Workshop on Stochastics, Memory and Roughness 2024

University of Oslo and STORM Project

January 17–19, 2024

Contents

Abstracts

On Some Recent Aspects of Stochastic Controlled Dynamics driven by G-Brownian Motion

Jan 19
11:00

Redjil Amel

Badji Mokhtar University

In the last years, aspects of model ambiguity such as volatility uncertainty have been studied by S. Peng in 2006 who introduced the G-expectation space with a process called G-Brownian motion, in the G-framework, the corresponding Itô stochastic calculus was established. The existence and uniqueness of solution of a G-SDE was proved. This work is devoted to study stochastic differential equations driven by G-Brownian motion. Under the global Lipschitz and the linear growth conditions, the existence and uniqueness of solution are proved. We investigate the relaxed stochastic control problem under incertainty. We prove the temporal Hölder regularity of the solution, under the Linear growth and the global Lipschitz conditions of the coefficients with respect to the state variable uniformly in the time. An example is given to support the effectiveness of our main results.

TBA

Jan 18
11:00

Nacira Agram

KTH Royal Institute of Technology

TBA

Jan 17
11:35

On the Skew and Curvature of the Implied and Local Volatilities

Elisa Alòs
Universitat Pompeu Fabra

In this talk, we study the relationship between the short-end of the local and the implied volatility surfaces. Our results, based on Malliavin calculus techniques, recover the recent $1/(H + 3/2)$ rule (where H denotes the Hurst parameter of the volatility process) for rough volatilities (see F. Bourgey, S. De Marco, P. Friz, and P. Pigato, 2022), that states that the short-time skew slope of the at-the-money implied volatility is $1/(H + 3/2)$ of the corresponding slope for local volatilities. Moreover, we see that the at-the-money short-end curvature of the implied volatility can be written in terms of the short-end skew and curvature of the local volatility and vice-versa. Additionally, this relationship depends on H.

Jan 17
14:35

The Fokker-Planck equation of the time-changed fractional Brownian motion and Ornstein-Uhlenbeck processes

Giacomo Ascione
Scuola Superiore Meridionale

Nowadays, probabilistic techniques based on Feller processes are widely used to prove several properties of the solutions of some linear parabolic partial differential equations. Vice versa, different properties of Feller processes can be deduced by their forward and backward Kolmogorov equations. In [3] the authors successfully extended the theory of stochastic representations (or probabilistic solutions) to time-fractional abstract Cauchy problems. In particular, the stochastic process whose conditional expected value defines the solution of such an equation can be constructed as the time-change, by means of the inverse of a stable subordinator, of a suitable Feller process. This also allowed the use of time-fractional parabolic partial differential equations to study the properties of the aforementioned time-changed process, which not only constitute a large class of semi-Markov processes, but are also widely used in the applications to describe, for instance, trapping phenomena in hydrology or different scaling limits in queueing theorem. In the latter context, also time-changed fractional Brownian motions arise as suitable scaling limits. However, since the original process (the fractional Brownian motion) is not even a Markov process, this case is not covered by the results in [3]. Thanks to the particular structure of the one-dimensional density of Gaussian processes, a first presentation of Fokker-Planck equations for time-changed Gaussian processes has been given in [4], in terms of Laplace inversion. In this talk we will focus on the specific case of the time-changed fractional Brownian motion and Ornstein-Uhlenbeck processes. More precisely, we will show the relation between the one-dimensional densities of the aforementioned processes and some *generalized* time-fractional Fokker-Planck equations, involving, together with the Dzhrbashyan-Caputo derivative, a special integral operator that we call *weighted inverse subordination*. In particular, we will provide a version of the Fokker-Planck equation given in [4] in which the inverse Laplace transform is explicitly recognized. The content of the talk is based on [1,2].

REFERENCES

[1] G. Ascione, Y. Mishura, and E. Pirozzi. Time-changed fractional Ornstein-Uhlenbeck process. Fractional Calculus and Applied Analysis, 23(2):450–483, 2020.
[2] G. Ascione, Y. Mishura, and E. Pirozzi. The Fokker–Planck equation for the time-changed fractional Ornstein–Uhlenbeck stochastic process. Proceedings of the Royal Society of Edinburgh Section A: Mathematics, 152(4):1032–1057, 2022.
[3] B. Baeumer, M. M. Meerschaert, et al. Stochastic solutions for fractional Cauchy problems. Fractional Calculus and Applied Analysis, 4(4):481–500, 2001.
[4] M. Hahn, J. Ryvkina, K. Kobayashi, and S. Umarov. On time-changed Gaussian processes and their associated Fokker-Planck-Kolmogorov equations. Electronic Communications in Probability, 2011.

Multi-Fractional Stochastic Dominance

Jan 19
10:05

Ehsan Azmoodeh
University of Liverpool

This talk introduces a pioneering concept known as Multi-Fractional Stochastic Dominance (MFSD), expanding upon the existing notion of Fractional Stochastic Dominance (FSD) as initially proposed by Müller et al. (Manag. Sci. 63, 2933–2947, 2017) to interpolate between integer degree dominance relations. The MFSD family is characterised by an arbitrary non-decreasing function that has a dual significance: (i) it captures the preferences of decision-makers in terms of risk aversion and greediness at a local level, and (ii) it enables a local interpolation between first and second stochastic dominance on different parts of the distributions' supports. Our generalization allows for the ordering of distribution functions that cannot be ordered by FSD in non-trivial cases. If time permits, we will discuss both the mathematical and economic properties of MFSD and demonstrate how it offers a more comprehensive framework for decision analysis.

Recent advances on forward curve modeling and applications

Jan 17
15:55

Fred Espen Benth
University of Oslo

We review some models for the evolution of forward term structures in commodity and power markets. Based on infinite dimensional stochastic processes, we introduce appropriate non-Gaussian models with stochastic volatility explaining features of forward term structures. An application. to option pricing is discussed, including a neural network approach. Joint work with Nils Detering (Duesseldorf), Luca Galimberti (London), Paul Kruhner (Vienna), Carlo Sgarra (Bari).

Jan 18
09:30

Recent advances on forward curve modeling and applications

Dan Crisan

Imperial College London

I will discuss a natural generalization of McKean-Vlasov (McV) processes to infinite dimensional state spaces where the classical global Lipschitz condition imposed on the coefficients may not hold in general. As an application, I will introduce an idealized Atmosphere-Ocean model that rests upon Hasselmann's paradigm for stochastic climate models where stochasticity is incorporated into the fast moving atmospheric component of an idealized coupled model by means of stochastic Lie transport, while the slow moving ocean model remains deterministic. In the MKV version of the model, the drift velocity of the stochastic vector field is replaced by its expected value. This is based on paper [1].

REFERENCES

[1] D. Crisan, D. D. Holm, P. Korn, An Implementation of Hasselmann's Paradigm for Stochastic Climate Modelling based on Stochastic Lie Transport, Nonlinearity 36 (9), pp 4862–4903, 2023.

Jan 18
14:35

Limit distributions for the discretization error of stochastic Volterra equations with fractional kernel

Masaaki Fukasawa

Osaka University

Our study aims to specify the asymptotic error distribution in the discretization of a stochastic Volterra equation with a fractional kernel. It is well known that for a standard stochastic differential equation, the discretization error, normalized with its rate of convergence $1/\sqrt{n}$, converges in law to the solution of a certain linear equation. Similar to this, we show that a suitably normalized discretization error of the Volterra equation converges in law to the solution of a certain linear Volterra equation with the same fractional kernel.

Signatures for Images

Jan 18
12:10

Fabian Harang
BI Norwegian Business School

An analytic and algebraic understanding of iterated integral signatures associated to continuous paths has played a central role in in a wide range of mathematical areas, such as the construction of stochastic integration for non-martingales with rough paths theory, to formal representations and expansions of solutions to (partial) differential equations. In recent years, the signature has proven to be an efficient feature map for machine learning tasks, where the learning task is related to time series data, or data streams. A distinct advantage of the path signature is that it has a rich algebraic and analytic structure associated with it. In contrast to time series data, image data can naturally be seen as two-parameter fields taking values in multi-dimensional space, and in recent years there has been some research into the extension of the path signature to multi-parameter fields (see e.g. Chouk/Gubinelli 14, Lee/Oberhauser (21 and 23)). In this talk I will propose an extension of the path signature to two-parameter fields. The algebraic structure of this object turns out to be rather complicated and I will discuss our current understanding of the challenges with going from 1 to 2 parameters, and provide some interesting observations related to a Chen type relation and a Shuffle type relation. At last I will briefly discuss the universality of the 2D signature, providing a universal approximation theorem, and some open problems. This talk is based on forthcoming joint work with Joscha Diehl, Kurusch Ebrahimi-Fard, and Samy Tindel, and is part of the Signatures for Images project for 2023/2024 at CAS.

Backward Euler method for stochastic differential equations with non-Lipschitz coefficients driven by fractional Brownian motion

Jan 18
14:00

Yaozhong Hu
University of Alberta

We study the traditional backward Euler method for stochastic differential equation driven by fractional Brownian motion whose drift coefficient satisfies the one-sided Lipschitz condition. The backward Euler scheme is proved to be of order one and this rate is optimal by showing the asymptotic error distribution result. Numerical experiments are performed to validate our claims about the optimality of the rate of convergence. This is joint work with Yanghui Liu and Hao Zhou.

Jan 17
15:20

Asymptotic analysis of integral equations with fractional covariance operators

Marina Kleptsyna
Le Mans Université

Some problems in the theory and applications of stochastic processes reduce to solving integral equations with their covariance operators. Usually such equations do not have explicit solutions but still useful information can be extracted through asymptotic analysis with respect to relevant parameters. In this talk I will survey some recent results on such equations for processes related to the fractional Brownian motion. Applications include the problem of small deviations, linear filtering and statistical inference.

REFERENCES

[1] D. Crisan, D. D. Holm, P. Korn, An Implementation of Hasselmann's Paradigm for Stochastic Climate Modelling based on Stochastic Lie Transport, Nonlinearity 36 (9), pp 4862–4903, 2023.

Jan 17
10:05

TBA

Jacques Lévy Véhel
Case Law Analytics

TBA

Jan 17
09:30

Standard and fractional Bessel and Cox-Ingersoll-Ross processes

Yuliya Mishura
Taras Shevchenko National University of Kyiv and Mälardalens universitet

We consider standard and fractional Bessel and Cox-Ingersoll-Ross processes on the entire spectrum of possible relationships between the values of the parameters of drift, diffusion, and (for fractional processes) the Hurst index. Statistical parameter estimation is also being studied, although these results are not known over the entire range of values.

Optimal control of SPDEs driven by the Brownian sheet

Jan 19
12:00

Bernt Øksendal
University of Oslo

We study the optimal control of a system where the state dynamics satisfy a stochastic partial differential equation (SPDE) driven by a two-parameter (time-space) Brownian motion (also called Brownian sheet). We first discuss some properties of a Brownian sheet - driven linear SPDE which models the growth of an ecosystem. Applying time-space white noise calculus we derive a sufficient maximum principle and a necessary maximum principle for optimality of the control. Finally, we illustrate our results by solving an optimal harvesting problem and a linear quadratic control problem in the plane. We also study possible applications to machine learning.

The presentation is based on joint work with Nacira Agram (KTH), Frank Proske (UiO) and Olena Tymoshenko (UiO and NTUU).

On a fractional Ornstein-Uhlenbeck process and its time-changed version

Jan 17
14:00

Enrica Pirozzi
Università di Napoli Federico II Italia

We consider a fractional Ornstein-Uhlenbeck process as a solution of a linear stochastic differential equation driven by a fractional Brownian motion. Under specified hypotheses on the forcing process involved in the drift, we can show a sort of short- or long-range dependence useful in some applications. Then, we define a time-changed fractional Ornstein-Uhlenbeck process by composing the fractional Ornstein-Uhlenbeck process with the inverse of a subordinator and we show some convergence results and properties.

Computation of Greeks under rough Volterra stochastic volatility models using the Malliavin calculus approach

Jan 17
11:00

Jan Pospíšil
University of West Bohemia

Using Malliavin calculus techniques we obtain formulas for computing Greeks under different rough Volterra stochastic volatility models. In particular we obtain formulas for rough versions of Stein-Stein, SABR and Bergomi models and numerically demonstrate the convergence.

This is a joint work with Mishari Al-Foraih and Josep Vives.

Jan 18
11:35

Drift parameter estimation in Vasicek-type model driven by tempered fractional Brownian motion

Kostiantyn Ralchenko

TarasShevchenko National University of Kyiv

We establish asymptotic bounds with probability of 1 for the growth rate of trajectories of tempered fractional Brownian motion (TFBM) and tempered fractional Brownian motion of the second kind (TFBM-II). Further, we construct least-squares-type estimators for the unknown drift parameters in the Vasicek model, driven either by TFBM or TFBM-II. By applying the derived asymptotic bounds, we demonstrate the strong consistency of the estimators in the nonergodic case.

Jan 18
10:05

Effective fractional wave equations in random multiscale media

Knut Sølna

University of California, Irvine

Experiments show that seismic waves propagating through the earth's crust experience frequency-dependent attenuation. Three regimes have been experimentally identified with specific attenuation properties: the low-, mid-, and high-frequency regimes. We show that the observed behavior can be explained via theory for waves in random media with short- and long-range correlation. Using stochastic multiscale analysis it is possible to show that wave propagation is described by effective fractional equations that exhibit fractional damping exponents, which reproduce qualitatively the observed behavior.

Jan 18
15:20

Integration-by-parts characterizations of Gaussian processes

Tommi Sottinen

University of Vaasa

The Malliavin integration-by-parts formula is a key ingredient to develop stochastic analysis on the Wiener space. In this article we show that a suitable integration-by-parts formula also characterizes a wide class of Gaussian processes, the so-called Gaussian Fredholm processes.

Long run stochastic control problems with general discounting: How to overcome time inconsistency

Jan 19
09:30

Łukasz Stettner

Institute of Mathematics, Polish Academy of Sciences

Majority of stochastic control problems concern constant discounting. Then so called Bellman optimality principle works and we are able to find and solve suitable Bellman equation. In the case of general discounting situation is more complicated - we don't have time consistency. We can overcome such problem considering additional time variable which unfortunately increases dimension of the problem. In particular long run problems however it appears that optimal control for suitable time consistent problem is also optimal for the problem with generalized discounting. This is shown for average reward per unit time in discrete time and for impulse control problems in continuous time in the papers mentioned below.

REFERENCES

[1] Ł. Stettner, Long run stochastic control problems with general discounting, arXiv:2306.14224v1

[2] D. Jelito, Ł. Stettner, Impulse control with generalized discounting, SIAM J. Control Optim. accepted, arXiv:2306.17448v2

Continuity of data-to-solution map for stochastic Camassa-Holm type equations

Jan 19
11:20

Hao Tang

University of Oslo

The analysis on continuous dependence on initial data for nonlinear stochastic partial differential equations has gained less attention in the literature so far. In this talk, we first show that the solution map is continuous for a stochastic Camassa–Holm type equation. Then we introduce a notion of stability of exiting time. We provide an example showing that one cannot improve the stability of the exiting time and simultaneously improve the continuity of the dependence on initial data. Hence we obtain a nearly optimal dependence result.

Jan 18
15:55

Sufficient variability of paths and differential equations with BV-coefficients

Lauri Vitasaari
Aalto University

Partial differential equations and differential systems play a fundamental role in many aspects of our daily life. However, in stochastic versions the underlying equation does not make sense in a classical way and one has to consider integral equations instead. Moreover, even the concept of integral is subtle for $\int_0^t \varphi(X_s)\, dY_s$, where φ is a given function and X, Y are non-differentiable objects. Several powerful techniques such as rough path theory have emerged to treat these situations, and it can be safely stated that nowadays differential systems driven by rough signals are already rather well understood. The idea in the rough path theory is, roughly speaking, that one assumes additional smoothness on the function φ in order to compensate bad behaviour of signal functions X and Y. As such, the methodology cannot be applied in any straightforward manner if one allows discontinuities in φ. In particular, this is the case if φ is a general BV-function.

In this talk we combine tools from fractional calculus and harmonic analysis to some fine properties of BV-functions and maximal functions, allowing us to give a meaningful definition for (multidimensional) integrals $\int_0^t \varphi(X_s)\, dY_s$ with a BV-function φ, provided that the functions X and Y are regular enough in the Sobolev sense. Here enough regularity means better regularity than what is customary assumed in the rough path theory, and this gain in regularity can be used to compensate ill behaviour of φ. The key idea is that the signal X should not spend too much time, in some sense, on the bad regions of φ. We quantify this in terms of potential theory and Riesz energies, and provide various probabilistic examples and methods how these conditions can be verified. We also discuss several consequences, and provide existence and uniqueness results for certain differential systems involving BV-coefficients. Extensions and further topics are discussed.

The talk is based on a joint work with Michael Hinz (Bielefeld University) and Jonas Tölle (Aalto University).

Jan 17
12:10

Decomposition formula for rough Volterra stochastic volatility models

Josep Vives
Universitat de Barcelona

The talk is devoted to present the paper [1], a joint work with Raúl Merino (Vida Caixa S. A. and Pompeu Fabra University), Jan Pospisil (West Bohemia University), Tomas Sobotka (Ernst and Young S. R. O.) and Tommi Sottinen (University of Vaasa).

In this paper we present an alternative approximate option pricing approach for a class of rough fractional stochastic volatility models. We provide a proof of the prediction law for general Gaussian Volterra processes. The prediction law is then utilized to obtain an adapted projection of the future squared volatility, a cornerstone of our proposed pricing approximation. Firstly, a decomposition formula for European option prices under general Volterra volatility models is introduced. Then we focus on particular models with rough fractional volatility and we derive an explicit semiclosed approximation formula. Numerical properties of the approximation for the rBergomi model are studied and we propose a hybrid calibration scheme which combines the approximation formula alongside MC simulations. This scheme can signicantly speed up the calibration to financial markets as illustrated on a set of AAPL options.

REFERENCES

[1] R. Merino, J. Pospisil, T. Sobotka, T. Sottinen and J. Vives (2021): *Decomposition formula for rough Volterra stochastic volatility models.* International Journal of Theoretical and Applied Finance 24 (2). 2150008. DOI: 10.1142/S0219024921500084.

Quadratic hedging in SVV-model

Jan 19
11:40

Anton Yurchenko-Tytarenko
University of Oslo

We consider stochastic volatility dynamics driven by a general Hölder continuous Volterra-type noise and with unbounded drift. For these so-called SVV-models, we consider the explicit computation of quadratic hedging strategies. While the theoretical hedge is well-known in terms of the non-anticipating derivative for all square integrable claims, the fact that these models are typically non-Markovian provides is a challenge in the direct computation of conditional expectations at the core of the explicit hedging strategy. To overcome this difficulty, we propose a Markovian approximation of the model which stems from an adequate approximation of the kernel in the Volterra noise. We study the approximation of the volatility, of the prices and of the optimal mean-square hedge. We provide the corresponding error estimates. The work is completed with numerical simulations.
