

DECOMPOSITION FORMULA FOR ROUGH VOLTERRA STOCHASTIC VOLATILITY MODELS

Josep Vives¹

joint work with R. Merino⁴, J. Pospíšil², T. Sobotka², T. Sottinen³

¹**Facultat de Economia i Empresa, Universitat de Barcelona**

²Faculty of Applied Sciences, University of West Bohemia

³Department of Mathematics and Statistics, University of Vaasa

⁴ Investment Risk Management Department, VidaCaixa S.A., Barcelona.

Workshop on Stochastics, Memory, and Roughness 2024
Oslo, Norway

January 17-19, 2024



UNIVERSITAT DE
BARCELONA



University of Vaasa
FINLAND



The talk is based on the paper

R. MERINO, J. POSPÍŠIL, T. SOBOTKA, T. SOTTINEN AND J. VIVES (2021):
Decomposition formula for rough Volterra stochastic volatility models. International Journal
of Theoretical and Applied Finance 24 (2): 2150008 (47 pages)



General decomposition formula

General stochastic volatility model

Decomposition formula

Approximation formula

Volterra volatility models

General Volterra volatility model

Exponential Volterra volatility model

Exponential fractional volatility model

Numerical results

Monte Carlo simulation of α RFSV model

Calibration of rBergomi ($\alpha = 1$) model

Conclusions

References

The End

Let $S = (S_t, t \in [0, T])$ be a strictly positive asset price process under a market chosen risk neutral probability measure $\mathcal{P}$ that follows the model:

$$dS_t = rS_t dt + \sigma_t S_t \left(\rho dW_t + \sqrt{1 - \rho^2} d\tilde{W}_t \right), \quad (1)$$

where S_0 is the current price, $r \geq 0$ is the interest rate, W_t and $\tilde{W}_t$ are independent standard Wiener processes defined on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and $\rho \in (-1, 1)$.

Let $\mathcal{F}^W$ and $\mathcal{F}^{\tilde{W}}$ be the filtrations generated by W and $\tilde{W}$ respectively and let $\mathcal{F} := \mathcal{F}^W \vee \mathcal{F}^{\tilde{W}}$.

The **volatility process** σ_t is a square-integrable process assumed to be adapted to the filtration generated by W and its trajectories are assumed to be a.s. càdlàg and strictly positive a.e..

For convenience we let $X_t = \log S_t$, $t \in [0, T]$, and consider the model

$$dX_t = \left(r - \frac{1}{2}\sigma_t^2 \right) dt + \sigma_t \left(\rho dW_t + \sqrt{1 - \rho^2} d\tilde{W}_t \right). \quad (2)$$

Recall that $Z := \rho W + \sqrt{1 - \rho^2} \tilde{W}$ is a standard Wiener process.

For any $t \in [0, T]$, $x \geq 0$ and $y \geq 0$ we denote by $BS(t, x, y)$ the so-called Black-Scholes function given by

$$BS(t, x, y) = e^x \Phi(d_+) - Ke^{-r\tau} \Phi(d_-),$$

where $\Phi(\cdot)$ denotes the cumulative distribution function of the standard normal law, $r \geq 0$ is a constant, $\tau = T - t$ and

$$d_{\pm}(y) = \frac{x - \ln K + (r \pm \frac{y^2}{2})\tau}{y\sqrt{\tau}}.$$

Recall that the price of an European plain vanilla call option under the classical Black-Scholes model with constant volatility σ , current log stock price X_t , time to maturity $\tau = T - t$, strike price K and interest rate r is given by $BS(t, X_t, \sigma)$.

In our setting, the call option price is given by

$$V_t = e^{-r\tau} \mathbb{E}_t[(e^{X_T} - K)^+]$$

where $\mathbb{E}_t$ is the conditional expectation respect to the σ -algebra $\mathcal{F}_t$.

Recall that from the Feynman-Kac formula for the model (2), the operator

$$\mathcal{L}_y := \partial_t + \frac{1}{2}y^2\partial_x^2 + \left(r - \frac{1}{2}y^2\right)\partial_x - r$$

satisfies $\mathcal{L}_y BS(t, x, y) = 0$, $t \in [0, T]$, $x \geq 0$ and $y \geq 0$.

It is well known that if the stochastic volatility process is independent of the price process, then the pricing formula of a plain vanilla European call is given by

$$V_t = \mathbb{E}_t[BS(t, S_t, \bar{\sigma}_t)]$$

where $\bar{\sigma}_t^2$ is the so called **average future variance** that is defined by

$$\bar{\sigma}_t^2 := \frac{1}{T-t} \int_t^T \sigma_s^2 ds.$$

This is the so called Hull and White formula. Naturally, $\bar{\sigma}_t$ is called the **average future volatility**.

Instead of the average future variance we can consider its adapted projection

$$v_t^2 := \mathbb{E}_t(\bar{\sigma}_t^2) = \frac{1}{T-t} \int_t^T \mathbb{E}_t[\sigma_s^2] ds$$

Our goal is to obtain a decomposition of V_t in terms of v_t . This switches an anticipative problem related with the anticipative process $\bar{\sigma}$ into a non-anticipative one related to the adapted process v . The idea comes from Alòs (2012), where the decomposition of V in terms of v was obtained for the Heston model.

We consider the martingale $M_t = \int_0^T \mathbb{E}_t[\sigma_s^2] ds$. We can write

$$dv_t^2 = \frac{1}{T-t} \left[dM_t + (v_t^2 - \sigma_t^2) dt \right].$$

We define the operators $\Lambda := \partial_x$, $\Gamma := (\partial_x^2 - \partial_x)$ and $\Gamma^2 = \Gamma \circ \Gamma$.
In particular, for the function BS we get

$$\begin{aligned}\Gamma BS(t, x, y) &:= \frac{e^x}{y\sqrt{2\pi\tau}} \exp\left(-\frac{d_+^2(y)}{2}\right), \\ \Lambda \Gamma BS(t, x, y) &:= \frac{e^x}{y\sqrt{2\pi\tau}} \exp\left(-\frac{d_+^2(y)}{2}\right) \left(1 - \frac{d_+(y)}{y\sqrt{\tau}}\right), \\ \Gamma^2 BS(t, x, y) &:= \frac{e^x}{y\sqrt{2\pi\tau}} \exp\left(-\frac{d_+^2(y)}{2}\right) \frac{d_+^2(y) - yd_+(y)\sqrt{\tau} - 1}{y^2\tau}.\end{aligned}$$

The following lemma is also useful in proofs:

Lemma

Let $0 \leq t \leq T$. Then, for every $n \geq 0$, there exists a constant C_n such that

$$|\Lambda^n \Gamma BS(t, X_t, v_t)| \leq \frac{C_n}{(\sqrt{T-t} \cdot v_t)^{n+1}}.$$



Extending the ideas of [Alòs \(2012\)](#) to jump diffusion models, [Merino, Pospíšil, Sobotka, and Vives \(2018\)](#) derived the following [generic decomposition formula](#) that is a consequence of the application of Itô and Feynman-Kac formulas.

Let

- ▶ B_t be a continuous semimartingale with respect to the filtration $\mathcal{F}^W$
- ▶ $A(t, x, y)$ be a $C^{1,2,2}([0, T] \times [0, \infty) \times [0, \infty))$ function
- ▶ v_t^2 and M_t be defined as above.
- ▶ $A_u := A(u, X_u, v_u^2)$

Theorem

$$\begin{aligned}
 \mathbb{E}_t \left[e^{-r(T-t)} A_T B_T \right] &= A_t B_t + \mathbb{E}_t \left[\int_t^T e^{-r(u-t)} \partial_y A_u B_u \frac{1}{T-u} (v_u^2 - \sigma_u^2) du \right] \\
 + \mathbb{E}_t \left[\int_t^T e^{-r(u-t)} A_u dB_u \right] &+ \frac{1}{2} \mathbb{E}_t \left[\int_t^T e^{-r(u-t)} (\partial_x^2 - \partial_x) A_u B_u (\sigma_u^2 - v_u^2) du \right] \\
 + \frac{1}{2} \mathbb{E}_t \left[\int_t^T e^{-r(u-t)} \partial_y^2 A_u B_u \frac{1}{(T-u)^2} d\langle M, M \rangle_u \right] \\
 + \rho \mathbb{E}_t \left[\int_t^T e^{-r(u-t)} \partial_{x,y}^2 A_u B_u \frac{\sigma_u}{T-u} d\langle W, M \rangle_u \right] \\
 + \rho \mathbb{E}_t \left[\int_t^T e^{-r(u-t)} \partial_x A_u \sigma_u d\langle W, B \rangle_u \right] &+ \mathbb{E}_t \left[\int_t^T e^{-r(u-t)} \partial_y A_u \frac{1}{T-u} d\langle M, B \rangle_u \right].
 \end{aligned}$$

In our setting, for function BS, we have the following corollary:

Corollary

Let $0 \leq t \leq T$. Then,

$$V_t = BS(t, X_t, v_t) + \mathbb{E}_t \left[\int_t^T e^{-r(u-t)} \Gamma^2 BS(u, X_u, v_u) d\langle M, M \rangle_u \right] \\ + \frac{\rho}{2} \mathbb{E}_t \left[\int_t^T e^{-r(u-t)} \Lambda \Gamma BS(u, X_u, v_u) \sigma_u d\langle W, M \rangle_u \right],$$

This formula is a consequence of previous theorem applied to function BS.

Note that, for the uncorrelated case we have

$$V_t = \mathbb{E}_t[BS(t, S_t, \bar{\sigma}_t)] = BS(t, X_t, v_t) + \mathbb{E}_t \left[\int_t^T e^{-r(u-t)} \Gamma^2 BS(u, X_u, v_u) d\langle M, M \rangle_u \right]$$

where

$$v_t = \sqrt{\mathbb{E}_t(\bar{\sigma}_t^2)}.$$

So, our formula is an alternative to Hull and White formula.

The idea of the **approximation** is to **freeze** the integrator in the two previous integral terms and write

$$V_t = BS(t, X_t, v_t) + \Gamma^2 BS(t, X_t, v_t) R_t + \Lambda \Gamma BS(t, X_t, v_t) U_t + \text{"error"},$$

where

$$R_t = \frac{1}{8} \mathbb{E}_t \left[\int_t^T d\langle M, M \rangle_u \right] \quad \text{and} \quad U_t = \frac{\rho}{2} \mathbb{E}_t \left[\int_t^T \sigma_u d\langle M, W \rangle_u \right]$$

where $\langle \cdot, \cdot \rangle$ denotes the quadratic covariation process.

- ▶ In case of Heston model, the terms R_t and U_t have nice analytic forms (Alòs, 2012), i.e. one can calculate them very quickly.
- ▶ Main goal: **find R_t and U_t for rough Volterra SV models.**

The error can be estimated applying the general decomposition another time. Using

$$A(t, X_t, v_t^2) = \Lambda \Gamma BS(t, X_t, v_t), \quad B_t = U_t$$

a decomposition of the term (I) can be found, and using

$$A(t, X_t, v_t^2) = \Gamma^2 BS(t, X_t, v_t), \quad B_t = R_t$$

a decomposition of the term (II) is obtained.

The terms that appear on the new decompositions can be bounded and the error of the previous approximation pricing formula can be controlled.



- ▶ The previous methodology has been applied up to now, by Alòs, Merino, V. and other collaborators to Heston model, Bates model, CEV model and more recently to other models with infinite activity jumps, by other authors.
- ▶ As far as we know the only application to a fractional model is the paper by Alòs, Chatterjee, Tudor and Wang (2019) where the methodology is applied to a lognormal fractional SABR model.
- ▶ The goal here is to apply the methodology to different Volterra type models, including rough Bergomi model.

We consider model (2) with a **general Volterra volatility process** defined as

$$\sigma_t := g(t, Y_t), \quad t \geq 0, \quad (3)$$

where $g : [0, +\infty) \times \mathbb{R} \mapsto [0, +\infty)$ is a deterministic function such that σ_t belongs to $L^1(\Omega \times [0, +\infty))$ and $Y = \{Y_t, t \geq 0\}$ is the Gaussian Volterra process

$$Y_t = \int_0^t K(t, s) \, dW_s, \quad (4)$$

where $K(t, s)$ is a kernel such that for all $0 \leq s < t \leq T$,

$$\int_s^T K^2(t, s) \, dt < \infty, \quad \int_0^t K^2(t, s) \, ds < \infty, \quad (A1)$$

and

$$\mathcal{F}_t^Y = \mathcal{F}_t^W. \quad (A2)$$

Let $r(t, s)$ denote the **autocovariance function** of Y_t and $r(t)$ the variance. That is,

$$r(t, s) = \mathbb{E}[Y_t Y_s], \quad t, s \geq 0, \quad r(t) = r(t, t) = \mathbb{E}[Y_t^2], \quad t \geq 0. \quad (5)$$

Theorem (Prediction law for Gaussian Volterra processes (Sottinen and Viitasaari, 2017))

Let $\{Y_t, t \geq 0\}$ be the Gaussian Volterra process (4) satisfying assumptions (A1) and (A2). Then, the conditional process $(Y_u | \mathcal{F}_t, 0 \leq t \leq u)$ is Gaussian with $\mathcal{F}_t$ -measurable mean function

$$\hat{m}_t(u) = \mathbb{E}_t[Y_u] = \int_0^t K(u, s) \, dW_s$$

and deterministic covariance function (with $u_1, u_2 \geq t$)

$$\hat{r}(u_1, u_2 | t) = \mathbb{E}_t[(Y_{u_1} - \hat{m}_t(u_1))(Y_{u_2} - \hat{m}_t(u_2))] = r(u_1, u_2) - \int_0^t K(u_1, v)K(u_2, v) \, dv.$$

Lemma (Auxiliary terms in the decomposition formula for the general volatility model)

Let $0 \leq t \leq u$ and $F(t, \hat{m}_t(u)) = \mathbb{E}_t(\sigma_u^2) = \mathbb{E}_t[g^2(u, Y_u)]$, then

$$dF(t, \hat{m}_t(u)) = \left(\partial_1 F(t, \hat{m}_t(u)) + \frac{1}{2} \partial_{22} F(t, \hat{m}_t(u)) K^2(u, t) \right) dt + \partial_2 F(t, \hat{m}_t(u)) d\hat{m}_t(u). \quad (6)$$

$$d\langle M, W \rangle_t = \int_0^T \partial_2 F(t, \hat{m}_t(u)) K(u, t) du dt.$$

$$d\langle M, M \rangle_t = \int_0^T \int_0^T \partial_2 F(t, \hat{m}_t(u_1)) \partial_2 F(t, \hat{m}_t(u_2)) K(u_1, t) K(u_2, t) du_1 du_2 dt.$$

Assume now that X_t is the log-price process (2) with σ_t being the **exponential Volterra volatility process**

$$\sigma_t = g(t, Y_t) = \sigma_0 \exp \left\{ \xi Y_t - \frac{1}{2} \alpha \xi^2 r(t) \right\}, \quad t \geq 0, \quad (7)$$

where $\{Y_t, t \geq 0\}$ is the Gaussian Volterra process (4) satisfying assumptions (A1) and (A2), $r(t)$ is its autocovariance function (5), and $\sigma_0 > 0$, $\xi > 0$ and $\alpha \in [0, 1]$ are model parameters.

Of course we could choose other examples changing function g or the model for σ_t . The treatment would be similar.

Lemma (Auxiliary terms in the decomposition formula for the exponential Volterra volatility model)

Let σ_t be as in (7) and $0 \leq t \leq u$. Then

$$F(t, \hat{m}_t(u)) = \sigma_0^2 \exp\{2\xi \hat{m}_t(u) + 2\xi^2 \hat{r}(u|t) - \alpha \xi^2 r(u)\},$$

$$d\langle M, W \rangle_t = 2\sigma_0^2 \xi \int_0^T \exp\{2\xi \hat{m}_t(u) + 2\xi^2 \hat{r}(u|t) - \alpha \xi^2 r(u)\} K(u, t) du dt,$$

$$\begin{aligned} d\langle M, M \rangle_t &= 4\sigma_0^4 \xi^2 \int_0^T \int_0^T \exp\{2\xi (\hat{m}_t(u_1) + \hat{m}_t(u_2))\} \\ &\cdot \exp\{2\xi^2 (\hat{r}(u_1|t) + \hat{r}(u_2|t))\} \cdot \exp\{-\alpha \xi^2 (r(u_1) + r(u_2))\} \cdot K(u_1, t) K(u_2, t) du_1 du_2 dt. \end{aligned}$$

Proposition (Terms in the approximation formula for the exponential Volterra volatility model)

Let σ_t be as in (7) and $0 \leq t \leq u$. Then we have detailed expressions for U_t and R_t . In particular,

$$U_0 = \rho \xi \sigma_0^3 \int_0^T \int_0^T e^{\alpha(s,u)} K(s,u) \, ds \, du$$

with

$$\alpha(s,u) = \frac{\xi^2}{2} \int_0^u [2K(s,z) + K(u,z)]^2 \, dz + 2\xi^2 \hat{r}(s|u) - \frac{1}{2} \alpha \xi^2 r(u) - \alpha \xi^2 r(s) \text{ and}$$

$$R_0 = \frac{1}{2} \sigma_0^4 \xi^2 \int_0^T \int_0^T \int_0^T e^{\beta(s,u,v)} K(s,u) K(v,u) \, ds \, dv \, du$$

with

$$\beta(s,u,v) = 2\xi^2 \int_0^u [K(s,z) + K(v,z)]^2 \, dz + 2\xi^2 (\hat{r}(s|u) + \hat{r}(v|u)) - \alpha \xi^2 (r(s) + r(v)).$$

Theorem (Upper error bound for the exponential Volterra volatility model)

Let X_t be a log-price process (2) with σ_t being the exponential Volterra volatility process (7). Then we can express the call option fair value V_t using the processes R_t, U_t from Proposition 7. In particular,

$$\begin{aligned} V_t = & BS(t, X_t, v_t) \\ & + \Lambda \Gamma BS(t, X_t, v_t) U_t \\ & + \Gamma^2 BS(t, X_t, v_t) R_t \\ & + \epsilon_t, \end{aligned}$$

where ϵ_t are error terms of order $O(\rho^2 \xi + \xi^3)$.

The error can be improved iterating the methodology one step more as it is done in Gulisashvili, Merino, Lagunas and V. (2020) for the Heston model. Then, more terms appear in the approximate formula.

Next step is to discuss possible elections for the kernel $K(t, s)$.

The most important example is the **standard fractional Brownian motion (fBm)** B_t^H :

$$B_t^H = \int_0^t K(t, s) \, dW_s, \quad (8)$$

where $K(t, s)$ is a kernel that depends on the Hurst parameter $H \in (0, 1)$.

Recall that the covariance function of B_t^H is given by

$$r(t, s) := \mathbb{E}[B_t^H B_s^H] = \frac{1}{2} \left(t^{2H} + s^{2H} - |t - s|^{2H} \right), \quad t, s \geq 0,$$

and in particular $r(t) := r(t, t) = t^{2H}$, $t \geq 0$.

As a first option, in α RFSV model we will assume that the volatility process

$$\sigma_t = \sigma_0 \exp \left\{ \xi B_t^H - \frac{1}{2} \alpha \xi^2 r(t) \right\}, \quad t \geq 0, \quad (9)$$

where $\sigma_0 > 0$, $\xi > 0$ and $\alpha \in [0, 1]$ are model parameters together with empirical $H < 1/2$.

Note that for $\alpha = 0$ we have the classical exponential fractional volatility RFSV model and for $\alpha = 1$ we have the rBergomi model.

Another representation of fractional Brownian motion, given by Weyl, is

$$B_t^H = \frac{1}{\Gamma(h + \frac{1}{2})} (Z_t + \int_0^t (t-s)^{H-\frac{1}{2}} dW_s)$$

where W is a standard Brownian motion and

$$Z = \int_{-\infty}^0 [(t-s)^{H-\frac{1}{2}} - (-s)^{H-\frac{1}{2}}] dW_s$$

has absolutely continuous trajectories.



Frequently in Finance the process

$$\hat{B}_t^H := \sqrt{2H} \int_0^t (t-s)^{H-\frac{1}{2}} dW_s$$

is used instead of the fractional Brownian motion.

Its behaviour is very similar. The only difference is that $\hat{B}^H$ has no stationary increments, but this is not crucial in our context.

Since fBm and $\hat{B}^H$ are not semimartingales, it is often useful to consider the so called **approximated fractional Brownian motion (afBm)**, i.e. a process with Volterra kernel

$$\tilde{K}(t, s) = \sqrt{2H}(t - s + \varepsilon)^{H-1/2} \mathbb{1}_{\{s \leq t\}}, \quad \varepsilon \geq 0, H \in (0, 1). \quad (10)$$

For every $\varepsilon > 0$ such a process is a semimartingale and as ε tends to zero it converges to $\hat{B}^H$ in L^2 uniformly on $t \in [0, T]$. In this case

$$\begin{aligned} r(t) &= \int_0^t \tilde{K}^2(t, v) dv = 2H \int_0^t (t - v + \varepsilon)^{2H-1} dv = (t + \varepsilon)^{2H} - \varepsilon^{2H}, \\ \hat{r}(t|u) &= r(t) - \int_0^u \tilde{K}^2(t, v) dv = r(t) - 2H \int_0^u (t - v + \varepsilon)^{2H-1} dv = (t - u + \varepsilon)^{2H} - \varepsilon^{2H} \end{aligned}$$



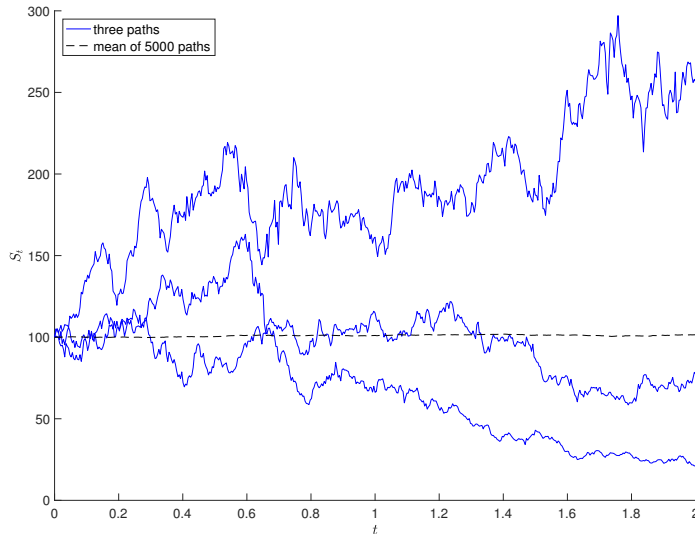
- ▶ For the *afBm* case we have obtained explicit formulas for U_0 and R_0 .
- ▶ A particular case of the *afBm* is the case $\epsilon = 0$ and $H = \frac{1}{2}$, that is, the case of exponential standard Brownian motion. In this case, Y is also a semimartingale.
- ▶ In the [αRFSV](#), if process Y is a semimartingale, the error of the approximation formula is of order $O(\rho^2 \xi^2 + \xi^4)$ because covariations between U , R , M and W are further specified.
- ▶ It is clear that choosing other functions g and other kernels $K(t, s)$, new rough volatility models can be proposed and it is a matter of algebraic operations to obtain the corresponding approximation formula.



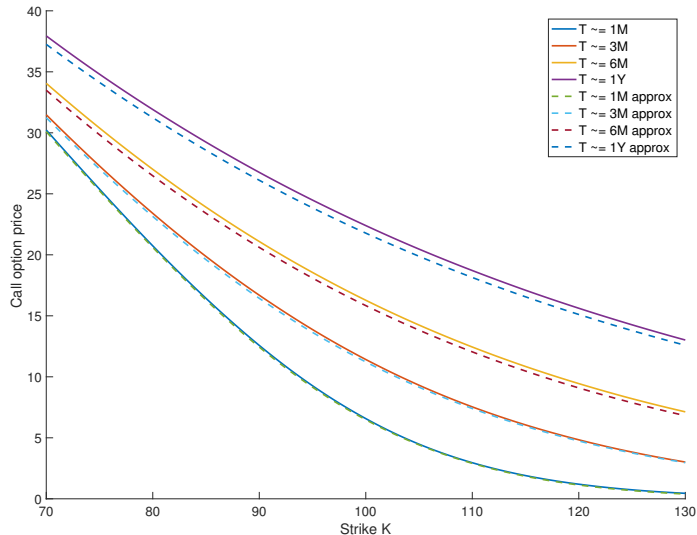
As a benchmark we use the MC method developed in [McCrickerd and Pakkanen \(2018\)](#).

- ▶ [McCrickerd and Pakkanen \(2018\)](#) applied a composition of variance reduction methods to the simulation of [rBergomi](#) model (Turbocharging Monte Carlo simulation) based on the first order [hybrid scheme](#) ([Bennedsen, Lunde, and Pakkanen, 2017](#)) for the Volterra process and applying antithetic sampling, conditional MC and control variates to achieve the variance reduction (and a speed-up by a factor of ca 20).
- ▶ We have modified the code for the [\$\alpha\$ RFSV model](#) defined by (2) and (7) with a Volterra process with kernel (10) with $\epsilon = 0$.

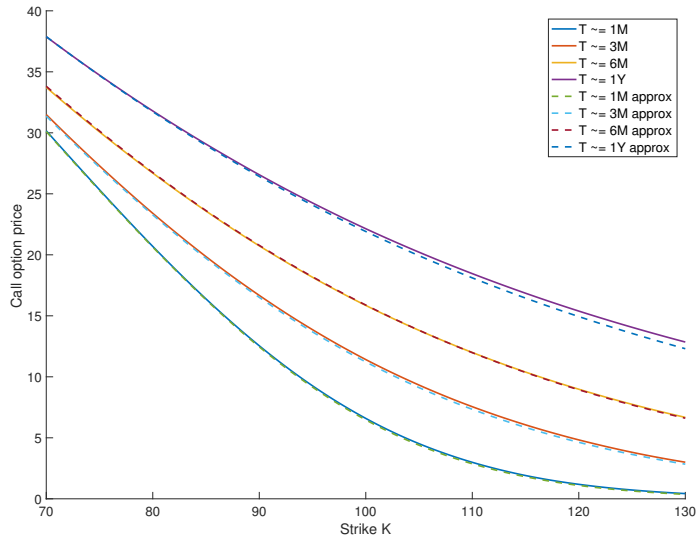
$H = 0.1, \alpha = 0, \rho = -0.2, \xi = 0.1, \sigma_0 = 0.3, S_0 = 100, r = 0.003, T = 2$



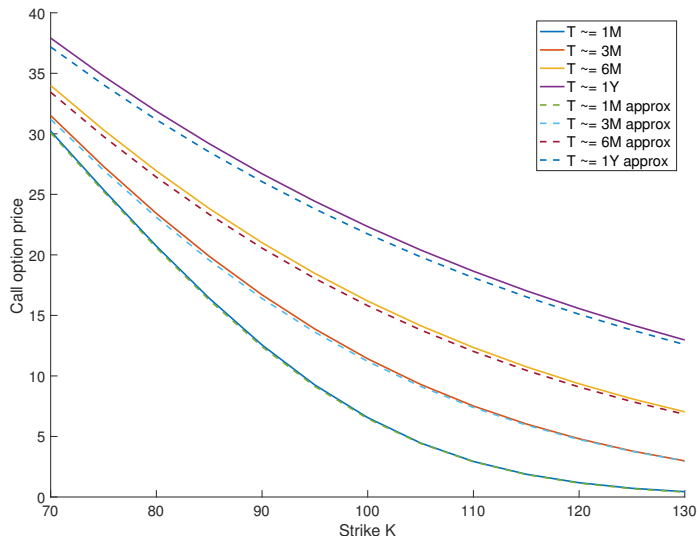
Example: $S_0 = 100$, $r = 0.003$, $\rho = -0.2$, $\sigma_0 = 0.3$, $\xi = 0.1$, $H = 0.1$.



Example: $S_0 = 100$, $r = 0.003$, $\rho = -0.2$, $\sigma_0 = 0.3$, $\xi = 0.1$, $H = 0.5$.



Example: $S_0 = 100$, $r = 0.003$, $\rho = -0.2$, $\sigma_0 = 0.3$, $\xi = 0.1$, $H = 0.1$.

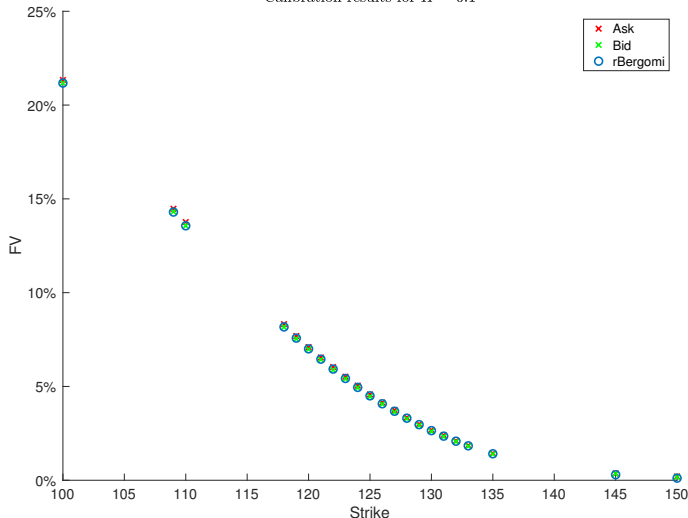


Market data and calibration technique used:

- ▶ options on AAPL from 2015-04-15 (Bloomberg),
- ▶ 158 options in total, 1M-maturity: 22 options,
- ▶ we focus on short-maturity, where other models (such as Heston) behave poorly (Pospíšil, Sobotka, and Ziegler, 2019),
- ▶ spot $S_0 = 126.78$.
- ▶ measured error: relative fair value $FV = \frac{|C^M - C^O|}{S}$ where C^M is the model call price, C^O is the observed call price and S is the spot price.

Calibrated parameters: $\sigma_0 = 0.0861, \xi = 0.2245, \rho = -0.5944$

Calibration results for $H = 0.1$





The following conclusions were drawn:

- ▶ The approximation error is well behaved for short maturities (typically for less than 1M) and the error increases with time to maturity and ξ parameter.
- ▶ For medium-term expires we obtain well approximated prices only under low ξ regimes.
- ▶ Although, the approximation under a rough Volterra process involves several numerical integration procedures, it is much faster than MC simulation approach implemented in the same environment.



- ▶ Considering that the modeling approach studied has only few parameters, we were able to fit the sample market data surprisingly well. For calibration to short maturity smiles, we use just the approximation formula. For calibration to the whole surface, we use a new hybrid scheme which consist of a combination of approximation and simulation techniques.
- ▶ The idea is to use approximation formula for low maturities or for low ξ values and MC simulations for the remaining computations. Suitability of this scheme was judged by a simple calibration to an ATMF-like backbone. We retrieved a well calibrated model, while saving a significant computational time compared to the calibration based on MC simulations only.

- E. ALÒS, R. CHATTERJEE, S. TUDOR AND T. H. WANG (2019): *Target volatility option pricing in lognormal fractional SABR model*. Quantitative Finance 19 (8): 1339-1356.
- E. ALÒS (2012): *A decomposition formula for option prices in the Heston model and applications to option pricing approximation*. Finance and Stochastics 16 (3): 403-422.
- M. BENNEDSEN, A. LUNDE AND M. PAKKANEN (2017): *Hybrid scheme for Brownian semistationary processes*. Finance and Stochastics 21 (4): 931-965.
- A. GULISASHVILI, M. LAGUNAS, R. MERINO AND J. VIVES (2020): *High-order approximations to call option prices in the Heston model*. Journal of Computational Finance 24(1), 1-20.
- R. MCCRICKERD AND M. PAKKANEN(2018): *Turbocharging Monte Carlo pricing for the rough Bergomi model*. Quantitative Finance 18 (11): 1877-1886.
- R. MERINO, J. POSPÍŠIL, T. SOBOTKA AND J. VIVES (2018): *Decomposition formula for jump diffusion models*. International Journal of Theoretical and Applied Finance 21 (8): 1850052.
- J. POSPÍŠIL, T. SOBOTKA AND P. ZIEGLER (2019): *Robustness and sensitivity analyses for stochastic volatility models under uncertain data structure*. Empirical Economics 57(6): 1935-1958.
- T. SOTTINEN AND L. VIITASAARI (2017): *Prediction law of fractional Brownian motion*. Statistics and Probability Letters 129 (Supplement C): 155-166.



Thank you for your attention!
Takk!
Gràcies!