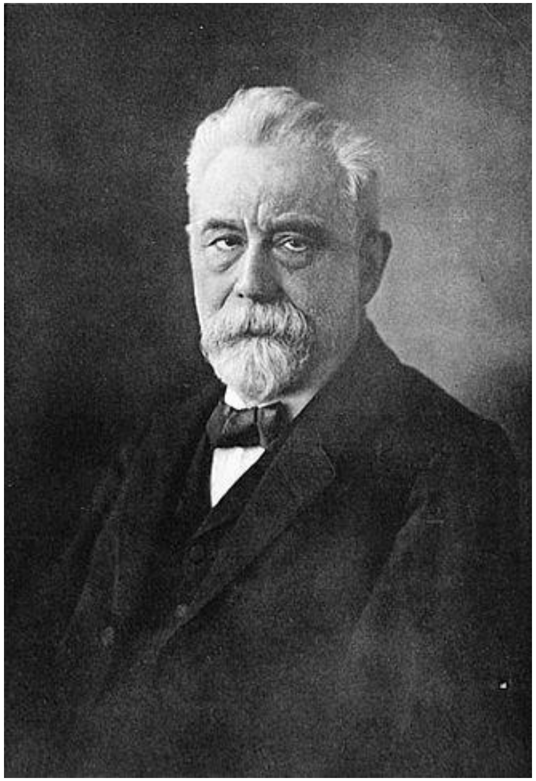
UNITED NATIONS DEVELOPMENT PROGRAMME

Human Development Reports

Life expectancy at birth is the number of years a newborn infant could expect to live if prevailing patterns of age-specific mortality rates at the time of birth stay the same throughout the infant’s life.

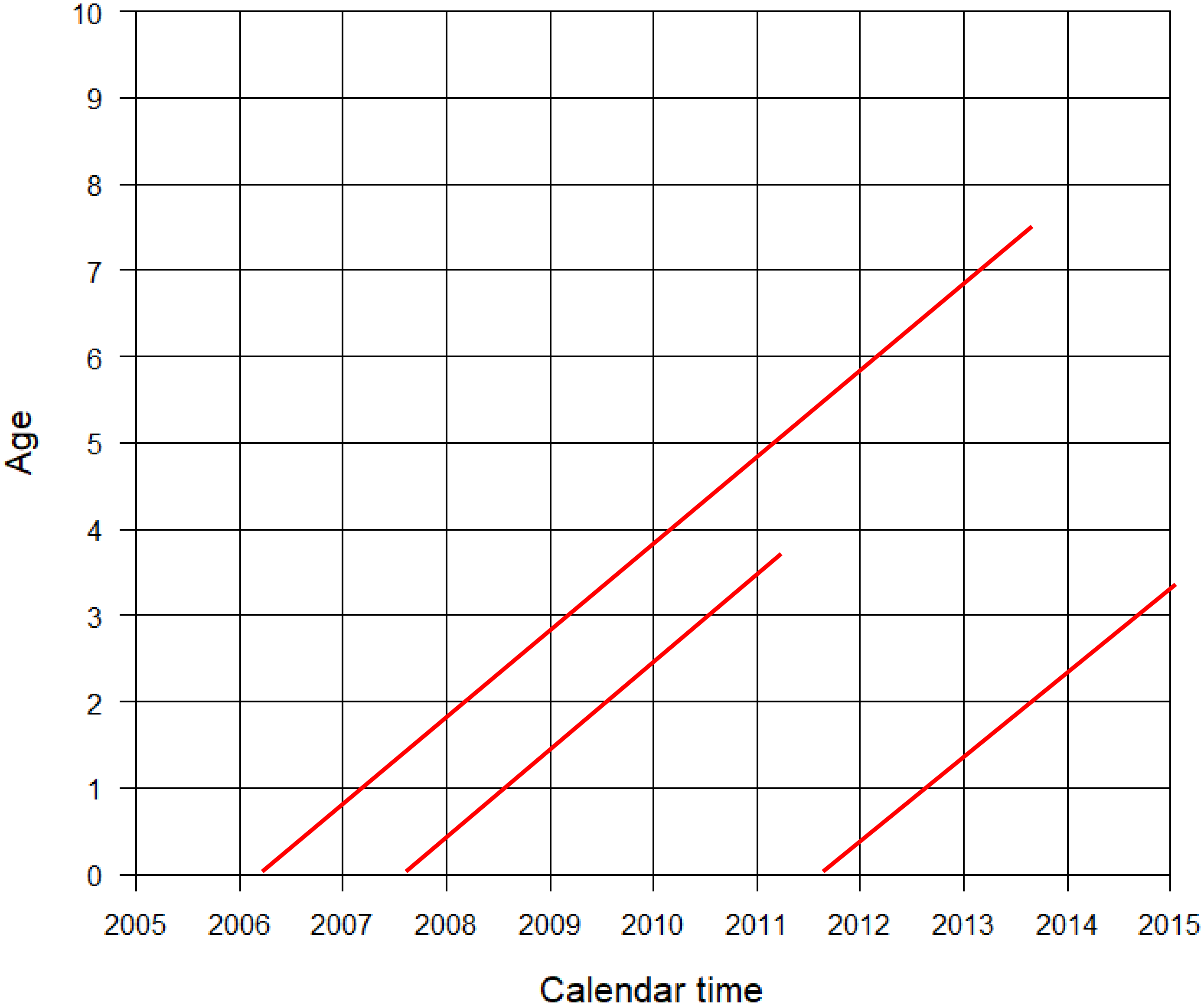
Lexis diagram

In order to explain how life expectancy at birth is computed, it is useful to consider the Lexis diagram, named after the German statistician, economist, and social scientist Wilhelm Lexis (1837-1914)



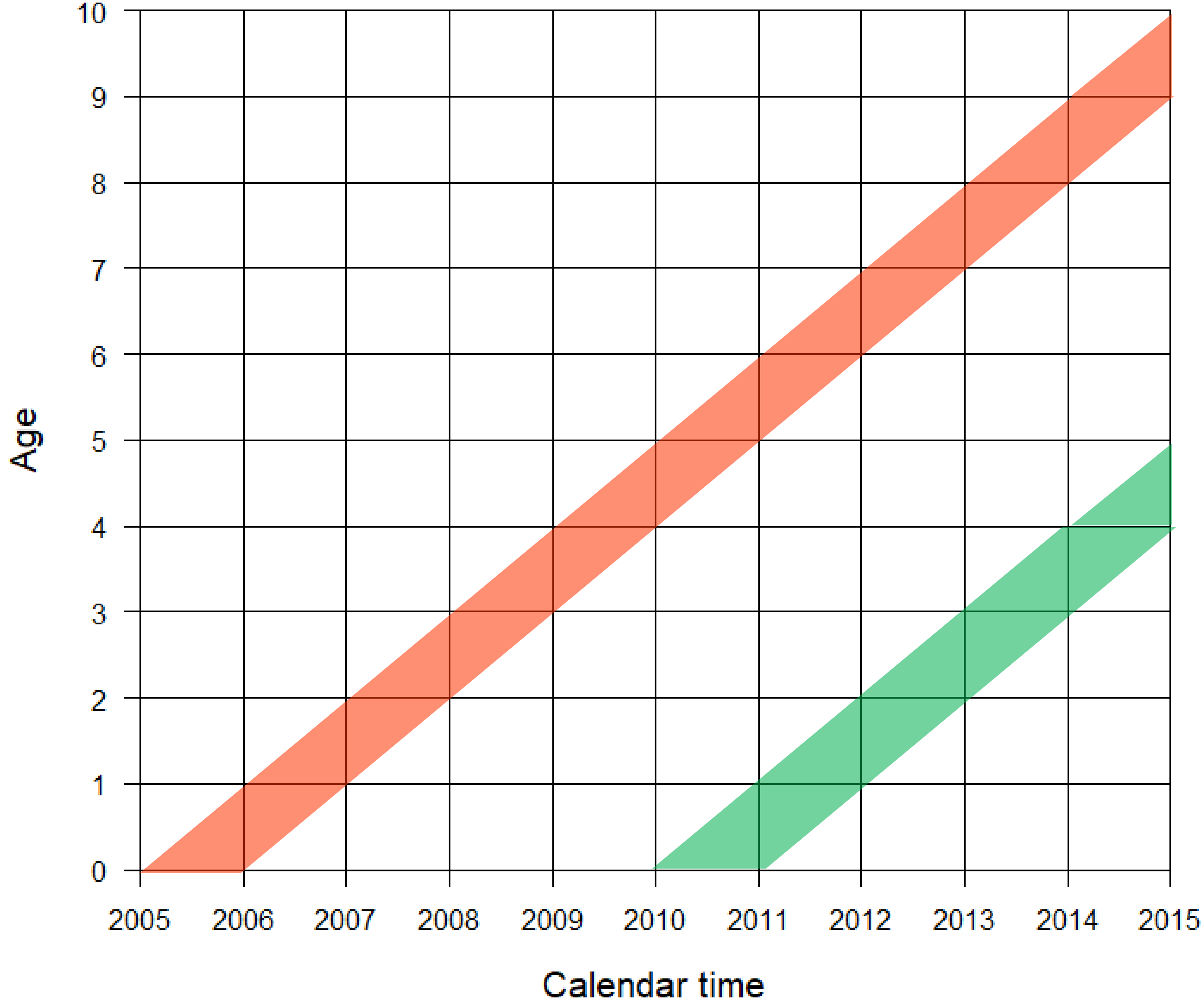
The Lexis diagram shows the relation between the three time scales calendar time, year of birth, and age

Lexis diagram (for a limited number of years and ages)

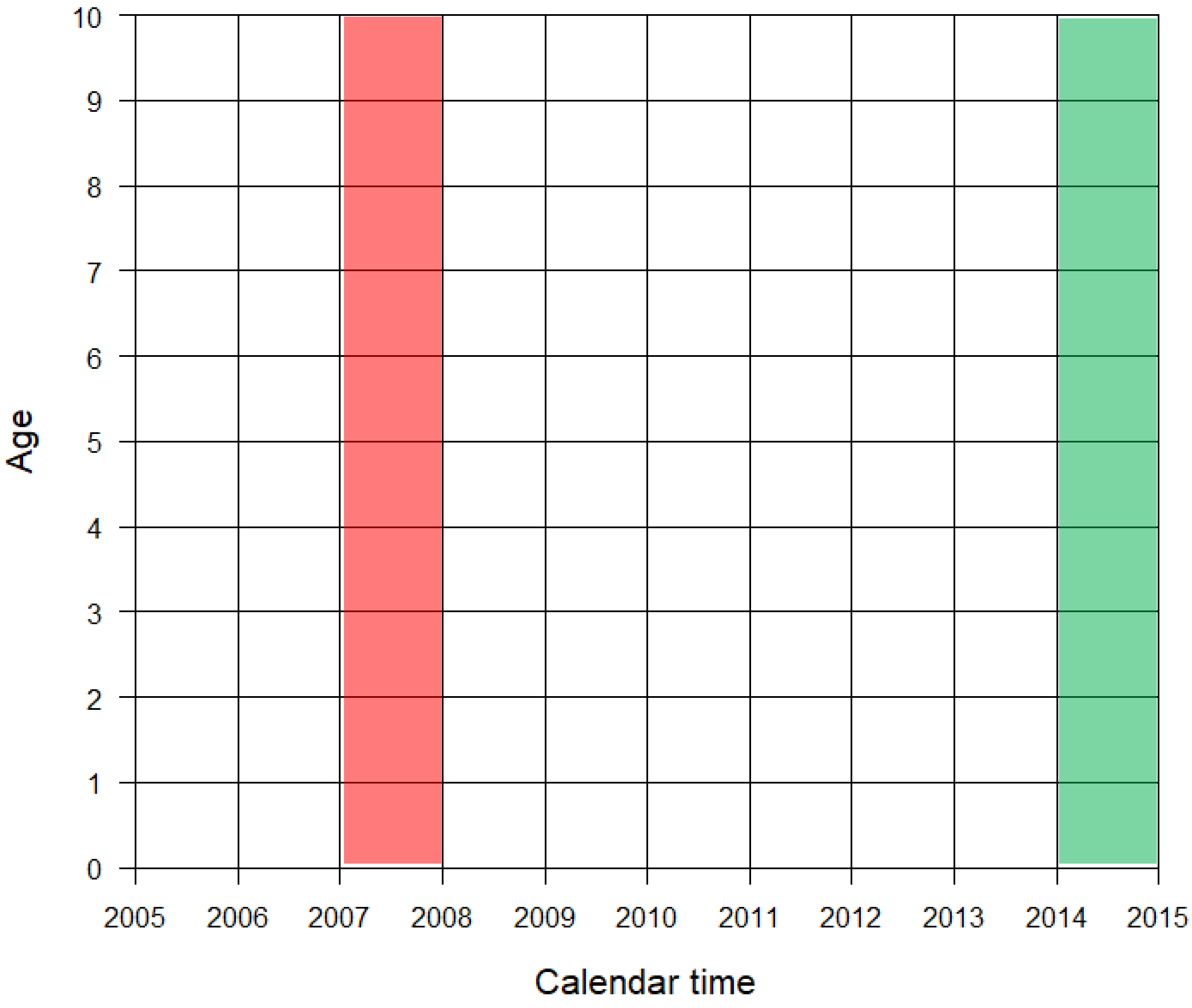


A «lifeline» is starting at birth and ending at death

Lexis diagram with the **cohorts** born in 2005 and 2010



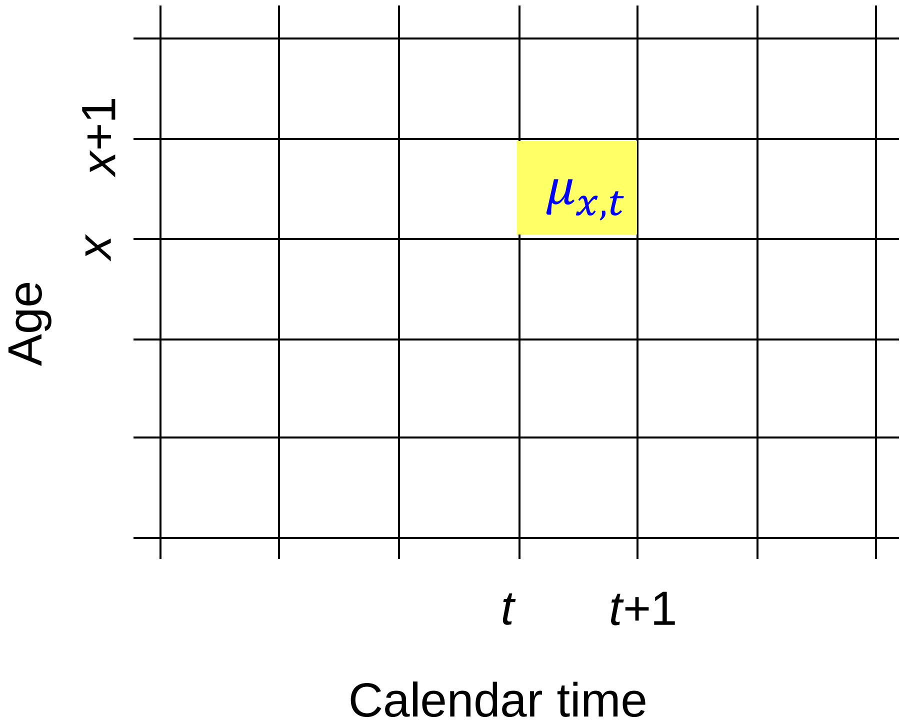
Lexis diagram with the **periods** (calendar years) 2007 and 2014



Assume that mortality rates are constant over each one-year age interval and calendar year (period)

The mortality rate for age *x* and year *t* is denoted $\mu_{x,t}$

The mortality rates are obtained as the number of deaths in the squares divided by the corresponding number of person-years lived (using approximation formulas)



From the mortality rates $\mu_{x,t}$ one-year probabilities of death $q_{x,t}$ for age x and year t may be obtained

We downloaded the probabilities $q_{x,t}$ for the years 1950-2014 from the period life tables given in the Human Mortality Database

Theoretical interlude I

Let X be the life length (in years) of a randomly selected woman from some population, and let ω be the highest possible age ($\omega = 110$ in the Human Mortality Database)

The pdf of X is $f(x), 0 < x < \omega,$ and the survival function is

$$S(x) = P(X > x) = \int_x^\omega f(u)du$$

Expected life length (life expectancy at birth):

$$e_0 = E(X) = \int_0^\omega xf(x)dx = \int_0^\omega S(x)dx$$

Expected number of years lived up to age $a < \omega$:

$$e_{0|a} = \int_0^a S(x)dx$$

Period life expectancy

The life expectancies at birth presented earlier are based on period data, i.e. on the $q_{x,t}$ for a given year *t*

From the $q_{x,t}$ one may compute a survival function $S_{(t)}(x)$ and from this one may obtain the expected life length at birth

More specifically one computes the period survival function $S_{(t)}(x)$ for $x = 1, 2, \dots, 110$ by

$$S_{(t)}(x) = \prod_{y=0}^{x-1} \left(1 - q_{y,t}\right)$$

and obtains $S_{(t)}(x)$ for non-integer values of x by interpolation

The period life expectancy at birth for year t is then found as

$$e_{(t)0} = \int_0^{110} S_{(t)}(x)dx$$

The period life expectancy at birth for year t may be interpreted as the expected life length of a girl born in year t if the age-specific period mortality rates would remain unchanged the next 110 years

The interpretation of the period life expectancy makes assumptions about the future

But period life expectancy at birth for year t is based on information that depends on the past, namely the mortality experience of the cohorts of women born in the years $t, t-1, t-2, \ldots, t-109$

Theoretical interlude II

The mortality differs between women in a birth cohort, some are «robust» and others are more «frail»

As simple model for the variation in mortality is as follows:

Let X be the life length of a randomly selected woman from the birth cohort, and let $Z > 0$ be an (unobserved) «frailty» of the woman

Given Z, we assume that X has hazard rate (mortality rate)

$$\alpha(x|Z) = Z \cdot \alpha(x)$$

The survival function of X, given Z, then becomes

$$S(x|Z) = \exp\{-Z \cdot A(x)\}$$

where $A(x) = \int_0^x \alpha(u)du$

The unconditional survival function, i.e., the population survival function, becomes

$$S(x) = E[\exp\{-Z \cdot A(x)\}]$$

The population mortality rate may be obtained by

$$\mu(x) = -\frac{d}{dx}\log S(x)$$

If we assume that Z is gamma distributed with mean 1 and variance δ we have

$$S(x) = \{1 + \delta A(x)\}^{-1/\delta}$$

and

$$\mu(x) = \alpha(x)/\{1 + \delta A(x)\}$$

A numerical illustration

Consider birth cohorts in two countries and assume that for women in country j we have baseline hazard $\alpha_j(x)$ and variance δ_j for the gamma frailty

The cohort life expectancy at birth for women country j is

$$e_{0j} = \int_0^\omega S_j(x)dx = \int_0^\omega \left\{1 + \delta_j A_j(x)\right\}^{-1/\delta} dx$$

and the cohort mortality rate at age x is given by

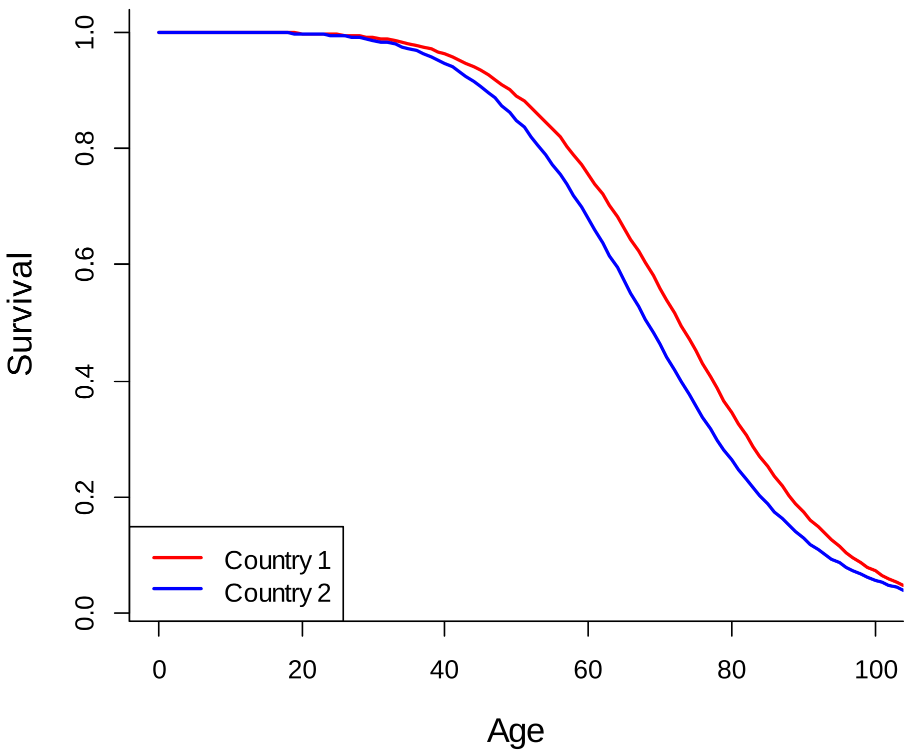
$$\mu_j(x) = \alpha_j(x)\left\{1 + \delta_j A_j(x)\right\}$$

For illustration we let

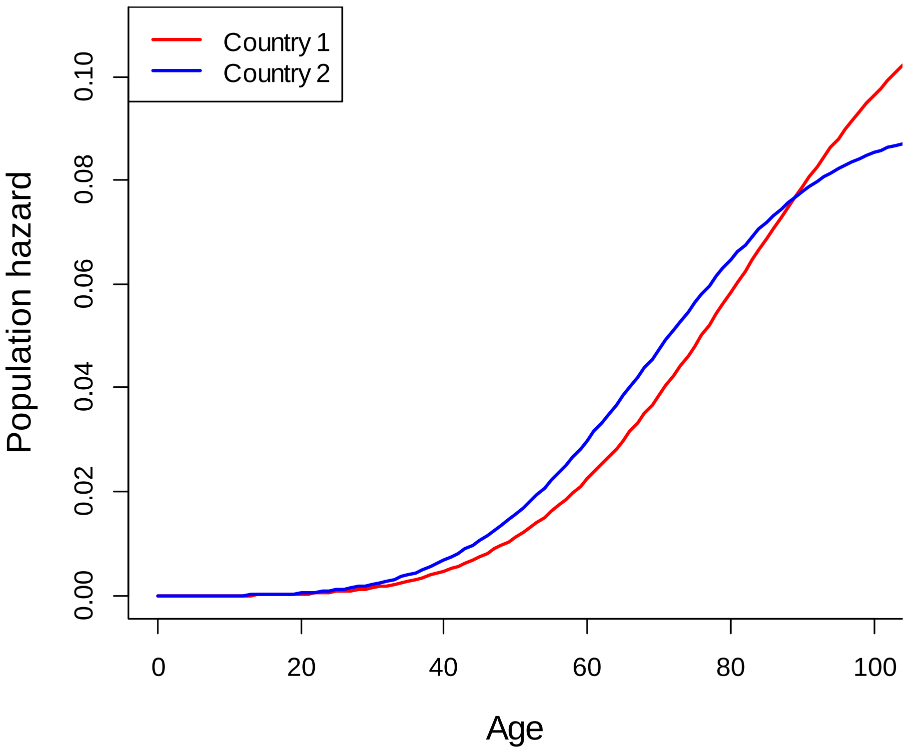
$$A_1(t) = (0.013 \cdot x)^5 \qquad \delta_1 = 0.25$$

$$A_2(t) = (0.014 \cdot x)^5 \qquad \delta_2 = 0.40$$

Population survival functions



Population mortality rates



Life expectancies at birth

e_{01} = 72.9 years

e_{02} = 69.1 years

Due to selection, women in country 2 have lower mortality than women in country1 for high ages

Consequences for period life expectancies

The «frail» members of a birth cohort tend to die earlier than those who are more «robust», which leaves a selected population at higher ages

When the living conditions improve, more of the «frail» cohort members will live longer

For a country («country 1») where the living conditions improved early, the elderly population in our days will contain more «frail» persons than it does in a country («country 2») where the living conditions improved later

Due to selection, we may therefore have $\mu_{x,2014}^{(2)} < \mu_{x,2014}^{(1)}$ for high ages x. But due to improved living conditions (in both countries) we may have $\mu_{x,2014}^{(2)} \approx \mu_{x,2014}^{(1)}$ for lower ages

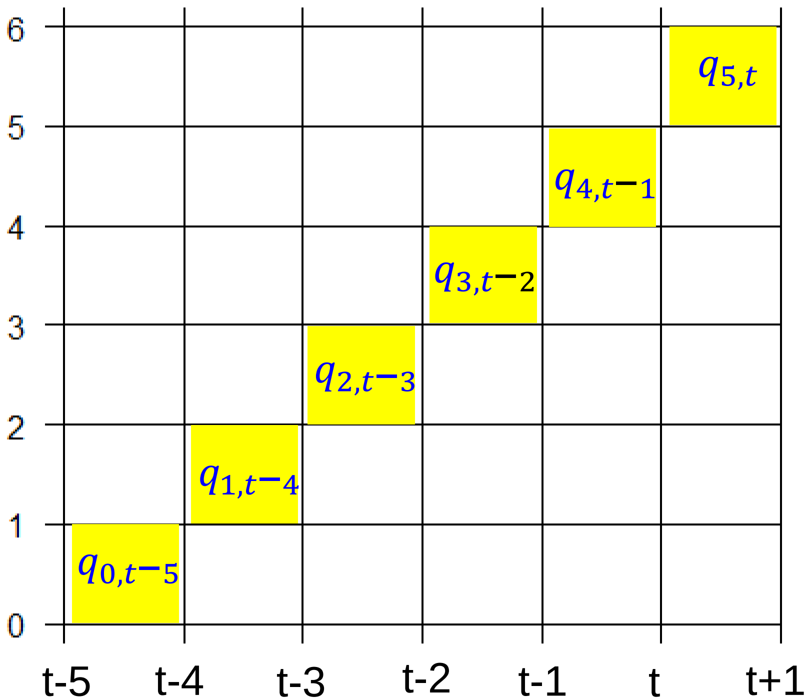
This makes it problematic to compare period life expectancies at birth between two countries when they have had different historical mortality developments

Cohort life expectancy

To avoid the problem with the period life expectancy, we may instead consider cohort life expectancies

But the cohort life expectancy at birth can only be computed for cohorts that were born more than 100 years ago

A (partial) solution is to consider the expected number of years lived for a woman in the cohort up to a given age a



Note that from the one-year death probabilities $q_{x,t}$ obtained from the period life tables, we may obtain the one-year death probabilities $q_x^{(c)}$ at age x for the cohort born in year c by

$$q_x^{(c)} = q_{x,c+x}$$

We may then compute the cohort survival function $S^{(c)}(x)$ for $x = 1, 2, \dots, 2014 - c$ by

$$S^{(c)}(x) = \prod_{y=0}^{x-1} \left(1 - q_y^{(c)}\right)$$

and obtain $S^{(c)}(x)$ for non-integer values of x by interpolation

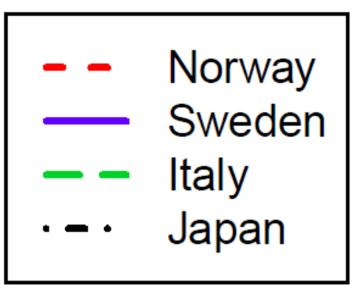
The expected number of years lived up to age a for a woman in the cohort born in year c is then obtained by

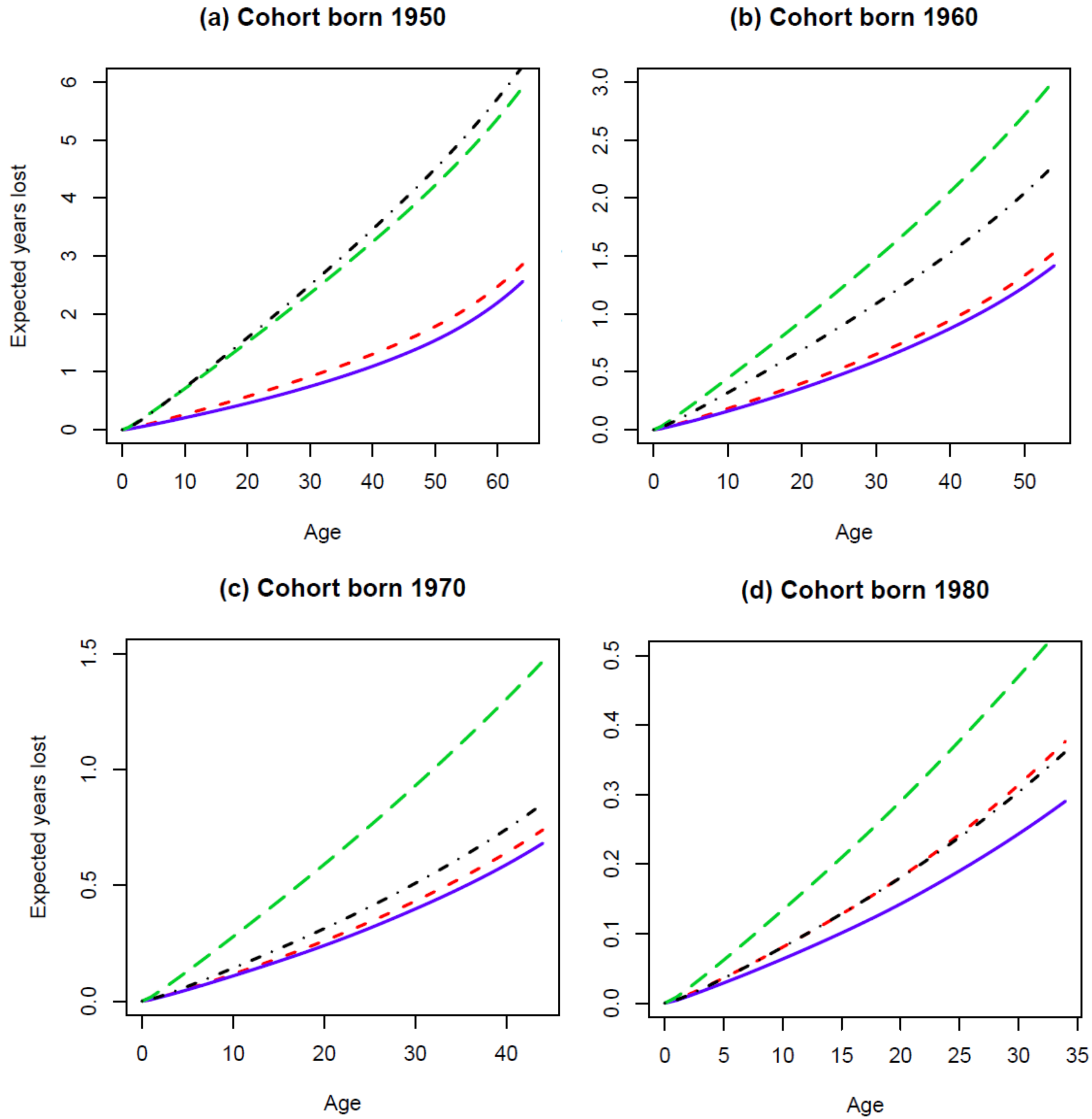
$$e_{0|a}^{(c)} = \int_0^a S^{(c)}(x)dx$$

The comparisons between countries become somewhat easier if we consider the expected number of years lost before age a for the cohort born in year c, i.e.

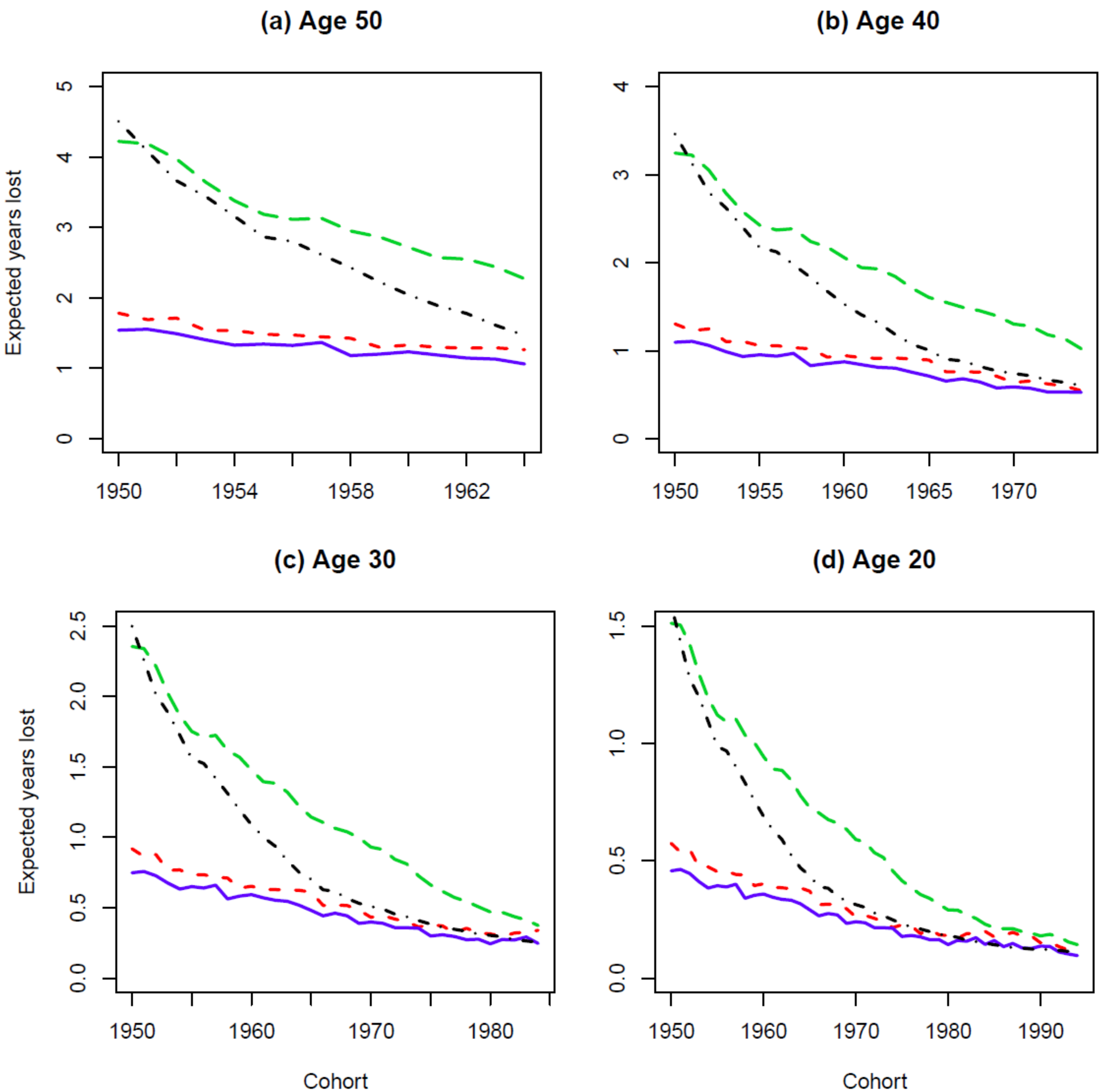
$$a - e_{0|a}^{(c)}$$

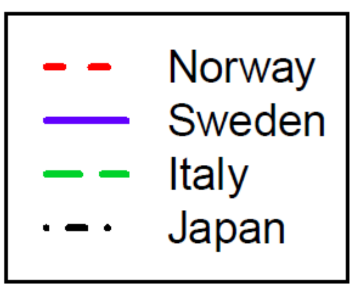
Expected number of years lost as a function of age a for the cohorts born in 1950, 1960, 1970, and 1980





Expected number of years lost before ages 50, 40 ,30, and 20 years as a function of birth cohort c





Conclusions

For all cohorts considered, the expected number of years lost for Italian women is higher than that for Swedish and Norwegian women

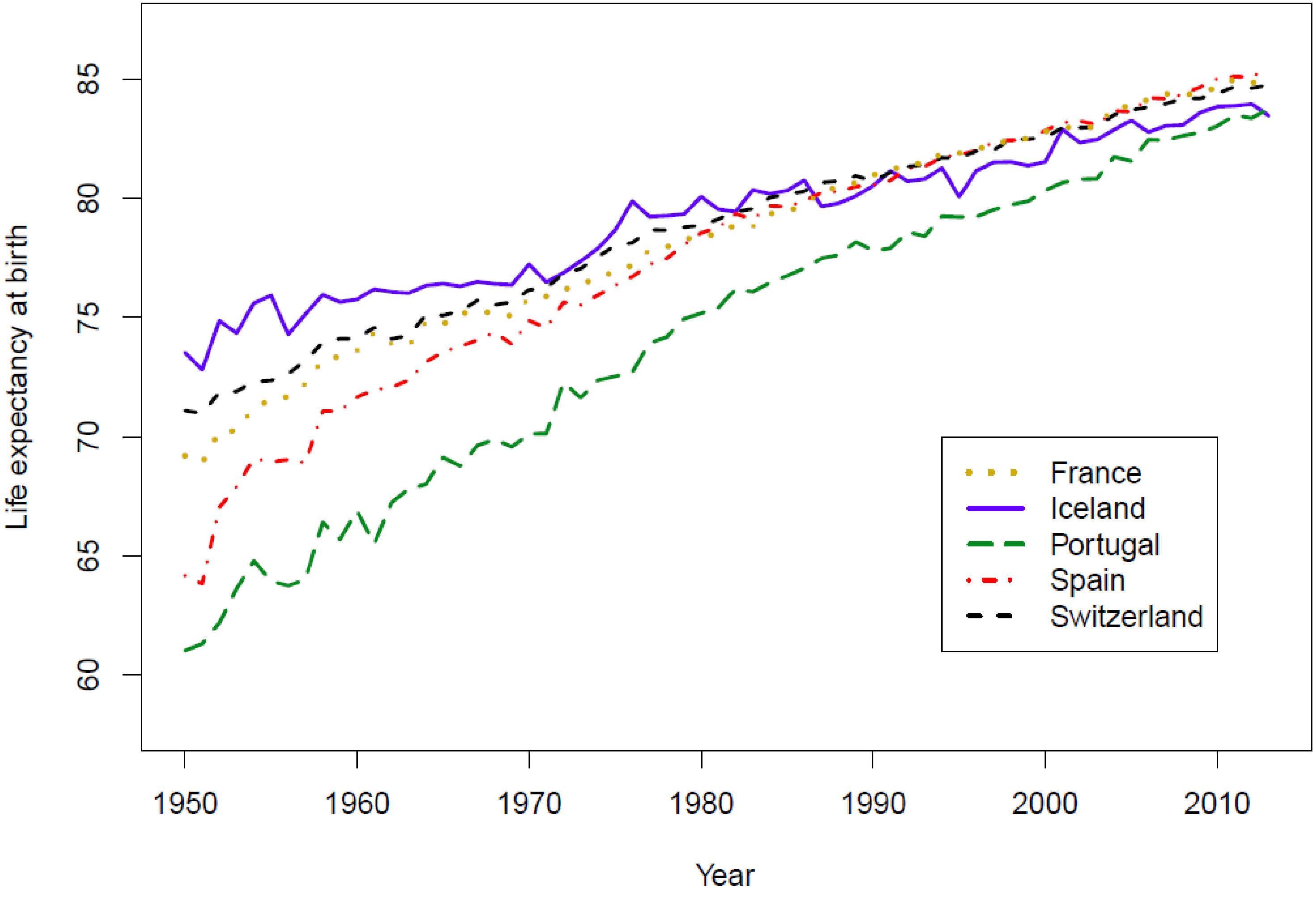
So the period life expectancies give a distorted view when they falsely give the impression that women in Italy may expect to live longer than Scandinavian women

The cohorts of Japanese women born before 1980 may expect to lose more years than women in Scandinavia, and there are only minor differences thereafter

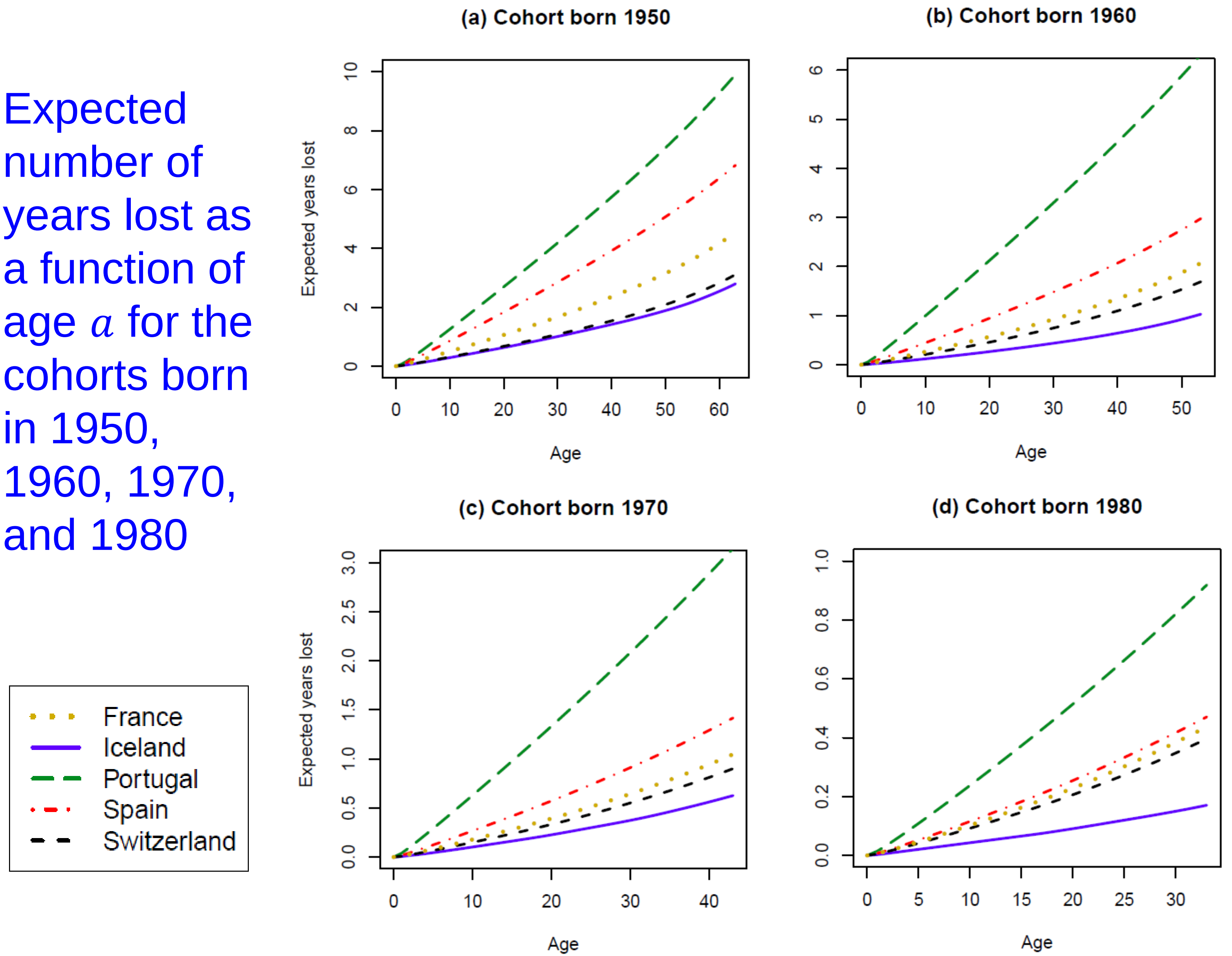
So the cohort results provide no indication that Japanese women may expect to live longer than women in Scandinavia

Other countries

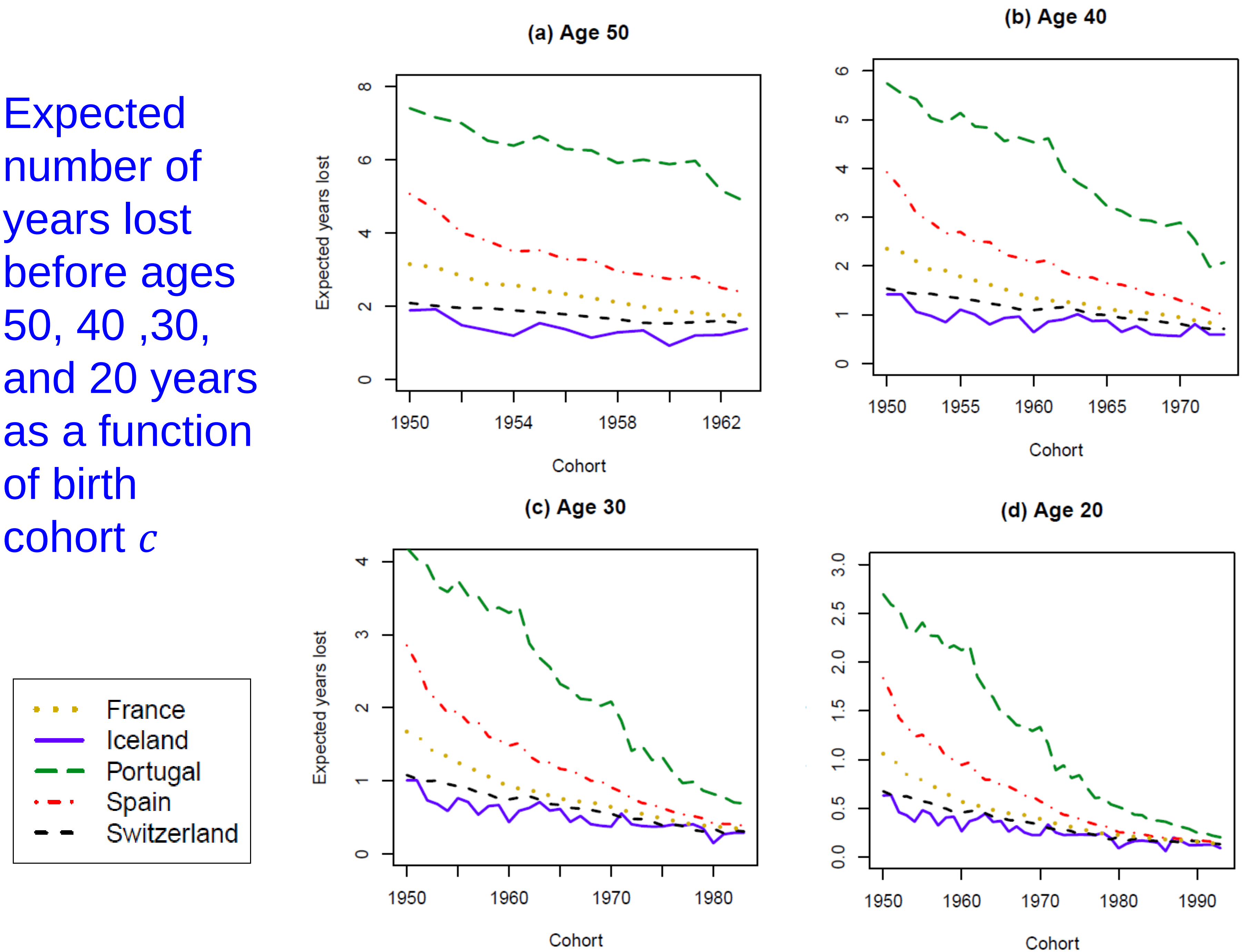
The figure give period life expectancy at birth 1950-2013 for women in five other countries



Expected number of years lost as a function of age a for the cohorts born in 1950, 1960, 1970, and 1980



Expected number of years lost before ages 50, 40 ,30, and 20 years as a function of birth cohort c



General comments

Consider country 1 and country 2, and assume that the living conditions improved earlier in country 1 than in country 2

If we consider life expectancies at birth based on period life tables, country 1 will typically have the longest life expectancy back in time

But the period life expectancy for country 2 will typically increase more rapidly than for country 1, and may eventually overtake the expected life length for country 1

However, if we consider the expected number of years lost for different cohorts c and ages a, it may be the case that people in country 2 may expect to lose more years than people in country 1 for all cohorts c and ages a

In such a situation, the period life expectancies give a distorted view when they give the impression that people in the country 2 may expect to live longer than people in country 1