Oka theory and Complex geometry conference

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1 Title & Abstract

On weighted Bergman spaces of a domain with Levi-flat boundary

Masanori Adachi (Shizuoka University)

Abstract: For each compact hyperbolic Riemann surface we may attach a canonical ruled surface over it using its uniformization. This ruled surface contains a Levi-flat real hypersurface that divides the surface into two 1-convex domains. A feature of this construction is that these two domains admit bounded psh exhaustions but no bounded holomorphic functions except for constants. We discuss structure theorems for the space of holomorphic functions on these domains.

On finitely generated Lie algebras of vector fields

Rafael Andrist (American University of Beirut and University of Ljubljana)

Abstract: Recently, infinitely transitive group actions with the group being generated by only finitely many unipotent groups have been investigated in algebraic geometry. In this talk we will look at this question from the complex-analytic viewpoint: If we can establish the density property for a complex-affine variety using only finitely many completely integrable vector fields, then the generated subgroup of holomorphic automorphisms will act infinitely transitively. Among the examples considered are $\mathbb{C}^n$, $\mathrm{SL}_2(\mathbb{C})$, and the singular quadric surface $xy = z^2$.

Surjective morphisms from affine space to its Zariski open subsets

Viktor Balch Barth (University of Oslo)

Abstract: Which varieties admit a surjective morphism from an affine space of the same dimension? This is a difficult problem to answer in general. In this talk, I will present recent constructive results for special cases of the above problem. In particular, I show that the complement in affine space of certain closed algebraic subsets of codimension at least two admit such surjective maps.

Bergman kernels and Paley-Wiener spaces

Bo Berndtsson (University of Gothenburg)

Abstract: The background of this talk is Nazarov's proof of the Bourgain-Milman theorem, where the basic ingredient is an estimate from below of the Bergman kernel of certain spaces of entire functions. I will give a proof of this estimate that uses subharmonic variation of Bergman kernels and explain the relation to the Bourgain-Milman theorem.

Invariants and Automorphisms for slice regular functions

Cinzia Bisi (University of Ferrara)

Abstract: Let A be one of the following algebras : $\mathbb{R}_2 \cong \mathbb{H}$, (the real algebra of quaternions) or the Clifford algebra $\mathbb{R}_3 \cong \mathbb{H} \oplus \mathbb{H}$. For the algebra A, the automorphism group Aut(A) and its invariants are well known. In this talk we will describe the invariants of the automorphism group of the algebra of slice regular functions over A. We recall that the notion of slice regularity generalizes the holomorphicity over C.

Universal entire curves in projective spaces with slow growth

Zhangchi Chen (Chinese Academy of Science))

Abstract: In 1929, Birkhoff constructed some universal entire functions. An entire function h is universal if any entire function g can be approximated by translations of h, in the topology of uniform convergence on compact sets. The universality inspires a research area in operator theory called hypercyclity.

The Nevanlinna characteristic function measures complexity of a transcendental entire function. Dinh-Sibony in their problem list (Problem 9.1) asked the minimal growth of the Nevanlinna characteristic function of universal entire maps in general complex manifolds.

At first glance, universal entire functions looks highly transcendental and highly complicated. However, in the recent work with Song-yan Xie and Dinh Tuan Huynh, we proved that the

existence of universal entire curves in any n-dim projective spaces with growth slower than any transcendental growth rate.

Our idea is motivated by Runge's approximation theorem and the theory of Oka manifolds. I will also present some open problems.

Versal families of real structures on complex spaces

Adrien Dubouloz (University of Bourgogne, CNRS)

Abstract: The aim of the talk will be to motivate and present the essential elements of a construction of a moduli refinement of Borel-Serre classification of anti-holomorphic involutions on a complex analytic space in terms of the first Galois cohomology of its automorphism group. In the algebraic case, it provides in particular a natural framework to study families of real forms of a given real algebraic variety depending algebraically on some real parameters. I will illustrate the notion with examples of such varieties admitting arbitrarily large moduli number of pairwise non isomorphic real forms.

The polyhedral type of a complex polynomial map on the plane

Boulos El Hilany (TU Braunschweig)

Abstract: We say that two continuous maps $f, g : X \to Y$ are topologically equivalent if there exist homeomorphisms $\varphi : X \to X$ and $\psi : Y \to Y$ such that $\psi \circ f \circ \varphi = g$. Given any positive integer d, it is known that there are finitely many topological equivalence classes of polynomial maps of degree d on the complex plane. Due to the lack of a systematic approach, their number, denoted by $\Theta_{\mathbf{C}}(d)$, is known only for the case $d = 2$.

The Newton tuple of a polynomial map is the collection of Newton polytopes corresponding to the respective polynomials. It is shown that a generic map (in the open Zariski sense) with fixed Newton pair contributes to one element in $\Theta_{\mathbf{C}}(d)$. I will present results relating topological invariants of generic maps to their Newton pairs. This gives rise to a polyhedra method that produces a lower bound on $\Theta_{\mathbf{C}}(d)$ for arbitrary d. We compute the latter by implementing a software package on OSCAR.

Joint work with Kemal Rose (MPI Leipzig).

Oka-1 manifolds

Franc Forstneric (University of Ljubljana)

Abstract: We introduce a new class of complex manifolds: Oka-1 manifolds. They are characterized by the property that holomorphic maps from any open Riemann surface satisfy the

Runge approximation and the Weierstrass interpolation condition. We prove that every complex manifold which is dominable by tubes of complex lines is an Oka-1 manifold. In particular, a manifold dominable by $\mathbb{C}^n$ at most points is an Oka-1 manifold. This provides many examples of Oka-1 manifolds among compact algebraic surfaces, including all Kummer and all elliptic K3 surfaces. We also show that every compact rationally connected manifold is an Oka-1 manifold. The class of Oka-1 manifolds is invariant under Oka maps inducing a surjective homomorphism of fundamental groups; this includes holomorphic fibre bundles with connected Oka fibres. In another direction, we prove that every bordered Riemann surface admits a holomorphic map with dense image in any connected complex manifold.

An upper bound for polynomial volume growth of automorphisms of zero entropy

Fei Hu (Nanjing University)

Abstract: Let X be a compact Kähler manifold of dimension d and f an automorphism of X. Suppose that the pullback $f^*|_{H^{1,1}(X)}$ of f on the Hodge cohomology group $H^{1,1}(X)$ is unipotent and denote the index of the eigenvalue 1 by k+1. We prove an upper bound for the polynomial volume growth plov(f) of f as follows:

$plov(f) \leq (k/2+1)d.$

Combining with the inequality $k \leq 2(d-1)$ due to Dinh–Lin–Oguiso–Zhang, we obtain an optimal inequality that

$plov(f) \leq d^2,$

which affirmatively answers questions of Cantat–Paris-Romaskevich and Lin–Oguiso–Zhang.

Symplectic holomorphic density property for Calogero-Moser spaces

Gaofeng Huang (University of Bern)

Abstract: The n-th Calogero-Moser space is the completed phase space of a system of n indistinguishable particles on a complex line, interacting through inverse square potential. By a work of Wilson, its mathematical model is known to be a smooth complex affine variety with a holomorphic symplectic structure, coming from complex symplectic reduction and this variety is diffeomorphic to the Hilbert scheme of n points on the affine plane. We consider four completely integrable symplectic holomorphic vector fields on this manifold, then compute new symplectic vector fields by taking linear combination and Lie bracket. It turns out that this process does generate all algebraic fields, thus giving the symplectic holomorphic density property of this Stein manifold. Joint work with Rafael B. Andrist.

On generic topological type of a complex plane polynomial mapping

Zbigniew Jelonek (University of Warsaw)

Abstract: We describe the topology of a generic polynomial mapping $f\mathbf{C}^2 \to \mathbf{C}^2$ of a given degree d.

Gromov's ellipticity of cones

Shulim Kaliman (University of Miami)

Abstract: We show that the notions of algebraic Gromov's ellipticity and subellipticity are equivalent. We prove that every complete uniformly rational variety is elliptic. We consider generalized cones over such varieties defined by ample divisors. We prove that every such cone punctured at the vertex is also elliptic.

Baum-Bott residue currents

Lucas Kaufmann (University of Orleans)

Abstract: Consider a (singular) holomorphic foliation F on a complex manifold. In the 1970s, Baum-Bott constructed residue classes associated with each singular component of F. These classes satisfy an index theorem, which computes characteristic classes of F.

A natural question since Baum-Bott's work is how to compute or find natural representatives of such cohomology classes. Apart from the case where the foliation is of rank one and has isolated singularities, no general result is known. In this talk I will show that each Baum-Bott class can be naturally represented by a residue current supported by the singular component of F. This is a joint work with Richard Lärkäng and Elizabeth Wulcan.

Surjective morphisms onto Gromov elliptic varieties

Yuta Kusakabe (Kyoto University)

Abstract: Gromov introduced ellipticity conditions for complex manifolds and algebraic varieties, and established the Oka principle for maps to elliptic complex manifolds. Roughly speaking, Gromov ellipticity means the existence of many dominating maps from affine spaces. Gromov's Oka principle was generalized to Oka manifolds by Forstnerič, who used it to prove that every Oka manifold admits a surjective holomorphic map from an affine space. In this talk, we will discuss the algebraic analogue of Forstnerič's theorem on surjective maps and its applications to algebraic Oka theory.

Generic dynamics on Oka-Stein manifolds and Stein manifolds with the density property

Finnur Larusson (University of Adelaide)

Abstract: I will describe joint work with Leandro Arosio (arXiv 2102.02195 and 2303.05002) on the dynamics of generic endomorphisms of Oka-Stein manifolds and generic automorphisms of Stein manifolds with the density property.

Factorization of holomorphic matrices and Gromov's Oka principle

Erik Løw (University of Oslo)

Abstract:

Geometry of complex polynomial maps at infinity

Thi Bich Thuy Nguyen (University of Estadual Paulista)

Abstract: In this talk, we present some recent results on complex polynomial maps approached by intersection homology, asymptotic sets and Newton polygons. In particular, a partial result of the 2-dimensional complex Jacobian conjecture will be presented: We prove that the conjecture is true until degree 104 (the degree 100 was proved by Moh in 1983).

The generalized Lelong numbers and intersection theory

Viet-Anh Nguyen (University of Lille)

Abstract: The notion of Lelong number $\nu(T, x)$ of a positive closed current T at a single point x in an ambient complex manifold X plays a fundamental role in Complex Analysis and Complex Geometry. In 1982 Henri Skoda formulated this notion for the more general class of positive plurisubharmonic currents.

In this talk we introduce a new concept of the generalized Lelong numbers $\nu_j(T, V)$, where V is a submanifold in X and T is a positive plurisubharmonic current in X. In general, we have $\dim V + 1$ generalized Lelong numbers associated to T along V. The classical case where $V = \{x\}$ corresponds to $\dim V = 0$. Our present research is inspired by two works. The first one is the theory of tangent currents for positive closed currents which were developed by Tien-Cuong Dinh and Nessim Sibony (2018). The second work is the theory of the Lelong number for positive plurisubharmonic currents along a complex linear subspace in $\mathbf{C}^n$ which were developed by Lucia Alessandrini and Giovanni Bassanelli (1996).

Next, we study these new numerical values and establish their basic properties. In particular, we obtain geometric characterizations as well as an upper-semicontinuity of the generalized Lelong numbers in the sense of Yum-Tong Siu (1974). When the current T is positive closed, we also establish some links between the generalized Lelong numbers and Dinh-Sibony cohomology classes.

Finally, as an application we give an effective condition (in terms of generalized Lelong numbers) ensuring that m positive closed currents $T_1, \ldots, T_m$ of possibly different bidegrees on a compact Kähler manifold X are wedgable in the sense of Dinh-Sibony.

Kahler-Einstein Bergman metrics on pseudoconvex domains of dimension two

Nikhil Savale (University of Cologne))

Abstract: This talk consists of two parts. In the first part, I will review the construction of a pointwise Boutet de Monvel-Sjostrand parametrix for the Szego kernel of a weakly pseudoconvex three dimensional CR manifold of finite type and Fefferman type boundary asymptotics of the Bergman kernel on weakly pseudoconvex domains in dimension two. The second part presents an application where we prove that a weakly pseudoconvex two dimensional domain of finite type with a Kahler-Einstein Bergman metric is biholomorphic to the unit ball. Based on joint works with C.Y. Hsiao and M. Xiao.

Degenerate J-flow on compact Kähler manifolds

Tat-Dat To (Sorbonne University)

Abstract: We study a degenerate twisted J-flow on compact Kähler manifolds. We show that it exists for all time, it is unique and converges to a weak solution of a degenerate twisted J-equation. In particular, this confirms an expectation formulated by Song-Weinkove for the J-flow. As a consequence, we establish the properness of the Mabuchi K-energy twisted by a certain semi-positive closed (1,1)-form for Kähler classes in a certain subcone.

Universal holomorphic maps with slow growth

Song-Yan Xie (Chinese Academy of Science)

Abstract: We design an Algorithm to fabricate universal holomorphic maps between any two complex Euclidean spaces, within preassigned transcendental growth rate. As by-products, universal holomorphic maps from $\mathbb{C}^n$ to $\mathbb{CP}^m$ ($n \leqslant m$) and to complex tori having slow growth

are obtained. We take inspiration from Oka manifolds theory, Nevanlinna theory, and hypercyclic operators theory. This is joint work with Bin Guo.

Green's function and partial Bergman kernels

Xu Wang (NTNU)

Abstract: We shall study the relation between Green's function and the partial Bergman kernels. For domains in the complex plane, similar to the proof of the Suita conjecture we obtain a lower bound of the partial Bergman kernel in terms of Green's function. In particular, our estimate gives a partial Bergman approximation of Green's function.

Chern currents

Elizabeth Wulcan (University of Gothenburg))

Abstract: I will discuss a joint work with Richard Lärkäng, partly in progress, which aims at finding explicit representatives of Chern classes in terms of currents. Given a locally free resolution of a coherent analytic sheaf $\mathcal{F}$, equipped with Hermitian metrics and connections, we construct an explicit current, obtained as the limit of certain smooth Chern forms, that represents the Chern class of $\mathcal{F}$ and has support on the support of $\mathcal{F}$. If $\mathcal{F}$ has pure dimension the first non-trivial component coincides with the fundamental cycle of $\mathcal{F}$. This can be seen as a generalization of the classical Poincaré-Lelong formula.