

COMPUTATION OF GREEKS UNDER ROUGH VOLTERRA STOCHASTIC VOLATILITY MODELS USING THE MALLIAVIN CALCULUS APPROACH

Jan Pospíšil¹

joint work with Josep Vives² and Mishari Al-Foraih³

¹Department of Mathematics, Faculty of Applied Sciences,
University of West Bohemia, Czech Republic

²Faculty of Economy and Business, University of Barcelona, Spain

³Department of Mathematics, University of Kuwait, Kuwait

Workshop on Stochastics, Memory and Roughness 2024
Oslo, Norway

January 17–19, 2024



UNIVERSITAT DE
BARCELONA



Little history

Calculation of Greeks using Malliavin calculus

Preliminaries

Methodology for computation of Greeks

Greeks formulas in general SV model

Greeks formulas in rough Volterra SV model

 AL-FORAIH, M., POSPÍŠIL, J., AND VIVES, J. (2023), *Computation of Greeks in rough Volterra stochastic volatility models using the Malliavin calculus approach*, DOI [10.48550/arXiv.2312.00405](https://doi.org/10.48550/arXiv.2312.00405), available at arXiv: <https://arxiv.org/abs/2312.00405>

Using Malliavin calculus techniques we obtain formulas for computing Greeks under different rough Volterra stochastic volatility models. In particular we obtain formulas for rough versions of Stein-Stein, SABR and Bergomi models and numerically demonstrate the convergence.

 FOURNIÉ, E., LASRY, J.-M., LEBUCHOUX, J., LIONS, P.-L., AND TOUZI, N. (1999), *Applications of Malliavin calculus to Monte Carlo methods in finance*. Finance Stoch. 3(4), 391–412, ISSN 0949-2984, DOI [10.1007/s007800050068](https://doi.org/10.1007/s007800050068), Zbl [0947.60066](https://zbmath.org/?q=sernum/0947.60066), [MR1842285](https://arxiv.org/abs/MR1842285)

 FOURNIÉ, E., LASRY, J.-M., LEBUCHOUX, J., AND LIONS, P.-L. (2001), *Applications of Malliavin calculus to Monte Carlo methods in finance II*. Finance Stoch. 5(2), 201–236, ISSN 0949-2984, DOI [10.1007/PL00013529](https://doi.org/10.1007/PL00013529), Zbl [0973.60061](https://zbmath.org/?q=sernum/0973.60061), [MR1841717](https://arxiv.org/abs/MR1841717)

- ▶ integration by parts in Malliavin calculus
- ▶ improved dramatically the efficiency in terms of computational time required for the numerical computation of Greeks, specially for discontinuous payoffs

 MONTERO, M. AND KOHATSU-HIGA, A. (2003), *Malliavin calculus applied to finance*. Phys. A 320(1-4), 548–570, ISSN 0378-4371, DOI [10.1016/S0378-4371\(02\)01531-5](https://doi.org/10.1016/S0378-4371(02)01531-5), Zbl [1010.91046](https://zbmath.org/?q=ri%2F1010.91046),
[MR1963722](https://arxiv.org/abs/MR1963722)

 NUALART, D. (2006), The Malliavin calculus and related topics. Probab. Appl., Berlin: Springer-Verlag, 2nd edn., ISBN 3-540-28328-5/hbk; 978-3-642-06651-1/pbk; 3-540-28329-3/ebook, DOI [10.1007/3-540-28329-3](https://doi.org/10.1007/3-540-28329-3), Zbl [1099.60003](https://zbmath.org/?q=ri%2F1099.60003), [MR2200233](https://arxiv.org/abs/MR2200233)

- ▶ surveys of the Malliavin Calculus technique under the [Black and Scholes \(1973\)](#) framework
- ▶ Chapter 6 in the Nualart's book, it was newly added in the 2nd edition

 DAVIS, M. H. A. AND JOHANSSON, M. P. (2006), *Malliavin Monte Carlo Greeks for jump diffusions*. Stochastic Process. Appl. 116(1), 101–129, ISSN 0304-4149, DOI [10.1016/j.spa.2005.08.002](https://doi.org/10.1016/j.spa.2005.08.002), Zbl [1081.60040](https://zbmath.org/?q=1081.60040), MR2186841

 EDDAHBI, M., LALAOU BEN CHERIF, S. M., AND NASROALLAH, A. (2015), *Computation of Greeks for jump-diffusion models*. Int. J. Theor. Appl. Finance 18(6), 1550039, 30, ISSN 0219-0249, DOI [10.1142/S0219024915500399](https://doi.org/10.1142/S0219024915500399), Zbl [1337.91094](https://zbmath.org/?q=1337.91094), MR3397252

 MHLANGA, F. J. (2015), *Calculations of Greeks for jump diffusion processes*. Mediterr. J. Math. 12(3), 1141–1160, ISSN 1660-5446, DOI [10.1007/s00009-014-0459-1](https://doi.org/10.1007/s00009-014-0459-1), Zbl [1321.60168](https://zbmath.org/?q=1321.60168), MR3376837

 EBERLEIN, E., EDDAHBI, M., AND LALAOU BEN CHERIF, S. M. (2016), *Computation of Greeks in LIBOR models driven by time-inhomogeneous Lévy processes*. Appl. Math. Finance 23(3), 236–260, ISSN 1350-486X, DOI [10.1080/1350486X.2016.1243013](https://doi.org/10.1080/1350486X.2016.1243013), Zbl [1396.91779](https://zbmath.org/?q=1396.91779), MR3572024

► jump-diffusion models, including local volatility models

 PETROU, E. (2008), *Malliavin calculus in Lévy spaces and applications to finance*. Electron. J. Probab. 13(27), 852–879, ISSN 1083-6489/e, DOI [10.1214/EJP.v13-502](https://doi.org/10.1214/EJP.v13-502), Zbl [1193.60075](https://zbmath.org/journals/EJP/v13/502),

[MR2399298](https://arxiv.org/abs/0904.3247)

 EL-KHATIB, Y. (2009), *Computations of Greeks in stochastic volatility models via the Malliavin calculus*, DOI [10.48550/arXiv.0904.3247](https://doi.org/10.48550/arXiv.0904.3247), available at arXiv: <https://arxiv.org/abs/0904.3247>

 KHEDHER, A. (2012), *Computation of the delta in multidimensional jump-diffusion setting with applications to stochastic volatility models*. Stoch. Anal. Appl. 30(3), 403–425, ISSN 0736-2994, DOI [10.1080/07362994.2012.668440](https://doi.org/10.1080/07362994.2012.668440), Zbl [1242.91188](https://zbmath.org/journals/StochAnal/v30/3), [MR2916862](https://arxiv.org/abs/0904.3247)

 YOLCU-OKUR, Y., SAYER, T., YILMAZ, B., AND INKAYA, B. A. (2018), *Computation of the delta of European options under stochastic volatility models*. Comput. Manag. Sci. 15(2), 213–237, ISSN 1619-697X, DOI [10.1007/s10287-018-0316-y](https://doi.org/10.1007/s10287-018-0316-y), Zbl [1417.91518](https://zbmath.org/journals/ComputManagSci/v15/2), [MR3817795](https://arxiv.org/abs/0904.3247)

- ▶ Petrou (2008) considered a general stochastic volatility jump-diffusion model with jumps both in the underlying and in the volatility process, Heston (1993) and Bates (1996) models can be considered as a special case
- ▶ different formulas for the same models (?!), more references in the paper

Consider a price process $S := \{S_t, t \geq 0\}$ with completed natural filtration $\{\mathcal{F}_t, t \geq 0\}$. A financial derivative is contingent claim with payoff F , where F is a random variable adapted to $\mathcal{F}_T$ for a fixed expiry date $T > 0$. Typically $F = f(S_T)$ for a certain function f . Assume a fixed instantaneous interest rate $r \geq 0$. Under the no arbitrage principle, the price at $t = 0$ of the derivative is given by

$$V_0 := \mathbb{E}_{\mathbb{Q}}[e^{-rT} f(S_T)],$$

where $\mathbb{Q}$ is a risk-neutral probability measure.

Malliavin calculus is based on two dual operators, the Malliavin derivative D and the Skorohod integral δ , also called divergence operator. Let $\mathbb{D}^{1,2}$ and $\mathbb{D}^{2,2}$ denote the Sobolev spaces properly defined in the paper.

Greeks Delta, Gamma, Rho, Vega and the derivative with respect to the Hurst parameter in rough volatility models are defined respectively as

$$\begin{aligned}\Delta &= \partial_{S_0} \mathbb{E} \left[e^{-rT} f(S_T) \right], \\ \Gamma &= \partial_{S_0} \Delta = \partial_{S_0, S_0}^2 \mathbb{E} \left[e^{-rT} f(S_T) \right], \\ \varrho &= \partial_r \mathbb{E} \left[e^{-rT} f(S_T) \right], \\ \mathcal{V} &= \partial_{V_0} \mathbb{E} \left[e^{-rT} f(S_T) \right], \\ \mathcal{H} &= \partial_H \mathbb{E} \left[e^{-rT} f(S_T) \right],\end{aligned}$$

where V_0 denotes the initial volatility, r the fixed interest rate and H the Hurst parameter associated to the fractional process underlying the rough Volterra process associated to the volatility.

Let I be an open interval of $\mathbb{R}$. Let $\{F^\theta, \theta \in I\}$ and $\{\Phi^\theta, \theta \in I\}$ be two families of functionals on $\mathbb{D}^{1,2}$, both continuously differentiable with respect to $\theta \in I$. Assume we have $h \in L^2(\Omega \times [0, T])$ such that $h_t \in \mathbb{D}^{1,2}$ for any $t \in [0, T]$,

$$\int_0^T D_t F^\theta h_t dt \neq 0 \text{ a.s. on } \{\partial_\theta F^\theta \neq 0\}$$

and

$$\frac{\Phi^\theta \partial_\theta F^\theta h}{\int_0^T D_t F^\theta h_t dt}$$

belongs to $Dom(\delta)$ and is continuous in θ . Then,

$$\partial_\theta \mathbb{E}[\Phi^\theta f(F^\theta)] = \mathbb{E} \left[f(F^\theta) \cdot \delta \left(\frac{\Phi^\theta \partial_\theta F^\theta h}{\int_0^T D_t F^\theta h(t) dt} \right) \right] + \mathbb{E}[f(F^\theta) \partial_\theta \Phi^\theta] \quad (1)$$

for any function $f \in C_b^1(\mathbb{R})$ such that $f(F^\theta) \in L^2(\Omega)$.

Let $W = (W_t, t \geq 0)$ be a standard Wiener process defined on a probability space $(\Omega, \mathcal{F}, \mathbb{Q})$ and let $\mathcal{F}^W = (\mathcal{F}_t^W, t \geq 0)$ be the filtration generated by W . A **Gaussian Volterra process** is defined as a process $Y = \{Y_t, t \geq 0\}$ such that it can be represented as

$$Y_t = \int_0^t K(t, s) \, dW_s,$$

where $K(t, s)$ is a kernel $K : [0, T] \times [0, T] \longrightarrow \mathbb{R}$ such that the following three integrals

$$\int_0^T \int_0^t K(t, s)^2 \, ds \, dt, \quad \int_0^t K(t, s)^2 \, ds, \quad \int_s^T K(t, s)^2 \, dt$$

are finite, and for any $t \in [0, T]$, $\mathcal{F}_t^Y = \mathcal{F}_t^W$. By $r(t, s)$ we denote the autocovariance function of Y_t and by $r(t) := r(t, t)$ its variance:

$$r(t, s) := \mathbb{E}[Y_t Y_s], \quad t, s \geq 0,$$

$$r(t) := \mathbb{E}[Y_t^2], \quad t \geq 0.$$

Denote also $\kappa_t = \int_0^t K_H(t, s) \, ds$.

Different kernels K give different Gaussian Volterra processes. The most famous example is the **standard fractional Brownian motion** (fBm) B_t^H , that is a process with stationary increments and that corresponds to the representation

$$B_t^H = \int_0^t K_H(t, s) \, dW_s,$$

where $K_H(t, s)$ is a kernel that depends on the so-called Hurst parameter $H \in (0, 1)$. The autocovariance function of B_t^H is given by

$$r(t, s) := \mathbb{E}[B_t^H B_s^H] = \frac{1}{2} \left(t^{2H} + s^{2H} - |t - s|^{2H} \right), \quad t, s \geq 0,$$

and in particular $r(t) := r(t, t) = t^{2H}$, $t \geq 0$. For $H = 1/2$, fBm is the standard Brownian motion, for $H > 1/2$, the increments are positively correlated and the trajectories are more **regular** than Brownian ones, and for $H < 1/2$, the increments are negatively correlated and the trajectories are more **rough** than Brownian ones. The term **rough fractional model** therefore refers to the case where $H < 1/2$ is considered in the model.

- Simplified Riemann-Liouville kernel (Alòs, Mazet, and Nualart, 2000)

$$K_H(t, s) = \sqrt{2H}(t - s)^{H-1/2}.$$

- Historically, several other representations occurred (Mandelbrot and Van Ness, 1968; Molchan and Golosov, 1969; Muravlev, 2011) and relation between them was studied for example by Jost (2008).
- approximate fractional Brownian motion (afBm) (Thao, 2006)

$$K_H(t, s) = \sqrt{2H}(t - s + \varepsilon)^{H-1/2}, \quad \varepsilon \geq 0,$$

For every $\varepsilon > 0$ such a process B_t^H is a semimartingale and as ε tends to zero, B_t^ε converges to the Riemann-Liouville process in the L^2 -norm, uniformly in $t \in [0, T]$.

$$r(t, s) = \int_0^{t \wedge s} K_H(t, v) K_H(s, v) dv,$$

$$r(t) = \int_0^t K_H^2(t, v) dv = 2H \int_0^t (t - v + \varepsilon)^{2H-1} dv = (t + \varepsilon)^{2H} - \varepsilon^{2H},$$

Note that if $\varepsilon = 0$, we get exactly the variance $r(t) = t^{2H}$.

The price process is assumed to follow the equation

$$dS_t = rS_t dt + \sigma(V_t)S_t dW_t, \quad t \in [0, T],$$

where $r \geq 0$ is the constant instantaneous interest rate and V denotes the volatility or the variance process that we assume adapted to the completed filtration generated by W and $\widetilde{W}$. Processes S and V are assumed to be square-integrable with continuous trajectories and such that S_t and V_t belong to $\mathbb{D}^{2,2}$ for any $t \in [0, T]$. Note that

$$S_t = S_0 \exp \left\{ \int_0^t \left(r - \frac{1}{2} \sigma^2(V_s) \right) ds + \int_0^t \sigma(V_s) dW_s \right\}, \quad t \in [0, T].$$

The Malliavin derivative of S_T with respect to W is $\boxed{D_t S_T = S_T G(t, T)}$ where

$$\boxed{G(t, T) = \sigma(V_t) + \int_t^T \sigma'(V_s) D_t V_s dW_s - \int_t^T \sigma(V_s) \sigma'(V_s) D_t V_s ds.}$$

The Malliavin derivative of $G(t, T)$ is then

$$\begin{aligned} D_s G(t, T) &= \sigma'(V_{s \vee t}) D_{s \wedge t} V_{s \vee t} \\ &+ \int_{s \vee t}^T \sigma''(V_u) D_s V_u D_t V_u dW_u + \int_{s \vee t}^T \sigma'(V_u) D_s D_t V_u dW_u \\ &- \int_{s \vee t}^T (\sigma'(V_u))^2 D_s V_u D_t V_u du - \int_{s \vee t}^T \sigma''(V_u) \sigma(V_u) D_s V_u D_t V_u du - \int_{s \vee t}^T \sigma'(V_u) \sigma(V_u) D_s D_t V_u du \end{aligned}$$

Main result:

In order to find Malliavin weights and to compute the considered Greeks, it is essential to find

$$\boxed{\int_0^T G(t, T) dt} \quad \text{and} \quad \boxed{\int_0^T \int_s^T D_s G(t, T) dt ds}.$$

Let θ now be a model parameter and our goal is to calculate the corresponding Greek, in particular the partial derivative with respect to it. For example may write

$$\partial_{\theta} S_T^{\theta} = S_T^{\theta} \cdot \left\{ \int_0^T b(V_s) ds + \int_0^T a(V_s) dW_s \right\}, \quad \text{where}$$

$$a(V_s) = \partial_{\theta} (\sigma(V_s)) \quad \text{and} \quad b(V_s) = \partial_{\theta} \left(r - \frac{1}{2} \sigma^2(V_s) \right). \quad (2)$$

Direct consequence of the IBP formula applied to the constant process $h_t \equiv \mathbf{1}_{[0,T]}(t)$ then gives us the **general Greek formula**

$$\begin{aligned} \partial_{\theta} \mathbb{E}[e^{-rT} f(S_T^{\theta})] &= e^{-rT} \mathbb{E} \left[f(S_T^{\theta}) \cdot \delta \left(\frac{\partial_{\theta} S_T^{\theta}}{\int_0^T S_T^{\theta} G(u, T) du} \right) \right] + \mathbb{E} \left[f(S_T^{\theta}) \cdot \partial_{\theta} (e^{-rT}) \right] \\ &= e^{-rT} \mathbb{E} \left[f(S_T^{\theta}) \cdot \delta(c(V_s)) \right] + \mathbb{E} \left[f(S_T^{\theta}) \cdot \partial_{\theta} (e^{-rT}) \right], \end{aligned} \quad (3)$$

where we denoted

$$c(V_s) = \frac{\int_0^T b(V_s) ds}{\int_0^T G(u, T) du + \int_0^T a(V_s) dW_s}. \quad (4)$$

$$\begin{aligned}\Delta &= \mathbb{E} \left[f(S_T) \delta \left(\frac{e^{-rT} \partial_x S_T}{\int_0^T D_u S_T du} \right) \right] \\ &= \frac{e^{-rT}}{S_0} \mathbb{E} \left[f(S_T) \left(\frac{W_T}{\int_0^T G(t, T) dt} + \frac{\int_0^T \int_s^T D_s G(t, T) dt ds}{(\int_0^T G(t, T) dt)^2} \right) \right],\end{aligned}$$

$$\Gamma = \dots$$

$$\begin{aligned}&= \frac{e^{-2rT}}{S_0^2} \mathbb{E} \left[\frac{f(S_T)}{\int_0^T G(t, T) dt} \left(\frac{W_T}{\int_0^T G(t, T) dt} + \frac{\int_0^T \int_s^T D_s G(t, T) dt ds}{(\int_0^T G(t, T) dt)^2} \right) \right] \\ &\quad - \frac{e^{-rT}}{S_0^2} \mathbb{E} \left[f(S_T) \left(\frac{W_T}{\int_0^T G(t, T) dt} + \frac{\int_0^T \int_s^T D_s G(t, T) dt ds}{(\int_0^T G(t, T) dt)^2} \right) \right]\end{aligned}$$

$$\varrho = \dots$$

$$= rT e^{-rT} \mathbb{E} \left[F(S_T) \left(\frac{W_T}{\int_0^T G(t, T) dt} + \frac{\int_0^T \int_s^T D_s G(t, T) dt ds}{(\int_0^T G(t, T) dt)^2} \right) \right] - e^{-rT} \mathbb{E}[f(S_T)].$$

Let $Y = \{Y_t, t \geq 0\}$ be the Gaussian Volterra process correlated with the price process S in the following sense.

$$Y_t = \int_0^t K_H(t, s) dZ_s,$$

$$Z_s = \rho W_s + \sqrt{1 - \rho^2} \widetilde{W}_s, \quad \rho \in [-1, 1],$$

where $\widetilde{W}$ is also a Wiener process. We distinguish two qualitatively different cases:

1. V is some known functional of the Gaussian process Y , i.e.,

$$V_t = \varphi(t, Y_t), \quad (5)$$

where φ is a function differentiable in the second variable, or

2. V is a solution to the stochastic differential equation

$$V_t = V_0 + \int_0^t u(V_s) ds + \int_0^t v(V_s) K_H(t, s) dZ_s, \quad (6)$$

where functions u, v are assumed to be in $C^2(\mathbb{R})$. Moreover we will assume that $u(V_t)$ and $v(V_t)$ are square-integrable processes.

- (i). In the α RFSV model, first introduced by [Merino, Pospíšil, Sobotka, Sottinen, and Vives \(2021\)](#), $\sigma(x) = x$ and the process V_t is given explicitly by (5) with

$$\varphi(t, x) = V_0 \cdot \exp \left\{ \xi \cdot x - \frac{1}{2} \alpha \xi^2 r(t) \right\},$$

where $V_0 > 0$, $\xi \in \mathbb{R}$ and $\alpha \in [0, 1]$ are (together with the Hurst parameter H and correlation $\rho \in [0, 1]$) model parameters. In particular

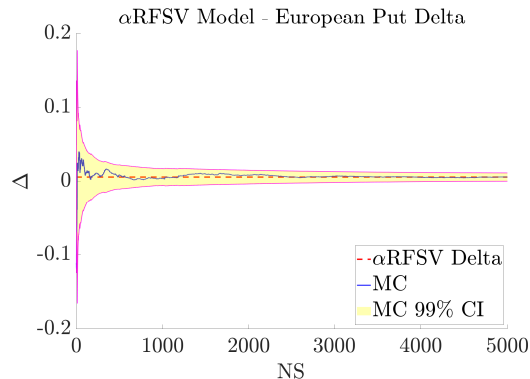
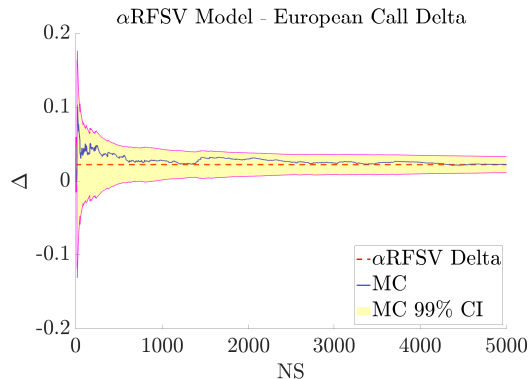
- ▶ for $\alpha = 1$ we get the *rough Bergomi model*, which in the $r = 0$ case coincide with the rough SABR($\beta = 1$) model, and
- ▶ for $\alpha = 0$ we get the non-stationary *RFSV model* ([Gatheral, Jaisson, and Rosenbaum, 2018](#)).

- (ii). A new *mixed α RFSV model* is a two factor fractional model, where two fractional volatility factors are considered, one with $H < \frac{1}{2}$ and another with $H' > \frac{1}{2}$. This model is a generalisation of the α RFSV model ([Merino, Pospíšil, Sobotka, Sottinen, and Vives, 2021](#)) and the mixed fractional Bergomi model ([Alòs and León, 2021](#)). The mixed α RFSV model has been studied only recently by [Al-Foraih, Pospíšil, and Vives \(2023\)](#).
- (iii). In the rough Stein and Stein model, $\sigma(x) = x$, $u(x) = \kappa(\theta - x)$ and $v(x) = \nu$. Here V is the volatility process.

formulas for the Greek Vega and for the derivative with respect the Hurst parameter H are

$$\begin{aligned}\mathcal{V} &= \mathbb{E} \left[f(S_T) \delta \left(\frac{e^{-rT} \partial_{V_0} S_T}{\int_0^T D_u S_T \, du} \right) \right] \\ &= e^{-rT} \mathbb{E} \left[f(S_T^\theta) \cdot \delta(c_{V_0}(V_s)) \right] \\ \mathcal{H} &= \mathbb{E} \left[f(S_T) \delta \left(\frac{e^{-rT} \partial_H S_T}{\int_0^T D_u S_T \, du} \right) \right] \\ &= e^{-rT} \mathbb{E} \left[f(S_T^\theta) \cdot \delta(c_H(V_s)) \right],\end{aligned}$$

where c_{V_0} and c_H are given by (4) with a and b being the corresponding partial derivatives (2).



$$\int_0^T G(t, T) dt = \int_0^T V_t dt + \rho \xi \left(\int_0^T V_r \kappa_r dW_r - \int_0^T V_r^2 \kappa_r dr \right)$$

$$\int_0^T \int_0^T D_s G(t, T) dt ds = 2\rho \xi \int_0^T V_t \kappa_t dt + \rho^2 \xi^2 \left(\int_0^T V_r \kappa_r^2 dW_r - 2 \int_0^T V_r^2 \kappa_r^2 dr \right).$$

- AL-FORAIH, M., POSPÍŠIL, J., AND VIVES, J. (2023), *Computation of Greeks in rough Volterra stochastic volatility models using the Malliavin calculus approach*, DOI [10.48550/arXiv.2312.00405](https://arxiv.org/abs/2312.00405), available at arXiv: <https://arxiv.org/abs/2312.00405>.
- ALÒS, E., MAZET, O., AND NUALART, D. (2000), *Stochastic calculus with respect to fractional Brownian motion with Hurst parameter lesser than $\frac{1}{2}$* . Stochastic Process. Appl. 86(1), 121–139, ISSN 0304-4149, DOI [10.1016/S0304-4149\(99\)00089-7](https://doi.org/10.1016/S0304-4149(99)00089-7), Zbl [1028.60047](https://zbmath.org/?q=ri%2F1028.60047), MR1741199.
- ALÒS, E. AND LEÓN, J. A. (2021), *An intuitive introduction to fractional and rough volatilities*. Mathematics 9(9), 994, ISSN 2227-7390, DOI [10.3390/math9090994](https://doi.org/10.3390/math9090994).
- BATES, D. S. (1996), *Jumps and stochastic volatility: Exchange rate processes implicit in Deutsche mark options*. Rev. Financ. Stud. 9(1), 69–107, DOI [10.1093/rfs/9.1.69](https://doi.org/10.1093/rfs/9.1.69).
- BLACK, F. AND SCHOLES, M. (1973), *The pricing of options and corporate liabilities*. J. Polit. Econ. 81(3), 637–654, ISSN 0022-3808, DOI [10.1086/260062](https://doi.org/10.1086/260062), Zbl [1092.91524](https://zbmath.org/?q=ri%2F1092.91524), MR3363443.
- DAVIS, M. H. A. AND JOHANSSON, M. P. (2006), *Malliavin Monte Carlo Greeks for jump diffusions*. Stochastic Process. Appl. 116(1), 101–129, ISSN 0304-4149, DOI [10.1016/j.spa.2005.08.002](https://doi.org/10.1016/j.spa.2005.08.002), Zbl [1081.60040](https://zbmath.org/?q=ri%2F1081.60040), MR2186841.

- EBERLEIN, E., EDDAHBI, M., AND LALAOUI BEN CHERIF, S. M. (2016), *Computation of Greeks in LIBOR models driven by time-inhomogeneous Lévy processes*. Appl. Math. Finance 23(3), 236–260, ISSN 1350-486X, DOI [10.1080/1350486X.2016.1243013](https://doi.org/10.1080/1350486X.2016.1243013), Zbl [1396.91779](https://zbmath.org/?q=ri%2F1396.91779), MR[3572024](https://arxiv.org/abs/13572024).
- EDDAHBI, M., LALAOUI BEN CHERIF, S. M., AND NASROALLAH, A. (2015), *Computation of Greeks for jump-diffusion models*. Int. J. Theor. Appl. Finance 18(6), 1550039, 30, ISSN 0219-0249, DOI [10.1142/S0219024915500399](https://doi.org/10.1142/S0219024915500399), Zbl [1337.91094](https://zbmath.org/?q=ri%2F1337.91094), MR[3397252](https://arxiv.org/abs/3397252).
- EL-KHATIB, Y. (2009), *Computations of Greeks in stochastic volatility models via the Malliavin calculus*, DOI [10.48550/arXiv.0904.3247](https://doi.org/10.48550/arXiv.0904.3247), available at arXiv: <https://arxiv.org/abs/0904.3247>.
- FOURNIÉ, E., LASRY, J.-M., LEBUCHOUX, J., AND LIONS, P.-L. (2001), *Applications of Malliavin calculus to Monte Carlo methods in finance II*. Finance Stoch. 5(2), 201–236, ISSN 0949-2984, DOI [10.1007/PL00013529](https://doi.org/10.1007/PL00013529), Zbl [0973.60061](https://zbmath.org/?q=ri%2F0973.60061), MR[1841717](https://arxiv.org/abs/1841717).
- FOURNIÉ, E., LASRY, J.-M., LEBUCHOUX, J., LIONS, P.-L., AND TOUZI, N. (1999), *Applications of Malliavin calculus to Monte Carlo methods in finance*. Finance Stoch. 3(4), 391–412, ISSN 0949-2984, DOI [10.1007/s007800050068](https://doi.org/10.1007/s007800050068), Zbl [0947.60066](https://zbmath.org/?q=ri%2F0947.60066), MR[1842285](https://arxiv.org/abs/1842285).
- GATHERAL, J., JAISSON, T., AND ROSENBAUM, M. (2018), *Volatility is rough*. Quant. Finance 18(6), 933–949, ISSN 1469-7688, DOI [10.1080/14697688.2017.1393551](https://doi.org/10.1080/14697688.2017.1393551), Zbl [1400.91590](https://zbmath.org/?q=ri%2F1400.91590), MR[3805308](https://arxiv.org/abs/3805308).

- HESTON, S. L. (1993), *A closed-form solution for options with stochastic volatility with applications to bond and currency options*. Rev. Financ. Stud. 6(2), 327–343, ISSN 0893-9454, DOI [10.1093/rfs/6.2.327](https://doi.org/10.1093/rfs/6.2.327), Zbl [1384.35131](https://zbmath.org/?q=1384.35131), MR3929676.
- JOST, C. (2008), *On the connection between Molchan-Golosov and Mandelbrot- van Ness representations of fractional Brownian motion*. J. Integral Equations Appl. 20(1), 93–119, ISSN 0897-3962, DOI [10.1216/JIE-2008-20-1-93](https://doi.org/10.1216/JIE-2008-20-1-93), Zbl [1147.60024](https://zbmath.org/?q=1147.60024), MR2396956.
- KHEDHER, A. (2012), *Computation of the delta in multidimensional jump-diffusion setting with applications to stochastic volatility models*. Stoch. Anal. Appl. 30(3), 403–425, ISSN 0736-2994, DOI [10.1080/07362994.2012.668440](https://doi.org/10.1080/07362994.2012.668440), Zbl [1242.91188](https://zbmath.org/?q=1242.91188), MR2916862.
- MANDELBROT, B. B. AND VAN NESS, J. W. (1968), *Fractional Brownian motions, fractional noises and applications*. SIAM Rev. 10(4), 422–437, ISSN 0036-1445, DOI [10.1137/1010093](https://doi.org/10.1137/1010093), Zbl [0179.47801](https://zbmath.org/?q=0179.47801), MR0242239.
- MERINO, R., POSPÍŠIL, J., SOBOTKA, T., SOTTINEN, T., AND VIVES, J. (2021), *Decomposition formula for rough Volterra stochastic volatility models*. Int. J. Theor. Appl. Finance 24(2), 2150008, ISSN 0219-0249, 1793-6322/e, DOI [10.1142/S0219024921500084](https://doi.org/10.1142/S0219024921500084), Zbl [1466.91350](https://zbmath.org/?q=1466.91350), MR4257140.
- MHLANGA, F. J. (2015), *Calculations of Greeks for jump diffusion processes*. Mediterr. J. Math. 12(3), 1141–1160, ISSN 1660-5446, DOI [10.1007/s00009-014-0459-1](https://doi.org/10.1007/s00009-014-0459-1), Zbl [1321.60168](https://zbmath.org/?q=1321.60168), MR3376837.

- MOLCHAN, G. M. AND GOLOSOV, J. I. (1969), *Gaussian stationary processes with asymptotic power spectrum*. Sov. Math. Dokl. 10, 134–137, ISSN 0197-6788, translation from Dokl. Akad. Nauk SSSR 184, 546–549 (1969), Zbl [0181.20704](#), MR[0242247](#).
- MONTERO, M. AND KOHATSU-HIGA, A. (2003), *Malliavin calculus applied to finance*. Phys. A 320(1-4), 548–570, ISSN 0378-4371, DOI [10.1016/S0378-4371\(02\)01531-5](#), Zbl [1010.91046](#), MR[1963722](#).
- MURAVLEV, A. A. (2011), *Representation of a fractional Brownian motion in terms of an infinite-dimensional Ornstein-Uhlenbeck process*. Russ. Math. Surv. 66(2), 439–441, ISSN 0036-0279, DOI [10.1070/rm2011v066n02abeh004746](#), translation from Usp. Mat. Nauk 66, No. 2, 235-236 (2011), Zbl [1225.60064](#), MR[2847795](#).
- NUALART, D. (2006), *The Malliavin calculus and related topics*. Probab. Appl., Berlin: Springer-Verlag, 2nd edn., ISBN 3-540-28328-5/hbk; 978-3-642-06651-1/pbk; 3-540-28329-3/ebook, DOI [10.1007/3-540-28329-3](#), Zbl [1099.60003](#), MR[2200233](#).
- PETROU, E. (2008), *Malliavin calculus in Lévy spaces and applications to finance*. Electron. J. Probab. 13(27), 852–879, ISSN 1083-6489/e, DOI [10.1214/EJP.v13-502](#), Zbl [1193.60075](#), MR[2399298](#).
- THAO, T. H. (2006), *An approximate approach to fractional analysis for finance*. Nonlinear Anal. Real World Appl. 7(1), 124–132, ISSN 1468-1218, DOI [10.1016/j.nonrwa.2004.08.012](#), Zbl [1104.60033](#), MR[2183802](#).

YOLCU-OKUR, Y., SAYER, T., YILMAZ, B., AND INKAYA, B. A. (2018), *Computation of the delta of European options under stochastic volatility models*. Comput. Manag. Sci. 15(2), 213–237, ISSN 1619-697X, DOI [10.1007/s10287-018-0316-y](https://doi.org/10.1007/s10287-018-0316-y), Zbl [1417.91518](https://zbmath.org/?q=1417.91518), MR3817795.

Thank you for your attention!