CSA-2019

CONFECENCE IN STOCHASTIC ANALYSIS AND APPLICATIONS

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Book of Abstracts

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CSA2019 - Conference in Stochastic Analysis and Applications

Welcome to the CSA2019 - Conference in Stochastic Analysis and Applications. The purpose of the conference is to bring together leading researchers and students within stochastic analysis and applications, to discuss new results and research challenges, with emphasis on stochastic control and information, as well as stochastic analysis for random fields with applications to risk models in finance, biology, insurance and physics.

CSA 2019 - Conference in Stochastic Analysis and Applications

Confernce Program

		26-ago	27-ago	28-ago	29-ago	30-ago
Start	End	Monday	Tuesday	Wednesday	Thursday	Friday
09:00	09:35	Mishura	Crisan	Pham	Zhang	Dorogovtsev
09:35	10:00	Doukhan	Sgarra	Guambe	Kebiri	Harang
10:00	10:35	Bingham	Bion-Nadal	Sulem	Benth	Petterson
10:35	10:55	Coffe break	Coffe break	Coffe break	Coffe break	Coffe break
10:55	11:30	Zheng	Djehiche	Hilbert	Quenez	Pilipenko
11:30	11:55	Lim	Ilienko A.	Framstad	Cornelis Rosati	Ouknine
11:55	12:30	Ortiz-Latorre	Klesov	Ralchenko	Mlavets	Hu
12:30	14:00	Lunch	Lunch	Lunch	Lunch	Lunch
14:00	14:35	Melnykova	Agram	Tymoshenko	Ilienko M.	
14:35	15:00	Mastrolia	Hernandez Hernandez	Palmowski	Giordano	
15:00	15:25	Kharroubi	Draouil	Butko	Li	
15:25	15:45	Coffee break	Coffe break	Coffe break	Coffe break	
15:45	16:10	Dahl	Catellier		Bacouch	
16:10	16:35	Imkeller	Lobbe		Lagunas	
16:35	16:45					
		Posters-cocktails	Tour of Risør	Lecture by author and poet Arild Stubhaug	Boat tour and conference dinner	

Monday 26th of August

Fractional time-changed processes

Giacomo Ascione[‡], Yulia Mishura[†], Enrica Pirozzi[‡]

[†] Taras Shevchenko National University of Kyiv, Ukraine

[‡] Università di Napoli Federico II

We consider a fractional Ornstein-Uhlenbeck time-changed process, calculate its characteristics and study their asymptotical behavior. The results, to some extent, are based on the theorems from the paper [1], where the fractional Ornstein-Uhlenbeck process with forcing term was investigated, with application to neuronal modeling.

References

1 Giacomo Ascione, Yuliya Mishura, and Enrica Pirozzi. Fractional Ornstein-Uhlenbeck process with stochastic forcing and its applications. *Submitted*, 2019.

Discrete Trawl processes and their estimation

Paul Doukhan[†]

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We first recall the results included in([1]) coauthored with Silvia Lopes, Adam Jakubowski and Donatas Surgailis. Namely we introduced a new class of stationary stochastic processes

$$X_t = \sum_{j=0}^{\infty} \gamma_{t-j}(a_j)$$

with (a_j) a trawl sequence and (γ_j) an iid sequence of seeds. with different properties of short and long dependence. The main attraction of those discrete trawl time series is that they include long range dependent models with integer values, moreover such models admit a very interesting limiting behaviour. E.g. if the seeds are homogeneous Poisson and $a_j \sim cj^{-\alpha}$ for $1 < \alpha < 2$ then long range dependence holds true; namely in that case

$$\text{cov}\ (X_0, X_k) = \sum_{j \geq k} a_j.$$

In such a case, the partial sums processes do not necessarily converge towards a Brownian or fractional Brownian motion, once they are renormalised, but Lévy stable limits also occur. This is a really original situation

[1]Discrete-time trawl processes.
Stochastic Processes and their Applications, volume 129 (2019) 1326?1348.

In a recent preprint, with François Roueff and Joseph Rynkiewicz we investigate semi-parametric and parametric estimates of the long range dependence exponent of the given model. The specific form of the spectral density is fully used to consider adapted parametric frames. Indeed a more natural log-regression based estimator of α is proved to be quite inefficient. Numerical experiments also validate the study.

Gaussian Random Fields on Sphere and Sphere-cross-line

Nick Bingham[†]

[†]Imperial College London

We review the Dudley integral for the Belyaev dichotomy applied to Gaussian processes on spheres, and discuss the approximate (or restricted) continuity of paths in the discontinuous case. We discuss also the spatio-temporal case, of sphere cross line. In the continuous case, we investigate the link between the smoothness of paths and the decay rate of the angular power spectrum, following Tauberian work of the rst author, Malyarenko, and Lang and Schwab.

Constrained quadratic risk minimization via primal-dual FBSDEs and deep learning

Zheng Harry[†]

[†]Imperial College London

In this paper we study a stochastic control problem arising from mathematical finance. The goal is to minimize a quadratic cost function in a continuous time model with random market parameters and portfolio constraints. Following a convex duality approach, we prove the necessary and sufficient conditions for both the primal and dual problems in terms of fully coupled constrained FBSDEs. This allows us to explicitly characterise the primal control as a function of adjoint process coming from the dual FBSDEs in a dynamic fashion and vice versa. We show the usefulness of the dual approach with several examples, in particular, we give the explicit representation of the solution to the extended stochastic Riccati equation. Finally, we propose a deep learning primal-dual algorithm to solve fully coupled constrained FBSDEs.

Regulation of the exploitation of a natural resource by using a princal-agent approach

Idris Kharroubi[‡], Thomas Lim[†], Thibaut Mastrolia[°]
[†] ENSIIE-LaMMe
[‡] Université Paris Sorbonne-LPSM
[°] Ecole Polytechnique-CMAP

We investigate the impact of a regulation policy imposed on an agent exploiting a renewable natural resource. We adopt a principal/agent model in which the principal looks for a contract, i.e: taxes/compensations, leading the agent to a given level of exploitation. For a given contract, we first describe the agent's optimal effort using the BSDE theory. Under regularity and boundedness assumptions on the coefficients, we then express almost optimal contracts as solutions to HJB equations. We then extend the result to coefficients with less regularity and logistic dynamics for the natural resource. We end by numerical examples to illustrate the impact of the regulation in our model.

High order discretizations for the solution of the nonlinear filtering problem

Salvador Ortiz-Latorre[†]
[†]University of Oslo

The solution of the continuous time filtering problem can be represented as a ratio of two expectations of certain functionals of the signal process that are parametrized by the observation path. In this talk I will introduce a class of discretization schemes of these functionals of arbitrary order. For a given time interval partition, we construct discretization schemes with convergence rates that are proportional with the m-power of the mesh of the partition for arbitrary $m \in \mathbb{N}$. The result generalizes the classical work of Picard, who introduced first order discretizations to the filtering functionals. Moreover, the result paves the way for constructing high order numerical approximation for the solution of the filtering problem. This is a joint work with Dan Crisan.

Statistical testing of the covariance matrix in multidimensional neuronal models

Anna Melnykova[†], Patricia Reynaud-Bouret[‡], Adeline Samson[°]
[†] Université de Cergy-Pontoise and Université Grenoble Alpes
[‡] Université de Nice Sophia-Antipolis
[°]Université Grenoble Alpes

Stochastic diffusions became a classical tool for describing a neuronal activity, either of a one single neuron (Ditlevsen and Samson, 2012, Höpfner et al., 2016, Leon and Samson, 2017), or a large network of neurons (Ditlevsen and Löcherbach, 2017, Ableidinger et al., 2017). However, the techniques which would allow us to establish a rigorous link between a specific model and available neurophysiological data is often missing. The open question is the source of stochasticity in spiking activity. One point of view is that both the membrane and the ion channels of the neuron cell are affected by noise. Another position is that only the ion channels have a stochastic

behaviour and that their concentration in cell explicitly defines the membrane potential. The question is then how to test both hypotheses with extracellular recordings of the membrane potential. For network-scale neuronal models, the estimation of the noise rank is equivalent to estimating a number of populations of different types of neurons in the network. The question boils down to a problem of a covariance matrix rank estimation and constructing a statistical test of the rank. Our aim is to challenge this problem with the help of numerical approximation methods for stochastic diffusions and properties of matrix determinants, following works of Jacod et al. (2008), Jacod and Podolskij (2013).

References

1 Ableidinger, M., Buckwar, E., and Hinterleitner, H. (2017). *A stochastic version of the Jansen and Rit neural mass model: Analysis and numerics.* The Journal of Mathematical Neuroscience, 7(1):1.

2 Ditlevsen, S. and Löcherbach, E. (2017), *Multi-class oscillating systems of interacting neurons.* SPA, 127:1840–1869.

3 Fitzhugh, R. (1961). *Impulses and physiological states in theoretical models of nerve membrane.* Biophysical Journal, 1(6):445–466.

4 Höpfner, R., Löcherbach, E., Thieullen, M., et al. (2016). *Ergodicity for a stochastic Hodgkin–Huxley model driven by Ornstein–Uhlenbeck type input.* In Annales de l'Institut Henri Poincaré, Probabilités et Statistiques, volume 52, pages 483–501. Institut Henri Poincaré.

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6 Jacod, J. and Podolskij, M. (2013). *A test for the rank of the volatility process: the random perturbation approach.* The Annals of Statistics, 41(5):2391–2427.

7 Leon, J. R. and Samson, A. (2018). *Hypoelliptic stochastic FitzHugh-Nagumo neuronal model: mixing, up-crossing and estimation of the spike rate.* Annals of Applied Probability, 28:2243-2274

Optimal auction duration: price formation point of view

Paul Jusselin[†], Thibaut Mastrolia[†], Mathieu Rosenbaum[†]

[†] Ecole Polytechnique

We investigate an auction market in which competitors deal during a time period fixed by the exchange. We determine the clearing price of the auction as the price maximizing the exchanged volume at the clearing time according to the supply of each market participants. Then, we compute the error made between this clearing price and the fundamental price as a function of the auction duration. We study the impact of the behavior of market takers on this error by considering rational market takers playing a Nash equilibrium to minimize their transaction costs. We compute the optimal duration of the auction in this situation for several stocks traded at Euronext exchange and we compare it with unsophisticated market takers, who do not optimize their transaction costs.

Optimal exploitation of a resource with stochastic population dynamics and delayed renewal

Idris Kharroubi[†], Thomas Lim[‡], Vathana Ly Vath[‡]

[†] Sorbonne Université
[‡]ENSIIE

In this work, we study an optimization problem arising when managing a renewable resource in

finite time. The resource is assumed to evolve according to a logistic stochastic differential equation. The manager harvests partially the resource at any time and sell it at a stochastic market price. She equally decides to renew part of the resource but uniquely at deterministic times. However, we realistically assume that there is a delay in the renewing order. By using the dynamic programming theory, we characterize our value function as the unique solution to a PDE. To complete our study, we give an algorithm to compute the value function and optimal strategy. Some numerical illustrations are also provided.

Management of a hydropower system via convex duality

Kristina Rognlien Dahl[†]

[†] University of Oslo

We consider a stochastic hydroelectric power plant management prob lem in discrete time with arbitrary scenario space. The inflow to the system is some stochastic process, representing the precipitation to each dam. The manager can control how much water to turbine from each dam at each time. She would like to choose this in a way which maximizes the total profit from the initial time 0 to some terminal time T. The total profit of the hydropower dam system depends on the price of electricity, which is also a stochastic process. The manager must take this price process into account when controlling the draining process. However, we assume that the manager only has partial information of how the price process is formed. She can observe the price, but not the underlying processes determining it. By using the conjugate duality framework, we derive a dual problem to the management problem. This dual problem turns out to be simple to solve in the case where the profit rate process is a martingale or submartingale with respect to the filtration modeling the information of the dam manager. In the case where we only consider a finite number of scenarios, solving the dual problem is computationally more efficient than the primal problem.

On the geometry of some rough Weierstrass curves: SBR measure and local time

Peter Imkeller[†]

[†]HU Berlin

We investigate geometric properties of Weierstrass curves with one or two components, representing series based on trigonometric functions. They are Holder continuous, and not (para-)controlled with respect to each other. They can be embedded into smooth dynamical systems, where their graph emerges as a pullback attractor. It turns out that occupation measures and Sinai-Bowen-Ruelle (SBR) measures on its stable manifold are closely related to each other. A suitable version of approximate self similarity for deterministic functions allows to "telescope" small scale properties from macroscopic ones. As a consequence, absolute continuity of the SBR measure is obtained, as well as the existence of a local time. The link between rough Weierstrass curves and smooth dynamical systems can be generalized considerably. Applications to regularization of singular ODE by rough (Weierstrass type) signals are on our agenda. This is joint work with G. dos Reis (U Edinburgh) and O. Pamen (U Liverpool and AIMS Ghana).

Tuesday 27th of August

Well-posednesss for a class of fluid dynamics equation with stochastic transport noise

Dan Crisan[†]

[†] Imperial College London

A Gamma Ornstein-Uhlenbeck model driven by a Hawkes process

Guillaume Bernis[†], Simone Scotti[‡], Carlo Sgarra[†]

[†]Politecnico di Milano
[‡]University Paris Diderot

We propose an extension of the Gamma-OU Barndorff-Nielsen and Shephard model taking into account jumps clustering phenomena. We assume that the intensity process of the Hawkes driver coincides, up to a constant, with the variance process. By applying the theory of continuous-state branching processes with immigration, we prove existence and uniqueness of strong solutions of the SDE governing the asset price dynamics. We introduce a measure change of Esscher type in order to describe the relation between the risk-neutral and the historical dynamics. By exploiting the affine features of the model we provide an explicit form for the Laplace transform of the asset log-return, for its quadratic variation and for the ergodic distribution of the variance process. We show that the model proposed exhibits a larger flexibility in comparison with the Gamma-OU model, in spite of the same number of parameters required. In particular, we illustrate numerically that the left wing implied volatility could be first fit by using the original Gamma-OU model and then the right wing can be arranged by a trigger of the intensity and variance processes. Moreover, implied volatility of variance swap options is upward-sloped due to the self-exciting property of Hawkes processes.

Approximation of solutions to stochastic differential equations

Jocelyne Bion-Nadal[†], Denis Talay[‡]

[†]CNRS - CMAP, Ecole Polytechnique
[‡]Inria Sophia Antipolis

In many situations where stochastic modeling is used, one desires to choose the coefficients of a stochastic differential equation which represents the reality as simply as possible. For example one desires to approximate a diffusion model with high complexity coefficients by a model within a class of simple diffusion models.
To achieve this goal, we introduce [1] a new Wasserstein type distance on the set of laws of solutions to d-dimensional stochastic differential equations. This new distance $\widetilde{W}^2$ is defined similarly to the classical Wasserstein distance W^2 but the set of couplings is restricted to the set of laws of solutions of $2d$-dimensional stochastic differential equations. We prove that this new distance $\widetilde{W}^2$ metrizes the weak topology. Furthermore this distance $\widetilde{W}^2$ is characterized in terms of a stochastic control problem.
In the case $d = 1$ we can constuct an explicite solution. The multi-dimensional case is more tricky, and classical results do not apply to solve the HJB equation because of the degenaracy of the differential operator. Nevertheless, we prove that this HJB equation admits a regular solution.

References

1 Bion-Nadal J. and Talay D. (2019) *On a Wasserstein-type distance between solutions to stochastic differential equations.* Annals of Applied Probability29(3):1609-1639

Mean-field type games in pedestrian crowd dynamics

Djehiche Boualem[†]

[†]KTH Royal Institute of Technology

We will review some recent results on modeling and games of pedestrian crowds in two situations: We first consider a mean-field model for the movement of tagged pedestrians, distinguishable from a surrounding crowd, with a targeted final destination. The tagged pedestrians move through a dynamic crowd, interacting with it while optimizing their path. The model includes distribution-dependent effects like congestion and crowd aversion. Finally, we focus on modeling and controlling pedestrian motion in confined domains, where interaction with solid obstacles like walls and pillars is considered. This amounts to control a particular class sticky SDEs of mean-field type.

On the convergence of point processes associated with coupon collector's and dixie cup problems

Andrii Ilienko†

†Igor Sikorsky Kyiv Polytechnic Institute

The coupon collector's problem (CCP) as well as its generalization known as the Dixie cup problem (DCP) belong to the classics of combinatorial probability. Their statements are as follows: a person collects coupons, each of which belongs to one of n different types. The coupons arrive one by one at discrete times, the type of each coupon being equiprobable and independent of types of preceding ones. The most typical questions concern asymptotics of the (random) numberof coupons a person needs to collect in order to assemble r complete collections. The case $r = 1$ refers to CCP while $r \geq 2$ to DCP. CCP, DCP and their further generalizations have a long history going back to de Moivre, Euler and Laplace. In his seminal paper, Holst [1] proposed a fruitful poissonization idea which allowed to prove limit results avoiding intricate combinatorial calculations. In a very recent paper by Glavaš and Mladenović [2], the connections between CCP and Poisson processes were shown to be even more tight. It was proved that the point processes given by the times of first arrivals for coupons of each type, centered and normalized in a proper way, converge toward a non-homogeneous Poisson point process as $n \to \infty$. Inspired by this paper, the present talk pursues a threefold objective. Firstly, we generalize the above result to the case of DCP. Secondly, to this end, we develop a specific approach involving a poissonization technique in the spirit of [1] and some coupling-based depoissonization procedure. This allows for avoiding sophisticated combinatorial machinery used in [2]. Thirdly, we demonstrate the power of this result: it can be used to easily derive some generalizations and infinite-dimensional extensions of classical limit theorems on the topic.

References

1 Lars Holst. *On birthday, collectors', occupancy and other classical urn problems.* Int. Stat. Rev., 54:15-27, 1986.

2 Lenka Glavaš and Pavle Mladenović. *New limit results related to the coupon collector's problem.* Stud. Sci. Math. Hung., 55(1):115-140, 2018.

The sets of limit points of subordinators and inverse subordinators

Oleg Klesov†

†National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute"

Let X_t be a subordinator and let Y_t be the corresponding inverse subordinator. A very particular problem among those studied in [1] reads as follows: *find the asymptotic behavior of Y_t if you know that of X_t.* More precisely, assume that

$$\lim_{t\to\infty} \frac{X_t}{g_t} = 1 \qquad \text{almost surely} \tag{1}$$

for some function g_t. Then the problem is to find another function $\widetilde{g}_t$ such that

$$\lim_{t\to\infty} \frac{Y_t}{\widetilde{g}_t} = 1 \qquad \text{almost surely.} \tag{2}$$

This problem can be met throughout in probability theory and other branches of mathematics. This problem is solved in [1] rather completely in the framework of so-called *pseudo-regularly varying functions*.

For several classes of subordinators other results are known, namely

$$\limsup_{t\to\infty} \frac{X_t}{g_t} = 1 \qquad \text{almost surely} \tag{3}$$

or

$$\liminf_{t\to\infty} \frac{X_t}{g_t} = 1 \qquad \text{almost surely.} \tag{4}$$

For other classes of subordinators, similar results are known for Y_t rather than for X_t, that is

$$\limsup_{t\to\infty} \frac{Y_t}{g_t} = 1 \qquad \text{almost surely} \tag{5}$$

or

$$\liminf_{t\to\infty} \frac{Y_t}{g_t} = 1 \qquad \text{almost surely.} \tag{6}$$

Motivated by a strong relationship between (1) and (2) a natural question then is whether or not

(3) implies (5)?

or whether or not

(3) implies (6)?

There is a variety of other similar questions obtained by permuting (3), (4), (5), and (6) in the questions above. For example,

is it true that (5) implies (3)?

or

is it true that (5) implies (4)?

An even more general problem discussed in the talk is as follows: let $\mathcal{L}(X, g)$ be the set of limit points of the ratio X_t/g_t and $\mathcal{L}(Y, \widetilde{g})$ be the set of limit points of the ratio $Y_t/\widetilde{g}_t$.

How $\mathcal{L}(Y, \widetilde{g})$ is related to $\mathcal{L}(X, g)$?

Some answers to the last question will be given in the talk.

Acknowledgement. The results presented in the talk is a part of an ongoing research project with J. Steinebach (University of Köln, Germany) and N. N. Leonenko (University of Cardiff, Great Britain).

References

1 O. I. Klesov V. V. Buldygin, K.-H. Indlekofer and J. G. Steinebach. (2018), *Pseudo-Regularly Varying Functions and Generalized Renewal Processes*, Springer

SPDEs with Space-Mean Dynamics

Nacira Agram[†]

[†]Linnaeus Universitet

We Consider a new type of stochastic partial differential equations (SPDEs), namely SPDEs where the coefficients not only depend on the state of the solution at time t and space x, but also on space averages of the solution around the point x. We call these equations SPDEs with space averages (SPDE-SA). These equations constitute relevant models for population growth.

We obtain sufficient and necessary conditions for optimality for such systems. Then we apply the results to an optimal harvesting problem from a population whose density is modelled as a space-mean stochastic reaction-diffusion equation.

This talk is based on a joint work with Astrid Hilbert (Linnaeus University) and Bernt Oksendal (University of Oslo).

Stochastic Optimal control in continuous time for semimartingales

Ma. Elena Hernández-Hernández[†], Saul Jacka[†], Alexander Mijatovic[†]

[†]University of Warwick

We provide a verification theorem for infinite horizon stochastic control problems in continuous time for general stochastic processes. The control framework is given as an abstract "martingale formulation" which encompasses a broad range of standard control problems. Under appropriate conditions we are able to show that the set of admissible controls gives rise to controlled processes that are special semimartingales. As an illustration, we present an explicit solution to a control problem with a running cost function which is quadratic in the space variable and in the infinitesimal drift of the underlying dynamics.

Variational inequalities and optimal stopping of Hunt processes

Olfa Draouil[†]

[†]University of Oslo

In this paper we present a new verification theorem for optimal stopping problems for Hunt processes. The approach is based on the Fukushima-Dynkin formula and its advantage is that it allows us to verify that a given function is the value function without using the viscosity solution argument. Our verification theorem works in any dimension. We illustrate our results with some examples of optimal stopping of reflected diffusions and absorbed diffusions.

Mean-field rough differential equation

Ismaël Bailleul[‡], Rémi Catellier[†], François Delarue[†]
[†]Université Côte d'Azur
[‡]Université de Rennes 1

In this talk we provide a robust solution theory for random rough differential equations of mean field type

$$dX_t = V(X_t, \mathcal{L}(X_t))dt + F(X_t, \mathcal{L}(X_t))dW_t,$$

where W is a random rough path and $\mathcal{L}(X_t)$ stands for the law of X_t, with mean field interaction in both the drift and diffusivity. The analysis requires the introduction of a new rough path-like setting and an associated notion of controlled path. We use crucially Lions' approach to differential calculus on Wasserstein space along the way. As an application we address the propagation of chaos for interacting particle system driven by independent random rough path.

Deep Learning for Stochastic Filtering

Alexander Lobbe[†]
[†]University of Oslo

In this talk we will start with an overview over some developments relating stochastic analysis and machine learning. Specifically, the deep learning paradigm has yielded many successful results in the area of the numerical approximation of high-dimensional partial differential equations in terms of speed of computation and accuracy. A crucial ingredient here is the Feynman-Kac formula relating deterministic PDEs with stochastic equations. Thereafter, we will introduce a stochastic filtering problem and the related Zakai equation for the density of the filter

$$p_0(x) + \int_0^t A^* p_s(x)ds + \int_0^t h'(x)p_s(x)dY_s = 0, \tag{1}$$

where A^* is the adjoint of the infinitesimal generator of the given signal process. With this problem at hand we present a work-in-progress idea of using a deep learning method to compute approximate solutions for the Zakai equation based on the splitting method which is known in the literature. With this so-called predictor-corrector scheme one iteratively computes first a solution to the deterministic Fokker-Planck equation

$$\frac{\partial}{\partial t} p_t^n(z) = A^* p_t^n(z) \qquad p_{t_{n-1}}^n(z) = p_{t_{n-1}}^{n-1}(z) \tag{2}$$

over a slice $[t_{n-1}, t_n]$ of the time interval and, secondly, corrects for the stochastic term using the standard Kallianpur-Striebel formula from stochastic filtering. Here, the first step is where we wish to use deep learning to be able to compute the filter in high-dimensions. In order to apply the deep learning paradigm in this case we present a Feynman-Kac result derived for our case along with results from the literature that we adapted to the filtering problem.

Wednesday 28th of August

Markov decision processes with mean-field interaction under common noise and application to targeted avertising

Huyen Pham[†‡], Médéric Motte[†]

[†] LPSM, Université de Paris
[‡]CREST-ENSAE

We consider Markov decision process (MDP) on infinite horizon where the state transition function depends upon the conditional law (given some noise) of the state and of the action/policy. Such control problem, which extends the theory of MDP, and called conditional McKean-Vlasov MDP, is proven to be the limiting problem of an N-player cooperative game in discrete time under mean-field interaction and common noise, and can also be viewed as an influencer strategy or social planner problem over an interacting large population. We show equivalent formulations of the CMKV-MDP with open, closed-loop and stationary controls, and prove the law-invariance property of the value function together with the dynamic programming equation and the associated algorithms. Finally, we give an application with explicit results to a problem of targeted advertising via social networks.

Risk-Based optimal portfolio of an insurer with regime switching and noisy memory

Rodwell Kufakunesu[‡], Calisto Guambe[†], Lesed Mabitsela [‡]

[‡] University of Pretoria
[†]Eduardo Mondlane University

In this talk, we consider a risk-based optimal investment problem of an insurer in a regime-switching jump diffusion model with noisy memory. Using the model uncertainty modeling, we formulate the investment problem as a zero-sum, stochastic differential delay game between the insurer and the market, with a convex risk measure of the terminal surplus and the Brownian delay surplus over a period $[T - \varrho, T]$. Then, by the BSDE approach, the game problem is solved. Finally, we derive analytical solutions of the game problem, for a particular case of a quadratic penalty function and a numerical example is considered.

Option pricing in a nonlinear incomplete market model with default

Agnès Sulem[†], Marie-Claire Quenez[‡], Miryana Grigorova[°]

[†]INRIA Paris

[‡] LPSM, Université Paris-Diderot

[°] University of Leeds

We study superhedging of European options in a nonlinear incomplete market model with default, from both the seller's and the buyer's point of view. The underlying market model consists of a risk-free asset and a risky asset driven by a Brownian motion and a compensated default martingale. The portfolio processes follow nonlinear dynamics with a nonlinear driver f. By using a dynamic programming approach, we first provide a dual formulation of the seller's superhedging price for the European option as the supremum over a suitable set of equivalent probability measures Q of the f-evaluation under Q of the payoff. We also provide a characterization of the seller's superhedging price as the minimal supersolution of a constrained BSDE with default and a characterization in terms of the minimal weak supersolution of a BSDE with default. By a form of symmetry, we derive corresponding results for the buyer. The case of American options is then addressed.

A mean-field game of pure jump type with common noise

Astrid Hilbert[†]

[†]Linnaeus University

In this paper we study a mean field game under jump dynamics, where all the players are subject to the same additional Brownian noise. We study the well-posedness and the regularity for the jump version of the stochastic partial differential equation of McKean-Vlasov type. Finally, we show that the solution of the master equation, which is a type of second order partial differential equation in the space of probability measures, provides an approximate Nash-equilibrium.

Dynamic conservation contracts

Bard Harstad[†‡], Nils Chr. Framstad[†]

[†]University of Oslo

[‡]Work supported by Bård Harstad's ERC 683031 grant.

In order to preserve the environment, a conservation-minded principal offers owners of a stock (forests, wildlife, fossil fuels ...) state-contingent contracts which provide a running income as compensation for conserving the stock rather than extracting and selling in a market with linear demand. We formulate a stylized mathematical model for such dynamic contracts. For a class of problems, value functions are shown to be sufficiently regular for classical solutions of the corresponding Hamilton-Jacobi-Bellman-Isaacs equation systems (second-order when there is Brownian noise). For the deterministic case, the resulting Markov differential game could yield a stable or unstable dynamic system, where in the simplest unstable case, the smallest resource stocks will be depleted and only the largest conserved, except on the saddle path (or if starting in the steady-state region).

Existence and uniqueness of mild solution to fractional stochastic heat equation

Yulia Mishura†, Kostiantyn Ralchenko†, Georgiy Shevchenko†

†Taras Shevchenko National University of Kyiv

For a class of non-autonomous parabolic stochastic partial differential equations defined on a bounded open subset $D \subset \mathbb{R}^d$ and driven by an $L^2(D)$-valued fractional Brownian motion with the Hurst index $H \geq 1/2$, we establish a new result on existence and uniqueness of a mild solution. Compared to the existing results [1], we show uniqueness in a fully nonlinear case, not assuming the coefficient in front of the noise to be affine. Additionally, we establish existence of moments for the solution. Furthermore, we prove the existence and uniqueness of a mild solution to a similar stochastic partial differential equation involving standard and fractional $L^2(D)$-valued Brownian motions. We assume that the coefficients are homogeneous, Lipschitz continuous and the coefficient at the fractional Brownian motion is an affine function.

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Asymptotic Properties of Some Non-homogeneous Stochastic Differential Equations

Olena Tymoshenko†

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Consider non-homogeneous stochastic differential equation with separation of stochastic and deterministic variables

$$dX(t) = \varphi(t)g(X(t))dt + \theta(t)\sigma(X(t))dw(t).$$

We express an asymptotic behavior of solutions of such equations in terms of the corresponding ordinary differential equation.

The general results are discussed and applied to some particular equations, mainly in the field of mathematics of finance. For example, the asymptotic behavior of the solutions of differential equation with state-independent perturbation [1-2].

$$dX(t) = g\left(X(t)\right)dt + \theta\left(t\right)dw(t).$$

are studied. Population Growth Model [3]

$$dX(t) = \phi(t)X(t)dt + \beta X(t)dw(t),$$

and Rendleman–Bartter Model [4]

$$dX(t) = X(t)\phi(t)\,dt + X(t)\theta(t)\,dw(t)$$

is also considered.

We follow the setting by I.I. Gihman and A.V. Skorohod [5], however the results of the talk are more general.

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The Leland-Toft optimal capital structure model under Poisson observations

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[⋆]Kansai University

We revisit the optimal capital structure model with endogenous bankruptcy first studied by Leland [1] and Leland and Toft [2]. Differently from the standard case, where shareholders observe continuously the asset value and bankruptcy is executed instantaneously without delay, we assume that the information of the asset value is updated only at intervals, modeled by the jump times of an independent Poisson process. Under the spectrally negative Lévy model, we obtain the optimal bankruptcy strategy and the corresponding capital structure. A series of numerical studies are given to analyze the sensitivity of observation frequency on the optimal solutions, the optimal leverage and the credit spreads.

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Chernoff approximation of Markov evolution

Yana A. Butko[†]

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We present a method to approximate evolution semigroups generated by Markov processes and, therefore, transition probabilities of these processes. This method is based on the Chernoff theorem. In some cases, Chernoff approximations provide also Markov chains approximating the considered processes and Euler-Maruyama Schemes for the related SDEs. In some cases, Chernoff approximations have the form of limits of n iterated integrals of elementary functions as $n \to \infty$ (in this case, they are called Feynman formulae) and can be used for direct computations and simulations of Markov processes. The limits in Feynman formulae sometimes coincide with (or give rise to) path integrals with respect to probability measures (such path integrals are usually called Feynman-Kac formulae). Therefore, Feynman formulae can be used to approximate the corresponding path integrals and to establish relations between different path integrals.

In this talk, we discuss Chernoff approximations for (semigroups generated by) Feller processes in $\mathbb{R}^d$. We are also interested in constructing Chernoff approximations for Markov processes which are obtained from some original Markov processes, assuming that Chernoff approximations for the original processes are already known. In this talk, we present Chernoff approximations for such operations as: a random time change via an additive functional of a process, a subordination (i.e., a random time change via an independent a.s. nondecreasing 1-dim. Lévy process), killing of a process upon leaving a given domain, reflecting of a process. These results allow, in particular, to obtain Chernoff approximations for subordinate diffusion on strar graphs and compact Riemannian manifolds. Moreover, Chernoff approximations can be further used to approximate solutions of some time-fractional evolution equations and hence to approximate marginal densities of the corresponding non-Markovian stochastic processes.

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Thursday 28th of August

Brownian motion with singular drift: small time asymptotics and associated boundary value problems

Tusheng Zhang[†]

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We establish a small time large deviation principle and a Varadhan type asymptotics for Brownian motion with a singular drift which is a vector-valued signed measure. The associated boundary velue problems will also be discussed.

Importance sampling of rare events using forward-backward stochastic differential equations

Omar Kebiri[†]

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In this talk I will present adaptive importance sampling algorithm for rare events that is based on a dual stochastic control formulation of a path sampling problem. The idea is to transform this question to solve a stochastic optimal control problem, and then show that the associated semi-linear dynamic programming equations admit an equivalent formulation as a system of uncoupled forward-backward stochastic differential equations that can be solved efficiently by a least squares Monte Carlo algorithm.

Stochastic volatility in energy and commodity forward markets

Fred Espen Benth[†]

[†]University of Oslo

We introduce stochastic volatility models for the dynamics of forward curves in energy and commodity markets. These models are appropriately defined as operator-valued random processes, based on Ornstein-Uhlenbeck (OU) processes. We discuss Gaussian OU processes leading to Heston-type models, as well as non-Gaussian models extending the Barndorff-Nielsen & Shephard dynamics to infinite dimensions. Issues like simulation and estimation will be discussed, as well as pricing of options. The talk is based on collaborations with Heidar Eyjolfsson, Fabian Andsem Harang, Barbara Ruediger, Iben Simonsen, Andre Suess and Almut Veraart.

American options in a non-linear incomplete market model with default

Marie-Claire Quenez[†]

[†]Université de Paris, LPSM

We study the superhedging prices and the associated superhedging strategies for American options in a non-linear incomplete market model with default. The points of view of the seller and of the buyer are presented. The underlying market model consists of a risk-free asset and a risky asset driven by a Brownian motion and a compensated default martingale. The portfolio processes follow non-linear dynamics with a nonlinear driver f. We give a dual representation of the *seller's* (*superhedging*) price for the American option associated with a completely irregular payoff (ξ_t) (not necessarily càdlàg) in terms of the value of a non-linear mixed control/stopping problem. The dual representation involves a suitable set of equivalent probability measures, which we call f-martingale probability measures. We also provide an infinitesimal characterization of the dual value process (which might be interpreted as the seller's price process) in terms of the minimal supersolution of a constrained reflected BSDE with default (which involves a *Mertens process* due to the fact that xi is not necessarily càdlàg). Moreover, when ξ is càadlàg, we show a duality result for the *buyer's price* in terms of the value of a non-linear control/stopping *game problem*.

This presentation relies on a joint work with M. Grigorova and A.Sulem

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A rough super-Brownian motion

Tommaso Cornelis Rosati[†],Nicolas Perkowski[†‡]

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[‡]Max Planck Institute Leipzig

We consider the asymptotic behaviour, under an appropriate scaling, of the mass density associated to a system of particles, each of which performs a random walk and branches according to the value of a random (but fixed in time) potential at the particle's position. We study the relationship of this system with the parabolic Anderson model and use techniques from (singular) stochastic PDEs to prove a scaling limit. We eventually find an analogous of the Superbrownian motion in a random environment. Unlike it's non rough analogous we can prove that this process is persistent in low dimensions.

The space of random variables $\mathbf{F}_\psi(\Omega)$ and stochastic processes from this space. Conditions for the weak convergence

Yurii Kozachenko[†], Yurii Mlavets[‡]
[†]Taras Shevchenko National University of Kyiv
[‡]Uzhhorod National University

This report is devoted to the investigation of conditions for the weak convergence in the space $C(T)$ of the stochastic processes from the space $\mathbf{F}_\psi(\Omega)$. Using these conditions the limit theorem for stochastic processes from the space $\mathbf{F}_\psi(\Omega)$ is obtained. This theorem can be utilized for achieving the given approximation accuracy and reliability of integrals depending on parameter by Monte Carlo method.

Definition 0.1. *We say that the condition* $\mathbf{H}$ *is fulfilled for the Banach space of random variables* $B(\Omega)$*, if there exists an absolute constant* C_B *such that for any centered and independent random variables* $\xi_1, \xi_2, \ldots, \xi_n$ *from* $B(\Omega)$*, the following is true:*

$$\left\| \sum_{i=1}^{n} \xi_i \right\|^2 \leq C_B \sum_{i=1}^{n} \|\xi_i\|^2 .$$

The constant C_B is called a scale constant for the space $B(\Omega)$. For space $\mathbf{F}_\psi(\Omega)$ we shall denote the constants $C_{\mathbf{F}_\psi(\Omega)}$ as C_ψ.

Let $X = \{X(t),\ t \in T\}$ be a stochastic process from the space $\mathbf{F}_\psi(\Omega)$, $EX(t) = 0$. Let the condition $\mathbf{H}$ is fulfilled for this space.

Assume that compact pseudometric space (T, ρ_ψ), $\rho_\psi(t,s) = \|X(t) - X(s)\|_\psi$ is separable and the process $X = \{X(t),\ t \in T\}$ is separable as well. Let $X_k(t),\ k = 1, 2, \ldots, n$ be independent copies of $X(t)$. Consider a stochastic process

$$Y_n(t) = \frac{1}{\sqrt{n}} \sum_{k=1}^{n} X_k(t).$$

By Definition (0.1) we have

$$\|Y_n(t) - Y_n(s)\|_\psi^2 \leq C_\psi \frac{1}{n} \sum_{k=1}^{n} \|X_k(t) - X_k(s)\|_\psi^2 = C_\psi \rho_\psi^2(t,s).$$

The pseudometric space (T, ρ_ψ) is separable and the processes $Y_n(t)$ are separable in this space.

Theorem 0.2. *If the following condition holds*

$$\hat{\varepsilon}_0 = \sup_{t,s \in T} \|X(t) - X(s)\|_\psi < \infty,$$

and for any $\tau > 0$

$$\int_0^\tau \varkappa_\psi(\tilde{N}(u))du < \infty,$$

where $\varkappa_\psi(n)$ *is the* M*-characteristic of the space* $\mathbf{F}_\psi(\Omega)$*,* $\tilde{N}(\varepsilon)$ *is the metric massiveness of the space* (T, ρ_ψ)*, then* $Y_n(t)$ *converge weakly in* $\mathbf{C}(T, \rho_\psi)$ *to the Gaussian process* $X_\infty(t)$ *such that* $EX_\infty(t) = 0$*,* $EX_\infty(t)X_\infty(s) = EX(t)X(s)$*.*

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Some limit theorems for linear autoregression models

Maryna Ilienko†

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Let us consider the classical stochastic recurrence equation

$$X_t = A_t X_{t-1} + B_t, \ t = 1, 2..., \tag{3}$$

where A_t are random $d \times d$-matrices with real-valued entries, and B_t are $\mathbb{R}^d$-valued random vectors, and moreover, the pairs $(A_t, B_t)_{t\in\mathbb{N}}$ are independent for different t (in particular, form an i.i.d. sequence).

Being a powerful descriptional tool for many financial and physical processes, models of type (3) are of great interest over the last half a century. For instance, we refer to the recent book by D. Buraczewski, E. Damek, T. Mikosch (see [1]), which contains many important results on the topic as well as some their applications.

This talk is intended to just slightly touch some interesting types of classical results for the models of type (3), and, thereafter, concentrate mainly on the problems related to the sequence of partial sums, whose terms are elements of the model (3).

Thus, we introduce $S_n = \sum_{t=1}^n X_t$, $n \geq 1$, and study necessary and sufficient conditions for the convergence a.s. of the series

$$\sum_{t=1}^{\infty} X_t,$$

(see[3]), or the series

$$\sum_{n=1}^{\infty} \frac{S_n}{n^{1+\frac{1}{p}}},$$

with $p > 0$. Note that the convergence a.s. of the latter series immediately implies Marcinkiewicz -Zygmund type SLLN, [2].

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Maximum Principles for Volterra Time Change processes

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We establish a framework for the study of backward stochastic Volterra integral equations (BSVIE) driven by time-changed Lévy noises. In fact we shall consider the random measure μ:

$$\mu(\Delta) = B(\Delta \cap [0,T] \times \{0\}) + \widetilde{H}(\Delta \cap [0,T] \times \mathbb{R}_0),\ \Delta \in \mathcal{B}([0,T] \times \mathbb{R})$$

where B is a conditional Gaussian measure on $[0,T] \times \{0\}$, and $\widetilde{H}$ is a conditional centered Poisson measure on $[0,T] \times \mathbb{R}_0 := [0,T] \times \mathbb{R} \backslash \{0\}$.

In this paper we deal with two information flows

- $\mathbb{F} := \{\mathcal{F}_t,\ t \in [0,T]\}$, namely the smallest right continuous filtration to which μ is adapted,
- $\mathbb{G} := \{\mathcal{G}_t,\ t \in [0,T]\}$, generated by μ and the entire history of the time change processes,

and we shall consider the information $\mathbb{F}$ as partial with respect to $\mathbb{G}$.

Given a controlled dynamic:

$$X^u(t) = X_0 + \int_0^t b(t,s,\lambda,u,X)ds + \int_0^t \int_{\mathbb{R}} \kappa(t,s,z,\lambda,u,X)\mu(ds,dz),$$

we consider the optimization problem of finding

$$\sup_{u \in \mathcal{A}} J(u) = \sup_{u \in \mathcal{A}} \mathbb{E}\left[\int_0^T F(t,X^u(t),u(t))dt + G(X^u(T))\right]$$

for a suitable control set $\mathcal{A}$ which we consider to be either $\mathbb{F}$ or $\mathbb{G}$ predictable.

We prove both a sufficient and a necessary maximum principle for such performance functional, showing that in the $\mathbb{F}$-predictable case we can find a solution by projecting the results obtained for the $\mathbb{G}$-predictable case onto the $\mathbb{F}$-predictable one.

We shall make use of stochastic derivatives. We stress that we cannot use the classical Malliavin calculus as our integrators are not the Brownian motion nor the centered Poisson random measure. Indeed we could use a conditional form of such calculus as introduced by Yablonski [7]. However we resolve by using the *non-anticipating derivative* introduced in [4] and for martingale random fields as integrators in [2]. The use of the non-anticipating derivative has also the advantage that we do not require more restrictive conditions on domains, since it is already well defined for all $L^2(P)$ random variables.

When studying such problems, we come across a BSVIE of the form

$$\begin{aligned} Y_t &= \xi(t) + \int_t^T g(t,s,\lambda,Y,\phi)ds - \int_t^T \int_{\mathbb{R}} \phi(t,s,z)\mu(ds,dz) \\ Y_T &= \xi(T) \end{aligned}$$

where μ is as above.

We prove existence and uniqueness results for such BSVIE and we compute an explicit solution in the linear case.

Examples and applications will be presented.

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Multiple optimal stopping problem under nonlinear expectations

Hanwu Li[†]

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In this talk, we study the multiple optimal stopping problem under the so-called filtration consistent nonlinear expectations. The reward is given by a set of random variables satisfying some appropriate assumptions rather than an RCLL process. We first construct the optimal stopping time for the single stopping problem, which is no longer given by a first hitting time of processes. We then prove by induction that the value function of the multiple stopping problem can be interpreted as the one for the single stopping problem associated with a new re-ward family, which allows us to establish the multiple optimal stopping times. If the reward family satisfies some strong regularity conditions, we can show that the reward family and the value functions can be ag-gregated by some progressive processes. Hence, the optimal stopping times can be represented as hitting times. For some typical cases, we may weaken the assumption on the underlying nonlinear expectation.

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Deep neural networks algorithm for high-dimensional stochastic control problems on finite horizon

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In this talk, I will present an algorithm for high-dimensional stochastic control problems based on deep learning and dynamic programming. Unlike classical approximate dynamic programming approaches, we first approximate the optimal policy by means of neural networks in the spirit of deep reinforcement learning, and then the value function by Monte Carlo regression.

I will present theoretical convergence results for the control and value function estimates. These results are expressed in terms of the universal approximation error of the neural networks, and of the statistical error when estimating network function. I will also present some numerical results on various real-life applications that illustrate the performance of our algorithm. This talk is based on [1] and [2].

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Self-exciting multifractional processes

Fabian A. Harang[†], Marc Lagunas[†], Salvador Ortiz-Latorre[†]

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We propose a new multifractional stochastic process which allows for self-exciting behavior, similar to what can be seen for example in earthquakes and other self-organizing phenomena. The process can be seen as an extension of a multifractional Brownian motion, where the Hurst function is dependent on the past of the process. We define this through a stochastic Volterra equation, and we prove existence and uniqueness of this equation, as well as give bounds on the p-order moments, for all $p \geq 1$. We show convergence of an Euler-Maruyama scheme for the process, and also give the rate of convergence, which is depending on the self-exciting dynamics of the process. Moreover, we discuss different applications of this process, and give examples of different functions to model self-exciting behavior.

Friday 30th of August

Hitting moments for Gaussian integrators

Andrey Dorogovtsev[†], Olga Izyumtseva[†]

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In this talk we propose general method to find an expression for hitting moments and occupation measure of general Gaussian process in terms of multiple stochastic integrals.

Volterra equations driven by rough signals

Fabian A. Harang[†], Samy Tindel[‡]

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We will show existence and uniqueness of Volterra integral equations with singular kernels driven by rough signals. To this aim, we construct a Volterra signature, analogous to the signature used in Rough Path theory. This signature satisfies a Chens relation under a convolution product, but not under the regular tensor product. We will discuss properties of the signature, and the well-posedness of the convolution product. In the end we will give a definition of a space of controlled Volterra paths, in which we can solve non-linear Volterra equations driven by rough signals.

A study of a one-dimensional epidemic model with transmission from outside and recovery

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[°] Matematik Bölümü Nevşehir Haci Bektaş Veli Üniversitesi

We consider a one-dimensional model for epidemics described by a stochastic differential equation driven by a Brownian motion. Depending on different choices of the involved parameters, different properties of a solution concerning boundaries and ergodicity is obtained.

The particular SDE model is compared with a related diffusion approximation of a birth-death process driven by two Poisson processes. For huge populations the two models coincide with a one-dimensional ODE epidemic model. For semi-large populations some properties of the two models differ.

On a selection problem for small noise perturbations of ODEs with non-Lipschitz coefficients

Andrey Pilipenko[†]

[†]Institute of Mathematics of Ukrainian National Academy of Sciences

We study the limit behavior of an ordinary differential equation with non-Lipschitz coefficients that are perturbed by a small noise. Perturbed equations may have unique solutions while the initial ODE does not have a unique solution. Hence a limit of perturbed SDEs may be interpreted as a natural selection of a solution to the initial ODE.

The identification of the limit is closely related with a study of the exact growth rate of solutions to SDEs and also with the averaging principle.

Results are based on a joint research with F.N.Proske and with A.M.Kulik.

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Doubly Reflected BSDEs in the predictable setting

Ihsan Arharas[†], Siham Bouhadou[†], Youssef Ouknine[‡]

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[‡] Mohammed VI Polytechnic University, Complex Systems Engineering and Human Systems

In this paper, we introduce a specific kind of doubly reflected Backward Stochastic Differential Equations (in short DRBSDEs), defined on probability spaces equipped with general filtration that is essentially non quasi-left continuous, where the barriers are assumed to be predictable processes. We call these equations *predictable DRBSDEs*. Under a general type of *Mokobodzki's* condition, we show the existence of the solution (in consideration of the driver's nature) through a Picard iteration method and a Banach fixed point theorem. By using an appropriate generalization of Itô's formula due to Gal'chouk and Lenglart, we provide a suitable a priori estimates which immediately implies the uniqueness of the solution.

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Stochastic Heat Equation with general nonlinear spatial rough noise

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In this talk, we give a presentation on some recent results on a nonlinear one spatial dimensional stochastic heat equations driven by a Gaussian noise which is white in time and which has the covariance of a fractional Brownian motion with Hurst parameter $H \in (\frac{1}{4}, \frac{1}{2})$. This work is motivated by recent article of Hu, Huang, Lê, Nualart and Tindel [Ann. Probab. **45** (2017) 4561-4616] to loosen the important condition $\sigma(0) = 0$ there. At first we study the asympotics of $u_{\text{aff}}(t,x)$, the solution of SHE with additive noise. In particular, we find the sharp growth of $\sup_{|x|\le L} |u_{\text{aff}}(t,x)|$, $\sup_{|x|\le L} |\Delta_h u_{\text{aff}}(t,x)|$ and $\sup_{|x|\le L} |\Delta_\tau u_{\text{aff}}(t,x)|$. These asympotics enlighten us to introduce a specific decay $\lambda(x)$ in space in order to drop the condition $\sigma(0) = 0$. Next, after the study of characters of the stochastic convolutions, we show the weak existence of solutions on space $\mathcal{C}([0,T] \times \mathbb{R})$. In the end, with additional conditions on σ inspired by the spatial behavior of $\mathcal{N}_{\frac{1}{2}-H} u(t,x)$, we prove the pathwise uniqueness and strong existence.

Poster session

Martingale solution of nematic liquid crystals driven by pure jump noise in Marcus canonical form

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In this work we consider a stochastic evolution equation which describes the system governing the nematic liquid crystals driven by a pure jump noise in the Marcus canonical form. The existence of a martingale solution is proved for both two and three dimensions. The construction of the solution relies on a modified Faedo-Galerkin method based on the Littlewood-Paley-decomposition, compactness method and the Jakubowski version of the Skorokhod representation theorem for non-metric spaces. We prove that in the the martingale solution is pathwise unique in two dimensions and hence deduce the existence of a strong solution.

The relaxed stochastic maximum principle for G-SDE

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This work is concerned with stochastic optimal control problem in the G-frame work, we consider systems subject to model uncertainty or ambiguity due to incomplete or inaccurate information, or vague concepts and principles.

We study the stochastic maximum principle for stochastic differential equations driven by a G-Brownian motion, we establish existence of an optimal stochastic relaxed control and we derive the necessary conditions of optimality.

References

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Wong-Zakai approximation for the stochastic Landau-Lifshitz-Gilbert equations

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In this work we study stochastic Landau-Lifshitz-Gilbert equations (SLLGEs) in one dimension, with non-zero exchange energy only. Firstly, by introducing a suitable transformation, we convert the SLLGEs to a highly nonlinear time dependent partial differential equation with random coefficients, which is not fully parabolic. We then prove that there exists a pathwise unique solution to this equation and that this solution enjoys the maximal regularity property. Following regular approximation of the Brownian motion and using reverse transformation, we show existence of strong solution of SLLGEs taking values in a two-dimensional unit sphere $\mathbb{S}^2$ in $\mathbb{R}^3$. The construction of the solution and its corresponding convergence results are based on Wong-Zakai approximation.

Weak martingale solutions of stochastic cross-diffusion systems

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The so-called cross-diffusion systems appear in a multitude of different applications ranging from thermodynamics, population dynamics of interacting species to modelling the evolution of cancer. They are systems of strongly coupled parabolic equations whose diffusion matrix doesn't need to be positive semi-definite or symmetric. We considered a subclass of these equations and introduced a multiplicative (Stratonovich) noise term to account for random influences of the environment as well as intrinsic uncertainties,

$$du(t) = div(A(u)\nabla u)dt + \sigma(u) \circ dW_t.$$

The structure arising due to interactions at macroscopic level pose the main mathematical difficulty. To balance the loss of the positive-semi-definiteness of A, we require a certain (entropy) structure. We obtain global in time weak solutions, in stochastic and PDE sense, by combining results from both stochastic-and deterministic world through a Wong-Zakai type approximation of the driving noise. The existence proof is based on so called entropy estimates, the tightness criterion of Brzézniak and co-workers, and Jakubowski's generalization of the Skorokhod theorem.

Asymptotic properties of the least squares estimator in nonlinear regression with linear random noise

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Consider a regression model

$$X(t) = g(t, \alpha_0) + \varepsilon(t),\ t \geq 0 \tag{1}$$

where $g : (-\gamma, \infty) \times \mathcal{A}_\gamma \to \mathbb{R}$ is a continuous function, $\mathcal{A} \subset \mathbb{R}^q$ is a bounded open convex set, $\mathcal{A}_\gamma = \bigcup_{\|\epsilon\| \leq 1} (\mathcal{A} + \gamma e)$, γ is some number, $\alpha_0 \in \mathcal{A}$ is a true value of unknown parameter, and the process ε is a measurable casual linear process of the form

$$\varepsilon(t) = \int_{\mathbb{R}} \hat{a}(t-s) dL(s),\ t \in \mathbb{R} \tag{2}$$

where $\hat{a} : \mathbb{R} \to \mathbb{R}_+$ be a measurable function and a two-sides Lévy process L is such that $EL(1) = 0$, $\hat{a} \in L_2(\mathbb{R})$.

Sufficient conditions of consistency and asymptotic normality of the least squares estimator of unknown parameter α_0 of continuous-time nonlinear regression model (1) are presented in the talk. Asymptotic properties of the least squares estimator in nonlinear regression model were studied by many authors. Numerous results on the subject can be found in monograph by Ivanov and Leonenko[1], Ivanov[2].

Results of the talk cover the cases of nonlinear regression with Lévy-driven linear noise process of the type (2) that now is not necessary Gaussian.

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Regularity varying and regularity log-periodic functions in probability

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The class of regularly varying functions and other classes of functions that generalize regularly varying will be considered in this talk. The class of regularly log-periodic functions, that firstly appeared in work of V.V. Buldygin and V.V. Pavlenkov (2012), is one of these classes.

We will discuss the place of regularly log-periodic functions in the theory of regularly varying functions and different probabilistic problems where the regularly varying and regularly log-periodic functions are appears.

One of the part of this talk is based on a course of lectures "The application of regularly varying functions in probability theory", which prof. O.I. Klesov gives in our university.

The arbitrage condition and the value of the information

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The optimal portfolio problem is one of the most studied in financial mathematics in recent years. One of the fundamental works corresponds to Ioannis Karatzas who defines a model with a risky asset -under the expression of a Brownian geometric movement- and another one riskless -which plays the role of a bond that evolves under a certain interest rate-. He models the evolution of capital and tries to choose the strategy that optimizes the logarithm of profits at a future time horizon. After that, he develops the theory of insider trading based on enlargement of filtrations and manages to give the exact price of different privileged information in the financial market. Furthermore, the Arbitrage Theory, developed by Walter Schachermayer, focuses on studying the different options for achieving a positive amount of profit depending on the risk involved. By combining these works, we can have a much broader view of the problem and compare the deficiencies and benefits of each of the proposed models. After that, we study open problems proposed by the authors in their respective publications, such as valuing additional information not only about the risky asset but also about the interest rate, modeled through an OrnsteinUhlenbeck process. We also construct an example of enlargement of filtrations that does not satisfy with the arbitrage condition and analyze the differences that an investor who allows a partial bankruptcy against one who does not.

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Stability and Ergodicity Properties of the HJMM Equation

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We study the long time behaviour of forward rates in terms of the HJMM equation driven by Wiener and Poisson noise on a weighted Sobolev space. In economic theory interest rates are often assumed to have the property of mean reversion, that is they tend to an average over time. Some research was conducted to figure out this property for the mild solution of the HJMM equation within the aforementioned space, but it was not answered yet, how these long time equilibria precisely look like. Under assumptions, that are consistent with the necessary and sufficient conditions for providing a model that preserves positivity of the interest rates for positive initial data, we obtain exponential fast convergence of the mild solution to its long rate in mean square. Moreover, our prove is based on a Lyapunov function method, which allows for stating exponential stability on a certain subspace and exponential ultimate boundedness on related affine subspaces, whereby we can figure out the corresponding constants explicitly.

Stochastic data-driven model of the European power-grid frequency

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The energy system is rapidly changing to accommodate the increasing number of renewable generators and the general transition towards a more sustainable future. Simultaneously, new business models and market designs are proposed to stabilise the power grid and its frequency. Problems raised by this ongoing transition are increasingly addressed by transdisciplinary research approaches, ranging from purely mathematical modelling to applied case studies. These approaches require a stochastic description of consumer behaviour, fluctuations by renewables, market rules, and how they influence the stability of the power grid frequency. Here, we introduce an easy-to-use, data-driven, stochastic model for the power grid frequency and demonstrate how it reproduces key characteristics of the observed statistics of the Continental European and British power grids. We offer guidelines on how to use the model on any power grid for various mathematical or engineering applications.
