

Recent Developments in Stochastics 2021

Scientific program

November 12th, 2021

9:15 **Yuri Kondratiev, Bielefeld University**

From random times to fractional evolution equations

Abstract:

We will discuss the concept of random times in dynamical processes. There are two main situations under consideration.

There are random time changes in Markov processes. It leads to fractional in time Kolmogorov equations for the state development of initial distributions. The effect of random time is related with changes of the asymptotic behaviour of the system.

In the case of deterministic dynamical systems random time changes create fractional Liouville equations. In particular, fractional transport equations.

10:00 **Stefania Ugolini, University of Milan**

Bose-Einstein Condensation: relative entropy and related convergence results

Abstract:

A stochastic approach to various scaling limits in Bose-Einstein Condensation is briefly described as well as the related convergence of the ground state energy. In the mean-field case a strong form of Kac's chaos on path-space for the k -particles probability measures are derived from the previous energy convergence by purely probabilistic techniques notably using a simple chain-rule of the relative entropy. An overview on the Gross-Pitaevskii scaling limit is also provided.

This is joint work with S. Albeverio and F.C. De Vecchi, University of Bonn.

References

Albeverio, S., De Vecchi, F.C., Ugolini, S.: Entropy chaos and Bose-Einstein Condensation. Journal of Statistical Physics, Vol 168,3,483-507 (2017)
S. Albeverio, A. Romano, F.C. De Vecchi, S. Ugolini: Strong Kac's chaos in the mean-field Bose-Einstein Condensation. Stochastics and Dynamics (2019)

11:00 Peter Imkeller, Humboldt University

On the geometry of some rough Weierstrass and Takagi type curves: SBR measure and local time

Abstract:

We investigate geometric properties of graphs of Weierstrass or Takagi type functions, represented by series based on smooth functions. They are Hölder continuous, and can be embedded into smooth dynamical systems, where their graphs emerge as pullback attractors. It turns out that occupation measures and Sinai-Bowen-Ruelle (SBR) measures on their stable manifolds are dual by time reversal. A suitable version of approximate self similarity for deterministic functions allows to "telescope" small scale properties from macroscopic ones. As a consequence, absolute continuity of the SBR measure is seen to be dual to the existence of local time. The link between the rough curves considered and smooth dynamical systems can be generalised in various ways. Applications to regularization of singular ODE by rough signals are on our agenda.

This is joint work with O. Pamen, University of Liverpool and AIMS Ghana, A. Réveillac, University of Toulouse, and G. dos Reis, University of Edinburgh.

11:45 Olfa Draouil, University of Tunis El Manar

White noise approach to time changed processes

Abstract:

In this work, we investigate the time change process $\Lambda(t)$ in the framework of infinite dimensional analysis and especially in the theory of White Noise calculus. Using the Bochner-Milnos theorem, we introduce on the dual of the Schwartz space a conditional measure associated to the process of the time change $\Lambda(t)$. This conditional measure is a generalization of the classical Gaussian measure in the white Noise distribution theory when $\Lambda(t) = t$. Next we provide a generalization of Hida and Potthof distributions spaces associated with this conditional measure. Then we prove that the Brownian motion at the process $\Lambda(t)$ is a martingale with respect to an associated enlarged filtration and we obtain the associated Clark-Ocone formula.

13:30 Nicole El Karoui, Sorbonne University

Revealed utilities, an inverse forward problem

Abstract:

Decision making under uncertainty is generally considered as the selection of an optimal sequence of actions in an uncertain environment. Its calibration raises the “inverse” problem to recover the criterium from the data. A classical example in economy is the theory of “revealed preference” introduced by Samuelson in the 40’s. The observable or characteristic process at a given date t , $X_t(x)$, is an increasing function of a real parameter x (the wealth in economy). The objective is to recover a dynamic stochastic utility $\{U(t, z)\}$, “revealed” in the sense where its performance is without bias, more formally when “ $\{U(t, X_t(x))\}$ is a martingale”. The increasing of $X_t(x)$ in x , and the concavity of the utility leads to privilege the so-called adjoint process $Y_t(u_Z(x)) := U_Z(t, X_t(x))$ in the linearisation of the problem; the one to one correspondence between $U_Z(t, z)$ and $Y_t(u_Z(x))$ for a given characteristic process is used intensively. We focus on the (u, X, Y) triplets, bringing great attention to their initial conditions and find an equivalent intrinsic framework, where necessary the processes “ $\{X_t(x)\}, \{Y_t(y)\}, \{U(t, z)\}$ ” are supermartingales.

Itô’s semimartingale framework is used to illustrate this characterisation. The operational version ensures that the revealed utility is solution of a non-linear SPDE. Less obvious is its interpretation as stochastic value function of some optimisation problem. Financial markets framework appears as a special case, under stronger assumptions. This allows us to revisit the dynamic equilibrium problem as in He and Leland.

The talk is based on joint work with Mrad Mohamed (LAGA).

14:15 Paolo Guasoni, Dublin City University

Rogue Traders

Abstract:

Investing on behalf of a firm, a trader can feign personal skill by committing fraud that with high probability remains undetected and generates small gains, but that with low probability bankrupts the firm, offsetting ostensible gains. Honesty requires enough skin in the game: if two traders with isoelastic preferences operate in continuous-time and one of them is honest, the other is honest as long as the respective fraction of capital is above an endogenous fraud threshold that depends on the trader’s preferences and skill. If both traders can cheat, they reach a Nash equilibrium in which the fraud threshold of each of them is lower than if the other one were honest. More skill, higher risk aversion, longer horizons, and greater volatility all lead to honesty on a wider range of capital allocations between the traders.

This is joint work with Huayuan Dong, Dublin City University, Eberhad Mayerhofer, University of Limerick.

15:00 Bernt Øksendal, University of Oslo

Fokker-Planck PIDE for McKean-Vlasov diffusions with jumps, and applications to HJB equations and optimal control

Abstract:

We study optimal control of McKean-Vlasov (mean-field) stochastic differential equations with jumps. First we prove a Fokker-Planck equation for the law of the state. Then we study the situation when the law is absolute continuous with respect to Lebesgue measure. In that case the Fokker-Planck equation reduces to a deterministic integro-differential equation for the Radon-Nikodym derivative of the law. Combining this equation with the original state equation, we obtain a Markovian system for the state and its law. We apply this to formulate a Hamilton-Jacobi-Bellman (HJB) equation for the optimal control of McKean-Vlasov stochastic equations with jumps. Then we derive a Hamilton-Jacobi-Bellman-Isaacs (HJBI) equation for the Nash equilibrium of McKean-Vlasov jump diffusion games. Finally we apply these results to solve explicitly the following problems:

- (i) Linear-quadratic optimal control of stochastic McKean-Vlasov equations with jumps.
- (ii) Optimal consumption from a cash flow modelled as a stochastic McKean-Vlasov equation with jumps.

The presentation is based on joint work with Nacira Agram, Linnaeus University.