Workshop on the Standard Conjectures, (Weil's) Riemann Hypothesis, and Relations to Dynamical Systems

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Supported by RCN grant 300814

June 23-25, 2021

1 Title & Abstract

The standard conjecture of Hodge type for abelian fourfolds

Giuseppe Ancona (l'Université de Strasbourg)

Abstract: Let S be a surface, V be the $\mathbb{Q}$-vector space of divisors on S modulo numerical equivalence and d be the dimension of V. The intersection product defines a non degenerate quadratic form on V. The Hodge index theorem says that it is of signature $(1, d-1)$. In the Sixties Grothendieck conjectured a generalization of this statement to cycles of any codimension on a variety of any dimension. In characteristic zero this conjecture is a consequence of Hodge theory but in positive characteristic almost nothing is known. Instead of studying these quadratic forms at the archimedean place we will study them at p-adic places. It turns out that this question is more tractable, thanks to p-adic Hodge Theory. Moreover, using classical product formulas on quadratic forms, the p-adic result will give non-trivial informations on the archimedean place. For instance, we will prove the original conjecture for abelian fourfolds.

$\mathbb{Q}_\ell$- versus $\mathbb{F}_\ell$-coefficients for the Grothendieck–Serre and Tate (G–S/T) conjectures

Anna Cadoret (Institut de Mathématiques de Jussieu-Paris Rive Gauche)

Abstract: The conjectural formalism of pure motives predicts that every Weil cohomology should be the incarnation of a universal motivic cohomology. In particular, for a smooth projective variety over a finitely generated field the semisimplicity (Grothendieck–Serre) and

fullness (Tate) conjectures should be independent of the Weil cohomology one considers. I will discuss the relation between the (G–S/T) conjectures for $\mathbb{Q}_\ell$- and $\mathbb{F}_\ell$-coefficients and explain in particular that the (G–S/T) conjectures for $\mathbb{F}_\ell$-coefficients and ℓ large enough always imply the (G–S/T) conjectures for $\mathbb{Q}_\ell$-coefficients and that the converse holds if the base field is of positive characteristic. The natural explanation for these results is the existence of an ultraproduct variant of the motivic formalism. This work was the starting point for the development of a general ultraproduct formalism paralleling the one for ℓ-adic sheaves (see Weizhe Zheng's talk). It is joint work with Chun Yin Hui and Akio Tamagawa.

Spectral interpretations of dynamical degrees and degree growth problems

Nguyen-Bac Dang (Stony Brook University)

Abstract: This talk is based on a joint work with Charles Favre. I will explain how one can control the degree of the iterates of rational maps in arbitrary dimension using some particular norm on the space of b-divisors and on the spaces of b-classes.

Dynamical systems for arithmetic schemes

Christopher Deninger (University of Münster)

Abstract: We attach infinite dimensional dynamical systems to arithmetic schemes X and discuss some basic properties of these systems and in particular the relation between the periodic orbits and their lengths on the one hand and the closed points of X and their norms on the other. We construct these dynamical systems as the "points" of certain new ringed spaces involving sheaves of rational Witt vectors. In the zero dimensional case this is related to work of Kucharczyk and Scholze. If X is the spectrum of the p-adic integers and one takes p-adic "points" then a canonical subsystem is closely related to the Fargues–Fontaine curve.

The gamma filtration on K-theory

Matthias Flach (California Institute of Technology)

Abstract: Recently Barwick, Glasman, Mathew and Nikolaus have shown that the algebraic K-theory space of a stable infinity category is functorial under arbitrary polynomial rather than just exact functors. We use their work to generalize Grothendieck's gamma-filtration from K_0 to the K-theory space. The graded pieces are a new version of motivic cohomology which unlike all previous definitions is not based on $\mathbb{A}^1$-invariance.

A dynamical approach to Weil's Riemann hypothesis

Fei Hu (University of Oslo)

Abstract: Inspired by a result of Esnault and Srinivas on automorphisms of surfaces and recent advances in complex dynamics, Truong raised a question on the comparison of two dynamical degrees, which are defined using pullback actions of dynamical correspondences on numerical cycle class groups and cohomology groups, respectively. An affirmative answer to his question would surprisingly imply Weil's Riemann hypothesis. In this talk, we consider more comparison problems on the norms and spectral radii of the pullback actions of certain correspondences. I will talk about their connections with Truong's dynamical degree comparison and the standard conjectures. Under certain technical assumption, some partial results will be given. I will also discuss some applications to Abelian varieties and surfaces. This is based on joint work with Tuyen Truong.

Random walks on $\mathrm{SL}_2(\mathbb{C})$

Lucas Kaufmann (IBS Institute for Basic Science)

Abstract: Given a sequence of independent and identically distributed random 2-by-2 complex matrices, it is a classical problem to study the statistical properties of their product. In this talk I will show how methods from complex analysis can be used to obtain several new limit theorems for these random processes, often in their optimal version. This is based on joint works with Tien-Cuong Dinh and Hao Wu.

D-algebras: A general "formalism" for the study of connections in algebra

Helge Øystein Maakestad

Abstract: In 3 papers published on the arXiv preprint server in the period 2015-2019 I introduced the notion D-algebra. This is a notion "generalizing" the notion "Lie-Rinehart algebra" in the sense that a D-algebra is a Lie-Rinehart algebra equipped with the structure of a right A-module (or an $A \otimes A$-module). This construction is similar to the construction of the first order jet bundle: It has the structure of an $A \otimes A$-module. In the talk I will construct the enveloping algebra $U(L)$ of a D-algebra L, all non-abelian extensions of a D-algebra and Ext and Tor groups of connections on D-algebras.

The enveloping algebra $U(L)$ is universal in two ways: Any connection is a left $U(L)$-module and any "ring of differential operators" U is a quotient of $U(L)$. Given any affine groupoid scheme $G := G_1/G_0$, you may attach two objects to G: A Lie-Rinehart algebra $LR(G)$ and a D-algebra $D(G)$, and the Lie-Rinehart algebra $LR(G)$ is canonically a quotient of $D(G)$ by an ideal. Hence the D-algebra $D(G)$ of G has "more structure" than $LR(G)$. It may be interesting to study $D(G)$ and its relation to "moduli spaces" and "algebraic stacks".

Much work has been done on introducing general formalism and constructions related to connections, principal fiber bundles, Lie algebroids, differentiable stacks and one could try to introduce similar constructions in the algebraic category using D-algebras: Chern-Weil theory, gerbes, etc.

Topological Hochschild homology and Zeta-values

Baptiste Morin (Institut de Mathématiques de Bordeaux)

Abstract: We give a conjectural description of Zeta-values of arithmetic schemes at $s = n$ for any integer $n \in \mathbb{Z}$, in terms of two perfect complexes of abelian groups. The first complex is called Weil-étale motivic cohomology with compact support. The second complex can be thought of as derived de Rham cohomology relatively to the sphere spectrum, and is defined using topological Hochschild homology. The functional equation of zeta functions together with our description of Zeta-values implies a formula relating these complexes, special values of the archimedean Euler factors and Bloch's conductor. If time permits, we will state this formula, which can actually be proven. This is joint work with Matthias Flach.

The birational Torelli problem for Calabi–Yau 3-folds

Jørgen Vold Rennemo (University of Oslo)

Abstract: The birational Torelli problem for Calabi–Yau 3-folds asks whether two deformation equivalent Calabi–Yau 3-folds are necessarily birational if they have isomorphic Hodge structures on their middle cohomology. I'll explain an example, given by intersecting two translates of $\mathrm{Gr}(2, 5)$ inside $\mathbb{P}^9$, which shows that this question has a negative answer. This is joint work with John Christian Ottem.

The (non-uniform) Hrushovski–Lang–Weil estimates

Shuddhodan Kadattur Vasudevan (Purdue University)

Abstract: In 1996 using techniques from model theory and intersection theory, Hrushovski obtained a generalisation of the Lang–Weil estimates. Subsequently the estimate has found applications in group theory, algebraic dynamics and algebraic geometry. We shall discuss an ℓ-adic proof of the non-uniform version of these estimates and rationality of the associated generating function.

Higher Lelong numbers and full mass intersection

Duc-Viet Vu (University of Cologne)

Abstract: It was a long standing question in pluripotential theory asking whether one can recover higher Lelong numbers of a plurisubharmonic function by using its analytic data. Chi Li recently gave an explicit counter-example to this question. Motivated by the theory of non-pluripolar products in the compact setting, he conjectured that his example can be generalized to a much larger natural class of psh functions. We present in the talk an affirmative answer to this question. This is a joint work with Do Duc Thai.

Realization problems for degree sequences

Junyi Xie (Université de Rennes 1)

Abstract: This talk is based on a joint work with Cantat and Déserti. For a dominant rational self-map f of some projective space, its degree sequence is $(\deg f^n)_{n\geq 0}$. An interesting and difficult problem is to determine which sequences can be realized as a degree sequence. In this talk, we ask some questions relate to this problem and give some partial answer. In particular, we show that every bounded degree sequence over $\mathbb{C}$ can be realized as a degree sequence over $\overline{\mathbb{Q}}$.

Ultraproduct cohomology and the decomposition theorem

Weizhe Zheng (Morningside Center of Mathematics, Chinese Academy of Sciences)

Abstract: Ultraproducts of étale cohomology provide a large family of Weil cohomology theories for algebraic varieties. Their properties are closely related to questions of ℓ-independence and torsion-freeness of ℓ-adic cohomology. I will present recent progress in ultraproduct cohomology with coefficients, such as the decomposition theorem. This talk is based on joint work with Anna Cadoret.