

Scientificness in mathematics

<https://www.mn.uio.no/math/studier/aktuelt/arrangementer/2021/writingcourse-mi.html>

Karoline Moe
The Science Library
2021

Zoom rules

- It is nice to see you!
 - It is nice to hear you!
 - We're in this together!
 - Let me know if tech fails...
 - Raise hand to ask questions
 - Ask and comment in chat
 - No recording
- A variation:
 - Slides
 - Demonstrations
 - Plenary discussions
 - Student activity
 - Breakout rooms
 - Breaks

Today

Thursday January 14	Subject	Lecturer	About
09:15- 10:00	Scientificness in mathematics	Karoline Moe	Searching, source criticism, reading, ethics, cheating, citations and plagiarism Resources will appear here
10:15- 11:00	Writing in mathematics	Karoline Moe	Structure of thesis, mathematical subtleties and useful writing tips
11:15- 12:00	LaTeX for large documents BibLaTeX	Karoline Moe	How to organise the writing Requires some familiarity with LaTeX Efficient reference management with LaTeX Resources - Guide for large documents - Thesis template: zip , Overleaf - Thesis front page: package , documentation - UiO templates on Overleaf - Martin Helsø's Introduction to BibLaTeX

Tomorrow

Friday January 15	Subject	Lecturer	About
12:15- 13:00	Data management	Live Kvale and Ivana Malovic	File sense, meta data, data management plan and much more
13:15- 14:00	Simple design principles and Beamer	Karoline Moe	Applications of design principles to master's thesis, illustrations, figures and tables Beamer Resources - Martin Helsø's Introduction to Beamer - Department's Beamer theme: zip , Overleaf
14:15- 15:00	Oral presentation and Part of the research world	Karoline Moe	Presentation technique, talks, blackboard and slide shows What happens when you are done? Resources - AB123C

Who am I?

Who are you?

Menti

Outline of Scientificness 1

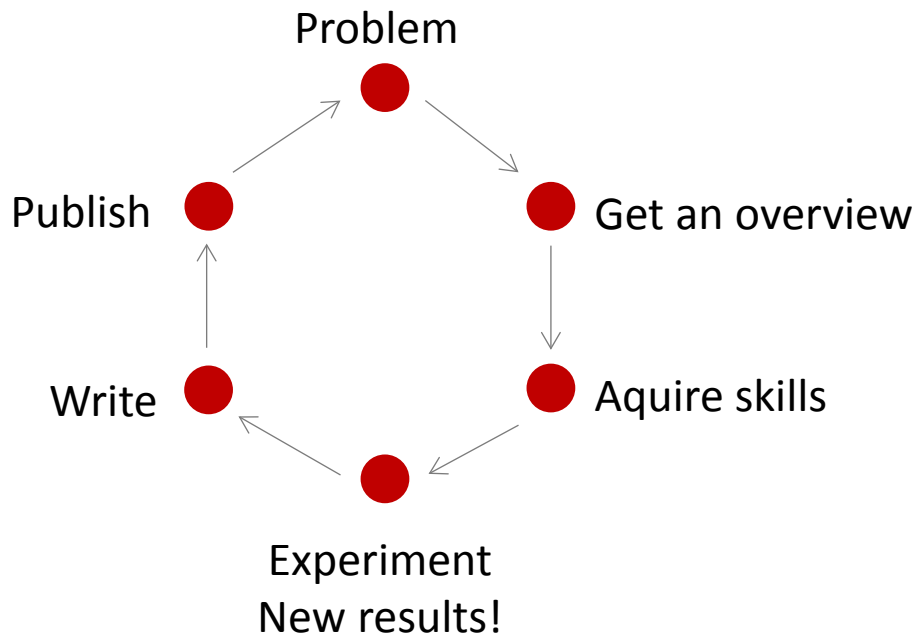
- Research in mathematics
- Searching and using the library
- Reading an article
- Citing and the reference list
- Ethics, cheating and plagiarism

Research in mathematics

How does a researcher work?

Research in mathematics

- Solve a problem
- Build a theory
- Programming
- Simulations
- Analyzing data
- Combinations



The publishing process 101

- Do great research
- Write results in a manuscript
- Publish pre-print in subject archive: www.arxiv.org
- Submit to journal for peer review – critical review by other experts
- Publish article in renowned journal
 - [Norwegian register for scientific journals](#) (NSD)
 - Research and publication bias
- Publish post-print (peer reviewed, but without journal's formatting) in institutional repository: www.duo.uio.no
- Get review in database: [MathSciNet](#), [zbMATH](#)
- Get noticed as researcher: citations, h-index, collaborations

Searching

What do you need?

You have to find literature about your topic

- Broad perspective
- History
- Motivation
- What have others done
- Core material / textbook
- Review article
- Specific results
- Your level
- Other points of view
- Have you missed anything?

Menti

How do you find the literature you need?

How to find literature?

- Get books or articles from supervisor
- Ask «older» students
- Ask a (subject) librarian
- Ask on a web page
- Google it
- Search online in a scientific database
- Systematic search
- Reference hunting – backwards and forwards
- Repeat from start

Scientific or alternative fact?

- In general quite difficult to decide
- A scientific article is evaluated by other experts prior to publication = peer review
 - An insurance – no guarantee
 - Tradition and integrity
 - Always small errors
 - Fraud and misconduct
- Important in all areas of your life and in the society

MMR vaccine controversy

From Wikipedia, the free encyclopedia

The **MMR vaccine controversy** started with the 1998 publication of a fraudulent research paper in *The Lancet* linking the combined measles, mumps, and rubella (MMR) vaccine to colitis and autism spectrum disorders.^[1] The claims in the paper were widely reported,^[2] leading to a sharp drop in vaccination rates in the UK and Ireland and increases in the incidence of measles and mumps, resulting in deaths and serious permanent injuries.^{[3][4]} Following the initial claims in 1998, multiple large epidemiological studies were undertaken. Reviews of the evidence by the Centers for Disease Control and Prevention,^[5] the American Academy of Pediatrics, the Institute of Medicine of the US National Academy of Sciences,^[6] the UK National Health Service,^[7] and the Cochrane Library^[8] all found no link between the MMR vaccine and autism.

https://en.wikipedia.org/wiki/MMR_vaccine_controversy

Wakefield's paper was described as "perhaps, the most damaging medical hoax of the last 100 years".^[16] Physicians, medical journals, and editors^{[17][18][19][20][21]} have described Wakefield's actions as fraudulent and tied them to epidemics and deaths.^{[22][23]}

MMR vaccine controversy

From Wikipedia, the free encyclopedia

The **MMR vaccine controversy** started with the 1998 publication of a fraudulent research paper in *The Lancet* linking the combined measles, mumps, and rubella (MMR) vaccine to colitis and autism spectrum disorders.^[1] The claims in the paper were widely reported,^[2] leading to a sharp drop in vaccination rates in the UK and Ireland and increases in the incidence of measles and mumps, resulting in deaths and serious permanent injuries.^{[3][4]} Following the initial claims in 1998, multiple large epidemiological studies were undertaken. Reviews of the evidence by the Centers for Disease Control and Prevention,^[5] the American Academy of Pediatrics, the Institute of Medicine of the US National Academy of Sciences,^[6] the UK National Health Service,^[7] and the Cochrane Library^[8] all found no link between the MMR vaccine and autism.

https://en.wikipedia.org/wiki/MMR_vaccine_controversy



Donald J. Trump
@realDonaldTrump



Follow

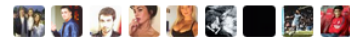
Healthy young child goes to doctor, gets pumped with massive shot of many vaccines, doesn't feel good and changes - AUTISM. Many such cases!

RETWEETS

8,212

LIKES

5,880



7:35 AM - 28 Mar 2014



8.2K



5.9K



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@realDonaldTrump



Follow

So many people who have children with autism have thanked me—amazing response. They know far better than fudged up reports!

RETWEETS
468

LIKES
502



The fast and safe lane to scientific literature

The library

- Physical
 - Online
- Research drowns on Google – if it is there...
 - Access to research is expensive
 - Most people don't have access
 - You have
 - 100 mill NOK per year at UiO!
 - New trend: Open Access publishing & Plan S
 - Free access to research, but publishing is expensive
 - Predatory journals

About the Science Library

Physical

- The books are mostly sorted by topic – browse the shelf
- Math collection
 - Entrance wall
- New book collection
- Undergraduate collection
- Collection 42
 - In order of appearance
 - Search online
- Deposited in storage rooms
 - SL-mag, VB-mag, GSH-mag

Online

- The University Library
 - ub.uio.no
 - Emneord
 - ub.uio.no/matematikk
- Databases
 - MathSciNet
 - zbMATH
 - Web of Science
 - Google Scholar
 - arXiv (pre-prints)



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Mathematics and Statistics

CATEGORIES

All Journals



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[Pure Mathematics](#)

[Statistics and Probability](#)

SORT [A-Z](#) / [JOURNAL RANK](#)



Abhandlungen
aus dem
Mathematische



Abstract and
Applied
Analysis



ACM
Transactions
on



Acta
Applicandae
Mathematicae



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Mathematica



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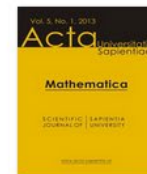
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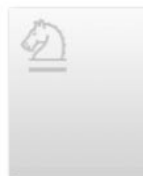
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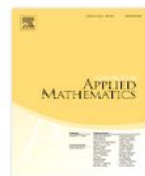
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Sapientiae,



Advanced
Modeling and
Simulation in



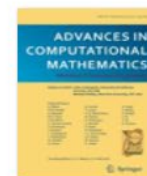
Advances in
Applied
Clifford



Advances in
Applied
Mathematics



Advances in
Applied
Probability



Advances in
Computational
Mathematics

What if you can't find it?

- 100 million NOK per year...
- Your student status gives you access to all research and published items
- Search the internet
 - If you can access it, then either we have payed for it or it is published Open Access
- Search the catalogue
- Can't find it?-button
- realfagsbiblioteket@ub.uio.no

NB! Scientificness in mathematics

- Specific terms
 - Even Google (Scholar) could be OK
- Dedicated mathematicians
 - Wikipedia and many webpages with high quality content
 - Long tradition of open access and pre-print publishing in some subfields
- Can be verified or debunked directly
- Publication bias is still a problem
- Science vs. insanity vs. jokes

Modular elliptic curves
and
Fermat's Last Theorem

By ANDREW WILES*

For Nada, Clare, Kate and Olivia

Cubum autem in duos cubos, aut quadratoquadratum in duos quadratoquadratos, et generaliter nullam in infinitum ultra quadratum potestatem in duos ejusdem nominis fas est dividere: cujus rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet.

Pierre de Fermat

Introduction

An elliptic curve over $\mathbf{Q}$ is said to be modular if it has a finite covering by a modular curve of the form $X_0(N)$. Any such elliptic curve has the property that its Hasse-Weil zeta function has an analytic continuation and satisfies a functional equation of the standard type. If an elliptic curve over $\mathbf{Q}$ with a given j -invariant is modular then it is easy to see that all elliptic curves with the same j -invariant are modular (in which case we say that the j -invariant is modular). A well-known conjecture which grew out of the work of Shimura and Taniyama in the 1950's and 1960's asserts that every elliptic curve over $\mathbf{Q}$ is modular. However, it only became widely known through its publication in a paper of Weil in 1967 [We] (as an exercise for the interested reader!), in which, moreover, Weil gave conceptual evidence for the conjecture. Although it had been numerically verified in many cases, prior to the results described in this paper it had only been known that finitely many j -invariants were modular.

In 1985 Frey made the remarkable observation that this conjecture should imply Fermat's Last Theorem. The precise mechanism relating the two was formulated by Serre as the ε -conjecture and this was then proved by Ribet in the summer of 1986. Ribet's result only requires one to prove the conjecture for semistable elliptic curves in order to deduce Fermat's Last Theorem.

Abstract

Employing only basic arithmetic and algebraic techniques that would have been known to Fermat, and utilizing alternate computation methods for arriving at $\sqrt[n]{c^n}$, we identify a governing relationship between $\sqrt{(a^2+b^2)}$ and $\sqrt[n]{(a^n+b^n)}$ (for all $n > 2$), and are able to establish that $c = \sqrt[n]{(a^n+b^n)}$ can never be an integer for any value of $n > 2$.

AMS SUBJECT CLASSIFICATION (2010): Primary, 11D41

KEYWORDS: Fermat's Last Theorem, Fermat's Equation, Pythagorean Triple

Contents

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	Theorem 1.1 For all $n > 2$ there are no solutions to the equation $(a^n + b^n) = c^n$ where a, b, c, n are all positive integers.	2
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	Appendices	4
A	Lemma 1: The positive integers a and b cannot be equal	4
B	Lemma 2: Only coprime positive integers need be considered for a and b	5
C	Lemma 3: The positive integers a and b cannot be of the same parity	6
D	Lemma 4: For all $n > 2$, $(a^n + b^n)$ contains primes not in $(a^2 + b^2)$. .	7

1 Introduction

"It is impossible for a cube to be the sum of two cubes, a fourth power to be the sum of two fourth powers, or in general, for any number that is a power greater than the second to be the sum of two like powers.

I have discovered a truly marvelous demonstration of this proposition that this margin is too narrow to contain" [6].

Pierre de Fermat (1637 [4, p. 139])

Operating on the premise that the construction and examination of equivalent restatements of an equation (and its elements and inverse operations) may reveal properties and relationships that might not otherwise be apparent, we construct such equivalent restatements and from their examination, are able to conclusively demonstrate that $\sqrt[n]{c^n}$ can be an integer only at $n = 2$.

Remark The proof of Fermat's Last Theorem rests upon the relationship between $\sqrt{c^2} = \sqrt{(a^2+b^2)}$, and for all $n > 2$, $\sqrt[n]{c^n} = \sqrt[n]{(a^n+b^n)}$, both of which are equally regarded as c . Where it is necessary to distinguish between the two values, either distinct side of these two equations may be used interchangeably.

Acknowledgement I can claim no credit for the insights and approaches contained within this paper. All were gifts from God. And it is only to him that praise is due.

*The work on this paper was supported by an NSF grant.

A SIMPLE PROOF THAT π IS IRRATIONAL

IVAN NIVEN

Let $\pi = a/b$, the quotient of positive integers. We define the polynomials

$$f(x) = \frac{x^n(a - bx)^n}{n!},$$

$$F(x) = f(x) - f^{(2)}(x) + f^{(4)}(x) - \cdots + (-1)^n f^{(2n)}(x),$$

the positive integer n being specified later. Since $n!f(x)$ has integral coefficients and terms in x of degree not less than n , $f(x)$ and its derivatives $f^{(j)}(x)$ have integral values for $x=0$; also for $x=\pi=a/b$, since $f(x)=f(a/b-bx)$. By elementary calculus we have

$$\frac{d}{dx} \{F'(x) \sin x - F(x) \cos x\} = F''(x) \sin x + F(x) \sin x = f(x) \sin x$$

and

$$(1) \quad \int_0^\pi f(x) \sin x dx = [F'(x) \sin x - F(x) \cos x]_0^\pi = F(\pi) + F(0).$$

Now $F(\pi) + F(0)$ is an *integer*, since $f^{(j)}(\pi)$ and $f^{(j)}(0)$ are integers. But for $0 < x < \pi$,

$$0 < f(x) \sin x < \frac{\pi^n a^n}{n!},$$

so that the integral in (1) is *positive, but arbitrarily small* for n sufficiently large. Thus (1) is false, and so is our assumption that π is rational.

PURDUE UNIVERSITY

Received by the editors November 26, 1946, and, in revised form, December 20, 1946.

A simple proof that π is rational*

Peter Rowlett

1st April 2013

Abstract

The number pi, written using the symbol π , is a mathematical constant that is the ratio of a circle's circumference to its diameter, and has been claimed since antiquity to be an irrational number, meaning that it cannot be expressed exactly as a ratio of two integers, and that therefore its decimal expansion never ends or settles into a permanent repeating pattern. Here a proof is given that π can indeed be expressed as a ratio of two integers, $\frac{4}{1}$, a fact that has unbelievably been overlooked until now. Moreover, this proof is understandable to anyone with a basic knowledge of algebra and calculus and arises from simply considering a standard integral at two values of x , $x = \frac{1}{4}$ and $x = 1$. Of course I doubted the result at first, given that it has been overlooked for so many years, but I have checked the proof and verified it to be correct. This is a crucial and important revelation that will significantly alter all of mathematics.

1 A formula for π

First we note that

$$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right)$$

and

$$\tan^{-1}(1) = \frac{\pi}{4}.$$

Since x can take any value, consider what happens when $x = \frac{1}{4}$. Then:

$$\int \frac{1}{x^2 + \left(\frac{1}{4}\right)^2} dx = 4 \tan^{-1}(1) = \pi. \quad (1)$$

*Cite this as: Rowlett, P., 2013. A simple proof that π is rational. *Travels in a Mathematical World*, 1st April. Available via: <http://travels.aperiodical.com/2013/04/a-simple-proof-that-pi-is-irrational.html>

Summing up

Use peer reviewed sources of information

- Books published by renowned publishing houses
- Articles published in renowned peer reviewed journals
- Reference hunting from good sources
- Ask your supervisor
- Beall's blacklist

Use subject databases

- Subject databases index renowned journals
- MathSciNet, zbMATH
- Google Scholar works well in mathematics
- Check the national database NSD
- Check reviews on MathSciNet

Format

- Mathematicians write in LaTeX
- Pretty LaTeX doesn't always mean correct mathematics

Stalk the author (Google Scholar, MathSciNet)

- Professor?
- Master's student?
- No university affiliation?

Beware of pre-print-archives

- arXiv.org

Stay away from alternative groups

- viXra.org
- <http://vixra.org/pdf/1312.0082v4.pdf>

Menti

Identify quality journals

Example: Fermat's last theorem

- Ask at the front desk or book a librarian
- Use the library catalogue online
 - Search your topic
 - Beware of special characters
 - Request the book or article
- Search in Emneord
- Use Wikipedia as a starting point
- Find your MSC and Dewey-number
- Search in MathSciNet
 - Andrew Wiles
- Search in Google Scholar (or Google)
 - Beware of special characters
 - Fermat's last theorem
- What if you can't find it?
 - Can't find it?-button
 - realfagsbiblioteket@ub.uio.no

Reading an article

Reading an article

- Read the abstract and the introduction first
- Be patient! It might take years to understand one single article
- Everything is easier in small bites
- Put it away – and pick it up again
- Ask for help from your supervisor
- Start with the most sentral source
- Find the most sentral result / technique
- Make notes in the margin and underline / highlight
- Work your way through important proofs with pen and paper
- Find other sources that can shed light on the result

Reading an article

Reading for an overview

- What is the main result in the article?
- How is it mentioned in...
 - The abstract
 - The introduction
 - The main part
- Sketch the structure of the article

Reading the details

- Which objects are involved in the main theorem?
- How and where are the objects defined?
- Can you apply the theorem to an example?
- Sketch the proof

Critical reading

- Stalk the author
- Check journal
- Am I able to verify this?
- Access to data?
- Details left to the reader?
- Ethical issues?
- Publication bias
- What is not commented?

Citing and the reference list

Give credit
and make your research verifiable

If I have seen further it is by standing on ye sholders of Giants.

Sir Isaac Newton, 1675

Using sources in your thesis

- All results, books, articles, other sources and programs that you use must be cited in the text

The second main result in this article is a formula for the number of sextactic points on cuspidal curves, for which the invariants involved are easily accessible. Indeed, for a cusp p with multiplicity m_p , there exists a unique line T_p such that $l_p = (T_p \cdot C)_p > m_p$, referred to as the tangent line to C at p . Similarly, applicable only to cusps where $l_p = 2m_p$, there exists a, not necessarily unique, conic O_p with $c_p = (O_p \cdot C)_p > 2m_p$ and $c_p \neq 3m_p, 4m_p$, referred to as an osculating conic to C at p , see [Lemma 3.2](#) on page [12](#).

THEOREM 1.2 (Sextactic point formula). *Let C be a cuspidal curve of genus g and degree $d \geq 3$. Let I denote the set of inflection points and cusps on C where $l_p \neq 2m_p$, and let J denote the set of cusps on C where $l_p = 2m_p$. Then the number of sextactic points s on C , counted with multiplicity, is given by*

$$s = 6(2d + 5g - 5) - \sum_I (4m_p + 4l_p - 15) - \sum_J (10m_p + c_p - 15).$$

REMARK 1. Although the formula in [Theorem 1.2](#) is a new restriction for cuspidal curves, it is superseded by other well known results.

The formula in [Theorem 1.2](#) has a classical touch and resembles the Plücker formula for inflection points; see [\[3\]](#), p. 586] for a formula for curves with any kind of singularities, attributed to Weierstrass and Noether.

- List of references with complete information about your sources at the end of your thesis

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Citing in mathematics

- Results and claims should be supported by
 - Your own mathematical argument
 - A citation of a result by someone else
- Different kinds of citations
 - The entire source
 - Specific results
 - Quotes are rare
 - Paraphrase
- Where do we cite
 - After a claim - at the end of sentences or paragraphs
 - In theorems, propositions and so on
- Different styles
 - Every journal has a preferred style
 - numerical [1]
 - alphabetic [Bor17]
- General citations without page numbers
 - [1] eller [Bor17]
 - Borgan i [Bor17]
- Specific results with numbering or page numbers
 - Maugesten og Moe [MM17, Theorem 1.1]
 - Maugesten og Moe [MM17, p. 12]
 - Maugesten og Moe [MM17, pp. 12--15]

What have you done yourself?

What has been done by others?

Give credit to the right person

- Who did this?
 - Clearly state this in the text!
 - Other people's results used directly
 - A combination of other people's results
 - You did it!
- Well known results
 - Not everything needs a citation
 - Depends on level, audience and goal
- Pictures, figures, graphs, tables and drawings belong to someone
 - Copyright
 - Ask for permission to use them
 - Cite them correctly
 - You may recreate graphs and figures, but cite the data / software that you use

Reference list

- Detailed information about all sources cited in the text
- Enough information to find exactly the text that has been used
- Surjective
 - Only these sources – no more, no less
- Proof read the reference list!
- Gives information about your text
 - What you work on
 - The level you work on
 - Where the text comes from
 - Where the text goes
 - Helps you get an overview

Reference list technical details

- Metadata
 - Information about the material
- Book
 - Author, title, publisher, place and year
- Article
 - Author, title, journal, volume, year, page numbers
- Web pages
 - Author, title, address, visiting date
 - arXiv-artikler has own set of rules
- Important to be precise
- Styles
 - numerical [1]
 - alphabetic [Ran17]
 - Many other styles (Borgan 2017)
 - Sorting

Verifiable research

- Main point with citations and the reference list is that the reader should be able to check and recreate or verify your results
- Always use the source in front of you!
- Be precise, be ware of editions, pre-prints etc.
- Use the primary source
- If you don't have the primary source or it can not be used, cite the statement about the result in the secondary source that you have in front of you!

Special problems in mathematics

- From general to special
 - Level on what you write vs. level on the source
 - A result that covers a more general case than you are working with
- Notation must be consequent!
 - Different notations must be merged
 - Declare notation early on and in running text
 - «In our notation, the theorem says...»
- Common to make changes / paraphrase
- But you still have to acknowledge the original work!

General or specific?



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Law of cosines

From Wikipedia, the free encyclopedia

This article is about the law of cosines in [Euclidean geometry](#). For the cosine law of optics, see [Lambert's cosine law](#).

In [trigonometry](#), the **law of cosines** (also known as the **cosine formula** or **cosine rule**) relates the lengths of the sides of a triangle to the [cosine](#) of one of its [angles](#). Using notation as in Fig. 1, the law of cosines states

$$c^2 = a^2 + b^2 - 2ab \cos \gamma,$$

where γ denotes the angle contained between sides of lengths a and b and opposite the side of length c .

The law of cosines generalizes the [Pythagorean theorem](#), which holds only for [right triangles](#): if the angle γ is a right angle (of measure 90° , or $\frac{\pi}{2}$ [radians](#)), then $\cos \gamma = 0$, and thus the law of cosines [reduces](#) to the [Pythagorean theorem](#):

$$c^2 = a^2 + b^2.$$

The law of cosines is useful for computing the third side of a triangle when two sides and their enclosed angle are known, and in computing the angles of a triangle if all three sides are known.

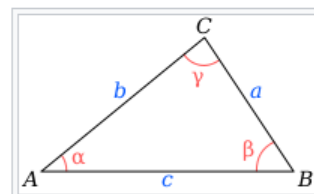


Figure 1 – A triangle. The angles α (or A), β (or B), and γ (or C) are respectively opposite the sides a , b , and c .

[Trigonometry](#)

Choosing notation

Definition [\[edit \]](#)

The Hirzebruch surface Σ_n is the $\mathbf{P}^1$ bundle over $\mathbf{P}^1$ associated to the sheaf

$$\mathcal{O}(0) + \mathcal{O}(-n).$$

The notation here means: $\mathcal{O}(n)$ is the n -th tensor power of the Serre twist sheaf $\mathcal{O}(1)$, the invertible sheaf or line bundle with associated Cartier divisor a single point. The surface Σ_0 is isomorphic to $\mathbf{P}^1 \times \mathbf{P}^1$, and Σ_1 is isomorphic to $\mathbf{P}^2$ blown up at a point so is not minimal.

Let H be any ample class on the Hirzebruch surface $\mathbb{F}_e = \mathbb{P}(\mathcal{O}_{\mathbb{P}^1} \oplus \mathcal{O}_{\mathbb{P}^1}(e))$, $e \geq 0$.

2.1. Hirzebruch surfaces. Let X_e , $e \geq 0$, be a Hirzebruch surface, regarded as a projectivisation of a rank 2 bundle $\mathcal{O} \oplus \mathcal{O}(-e)$ over $\mathbb{C}P^1$. Let L be a fibre and M_0 the special section, so that the intersections and self-intersections are as follows:

$$L^2 = 0, \quad M_0^2 = -e, \quad L \cdot M_0 = 1.$$

Numbering and cross references

- Numbering
 - Figures, tables, theorems, lemmas and so on should be numbered
 - Equations and formulas that you use different places in the text need a number
 - Central equations and formulas should have a number so that others can cite them
- What is cross references?
 - A way to connect different parts of the text using numbers
- Why use cross references?
 - Helps the reader navigate in your thesis
 - Use `\usepackage{hyperref}` for digital navigation

The second main result in this article is a formula for the number of sextactic points on cuspidal curves, for which the invariants involved are easily accessible. Indeed, for a cusp p with multiplicity m_p , there exists a unique line T_p such that $l_p = (T_p \cdot C)_p > m_p$, referred to as the tangent line to C at p . Similarly, applicable only to cusps where $l_p = 2m_p$, there exists a, not necessarily unique, conic O_p with $c_p = (O_p \cdot C)_p > 2m_p$ and $c_p \neq 3m_p, 4m_p$, referred to as an osculating conic to C at p , see [Lemma 3.2](#) on page [12](#).

THEOREM 1.2 (Sextactic point formula). *Let C be a cuspidal curve of genus g and degree $d \geq 3$. Let I denote the set of inflection points and cusps on C where $l_p \neq 2m_p$, and let J denote the set of cusps on C where $l_p = 2m_p$. Then the number of sextactic points s on C , counted with multiplicity, is given by*

$$s = 6(2d + 5g - 5) - \sum_I (4m_p + 4l_p - 15) - \sum_J (10m_p + c_p - 15).$$

REMARK 1. Although the formula in [Theorem 1.2](#) is a new restriction for cuspidal curves, it is superseded by other well known results.

The formula in [Theorem 1.2](#) has a classical touch and resembles the Plücker formula for inflection points; see [\[3\]](#), p. 586] for a formula for curves with any kind of singularities, attributed to Weierstrass and Noether.

Cross references code

- `\label{abel}`
- `\ref{abel}` – gives number only
- `\cref{abel}` – gives type and number
- `\vref{abel}` – gives type, number and page number
- `\usepackage{showkeys}`
- **Tip!** Use prefix `ex:` or `for:` or `thm:` or `lem:` when you make labels, and use `\usepackage{showkeys}` when you work on your document

Ethics, cheating and plagiarism

Cases of fraud and misconduct

MMR vaccine controversy

From Wikipedia, the free encyclopedia

The **MMR vaccine controversy** started with the 1998 publication of a fraudulent research paper in *The Lancet* linking the [combined measles, mumps, and rubella \(MMR\) vaccine](#) to [colitis](#) and [autism spectrum](#) disorders.^[1] The claims in the paper were widely reported,^[2] leading to a sharp drop in vaccination rates in the UK and Ireland and increases in the incidence of measles and mumps, resulting in deaths and serious permanent injuries.^{[3][4]} Following the initial claims in 1998, multiple large [epidemiological](#) studies were undertaken. Reviews of the evidence by the [Centers for Disease Control and Prevention](#),^[5] the [American Academy of Pediatrics](#), the [Institute of Medicine](#) of the [US National Academy of Sciences](#),^[6] the [UK National Health Service](#),^[7] and the [Cochrane Library](#)^[8] all found no link between the MMR vaccine and autism.

An investigation by journalist [Brian Deer](#) found that [Andrew Wakefield](#), the author of the original research paper linking the vaccine to autism, had multiple undeclared [conflicts of interest](#),^{[9][10]} had manipulated evidence,^[11] and had broken other ethical codes. The *Lancet* paper was partially retracted in 2004, and fully retracted in 2010, when *Lancet*'s editor-in-chief [Richard Horton](#) described it as "utterly false" and said that the journal had been "deceived".^[12] Wakefield was found guilty by the [General Medical Council](#) of serious professional misconduct in May 2010 and was struck off the [Medical Register](#), meaning he could no longer practice as a doctor in the UK.^[13] In 2011, Deer provided further information on Wakefield's improper research practices to the *British Medical Journal*, which in a signed editorial described the original paper as fraudulent.^{[14][15]} The [scientific consensus](#) is the MMR vaccine has no link to the development of autism, and that this vaccine's benefits greatly outweigh its risks.

Wakefield's paper was described as "perhaps, the most damaging medical hoax of the last 100 years".^[16] Physicians, medical journals, and editors^{[17][18][19][20][21]} have described Wakefield's actions as fraudulent and tied them to epidemics and deaths.^{[22][23]}

Cases of misconduct in science



[Home](#) / [News & Opinion](#)

The Top Retractions of 2019

A journal is forced to remove a record-breaking number of papers—and all in one go.

Dec 16, 2019

RETRACTION WATCH

Another year, another 1,433 (and counting) [retractions](#). The tenth year of [Retraction Watch](#)'s existence included—as is often the case—a new record, some impressive numbers, and some bizarre stories. But it also included some exemplary behavior. Here are some of the year's top retraction stories, in no particular order:

ABOVE: © ISTOCK.COM,
[TOMPROUT](#)

<https://www.the-scientist.com/news-opinion/the-top-retractions-of-2019-66852>

Menti

What is plagiarism?

Definition

Plagiarism is the representation of another author's language, thoughts, ideas, or expressions as one's own original work.

<https://en.wikipedia.org/wiki/Plagiarism>

Use of sources and citations

Norwegian

In order to write a good assignment, it is essential that you use sources correctly. Using other people's material without declaring it properly may be considered as cheating or attempted cheating.

Main rules

- Your assignment should be your own independent work, a result of your own learning and efforts.
- You may use other people's material:
 - as sources of information about your topic.
 - to support and illustrate your line of reasoning.
- You must cite the source if you obtain data, reasoning and arguments from the literature, syllabus, lecture notes, web pages, other students' papers or your own previously submitted papers.
- All sources should be specified in your assignment, both in the text and in the list of references. This also applies when referring to your own previous work.

Plagiarism traps in mathematics

- Often necessary to include many well known results
- What is common knowledge and what belongs to someone?
- Maybe your result is a twist on a previously known result
- Asked by supervisor to include a proof, based on a proof from a source

An example from the 16th century

Av https://wellcomeimages.org/indexplus/obf_images/ae/d3/8753d7ad74cb086ba79ccfc75e7f.jpgGallery:
<https://wellcomeimages.org/indexplus/image/V0001004.html>Wellcome Collection gallery (2018-03-29);
<https://wellcomecollection.org/works/pcd969qu> CC-BY 4.0, CC BY 4.0,
<https://commons.wikimedia.org/w/index.php?curid=36391939>



Gerolamo Cardano
1501-1576

Cardano's formula [\[edit \]](#)

Gerolamo Cardano is credited of the first formula for solving cubic equation. His formula applies to depressed cubics, but, as shown in [§ Depressed cubic](#), it allows solving all cubic equations.

Cardano's original result is that, if

$$x^3 + px + q = 0$$

is a cubic equation such that p and q are **real numbers** such that $4p^3 + 27q^2 > 0$, then the equation has the real root

$$\sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}.$$

https://en.wikipedia.org/wiki/Cubic_equation

A formula to solve the cubic

Av https://wellcomeimages.org/indexplus/obf_images/ae/d3/8753d7ad74cb086ba79ccfc75e7f.jpgGallery:
<https://wellcomeimages.org/indexplus/image/V0001004.html>Wellcome Collection gallery (2018-03-29):
<https://wellcomecollection.org/works/pcd969qu> CC-BY 4.0, CC BY 4.0,
<https://commons.wikimedia.org/w/index.php?curid=36391939>



Gerolamo Cardano
1501-1576

**1 - A solution for some cases
del Ferro**

**2 - A more general solution
Tartaglia**

Well known for claiming other people's
results as his own = plagiarism

**3 - The general solution
Cardano**

He **bought** Tartaglias proof...
...and promised not to share!
...but when he found out about del Ferros
solution, he published the proof anyway!
He cited both del Ferro and Tartaglia!
Tartaglia claimed plagiarism!



Niccolò Fontana Tartaglia
1500-1557

An example: Stealing from the textbook

- Textbook for an introduction to plane algebraic geometry
- Bachelor thesis where results are not properly cited

Theorem 9.7. *Let F be a reduced curve of degree $n \geq 3$. Let p be the characteristic of K , and assume that either $p = 0$ or $p > n$. Then*

- (a) $H_F \equiv 0 \pmod{F}$ if and only if F is a union of lines.
- (b) If $H_F \not\equiv 0 \pmod{F}$, then the intersection of F with its Hessian curve consists of the singular points of F and the flexes of F .
- (c) For every regular point P of F whose tangent line G at P is not a component of F we have

$$\mu_P(F, G) = \mu_P(F, H_F) + 2.$$

Theorem 4.8. *Let $C = V(F)$ be a curve of degree $n \geq 3$. Let p be the characteristic of k , and assume that either $p = 0$ or $p > n$. Then H_F and H_C have the following properties:*

1. H_F is a multiple of F if and only if C is a union of lines.
2. If H_F is not a multiple of F , then the intersection of C with its Hessian H_C consists of the singular points of C and the flexes of C .
3. For every point P of C whose tangent line L is not a component of C we have

$$\mu_P(C, L) = \mu_P(C, H_C) + 2.$$

Example 9.8. If one drops the assumption about the characteristic, then the theorem is no longer true in general. Suppose $\text{Char } K = 3$ and $F := X_0^2 X_2 - X_1^3$. This curve is irreducible, its singularity $\langle 0, 0, 1 \rangle$ is the only point at infinity, and one sees easily that $H_F = 0$.

The regular points of F are the points at finite distance. They satisfy the equation $Y = X^3$. Obviously $(0, 0)$ is a flex of F . For an arbitrary point (a, b) of F at finite distance,

$$Y - X^3 = Y - X^3 - (b - a^3) = (Y - b) - (X - a)^3$$

and therefore (a, b) is also a flex of F .

In contrast to this we have the following.

Corollary 9.9. *Under the assumptions of 9.7 let F be irreducible and let s be the number of singularities of F . Then F has at most $3n(n-2) - s$ flexes.*

Example. The assumption about the characteristic of k can not be dropped easily. Suppose $\text{char } k = 3$, and let $F = X_0^2 X_2 - X_1^3$ define the curve C . This curve is irreducible, its only point at infinity is the singularity $(0 : 0 : 1)$, and $H_F = 0$. The regular points are all at finite distance and satisfy $Y = X^3$. Thus we have a proper flex at $(0, 0)$. But even more, any point (a, b) on the curve at finite distance satisfies

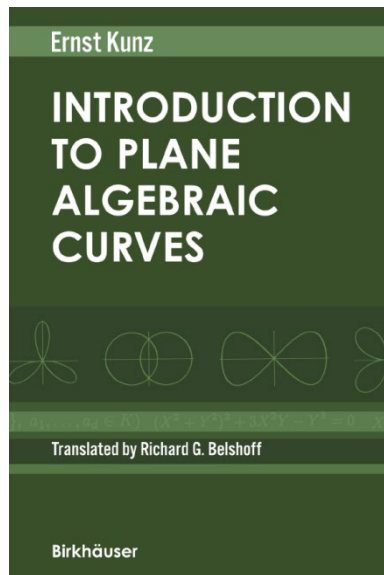
$$Y - X^3 = Y - X^3 - (b - a^3) = Y - b - (X - a)^3.$$

Therefore, any regular point is a flex.

Corollary 4.9. *Under the assumptions of 4.8 let C be irreducible of degree n with s singularities. Then C has at most $3n(n-2) - 2s$ flexes.*

An example: Stealing from the textbook

- Textbook with an introduction to plane algebraic geometry



- Bachelor thesis where results are not properly cited
- The book is mentioned in the bibliography, but each individual result is not correctly cited
- Rewritten, not copied, results and proofs, but they are not the student's results!
- Even examples are copied – and presented in the same way!

References

- [1] Gerd Fischer. *Ebene algebraische Kurven*. vieweg, 1994.
- [2] Ernst Kunz. *Introduction to Plane Algebraic Curves*. Birkhäuser, 2005.

How to avoid plagiarism

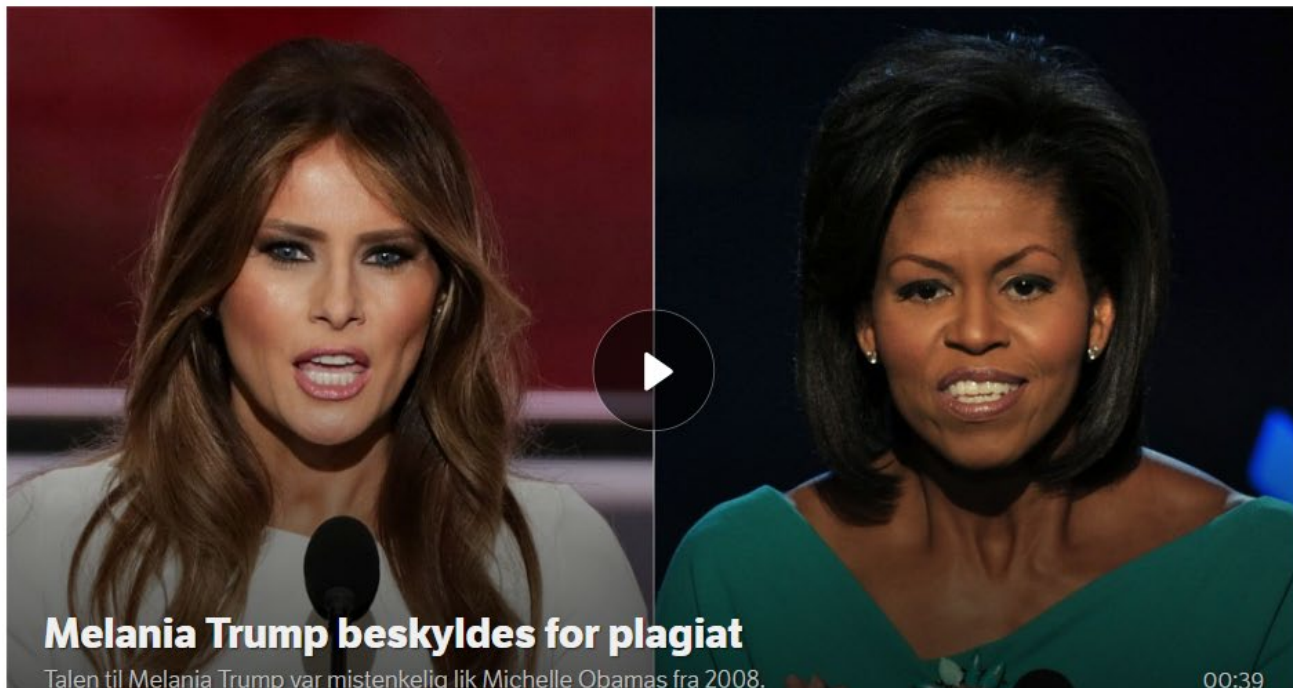
- Cite!
- It is always OK to cite (the source you have in front of you)!
- It is never OK to copy theorems, proofs etc., even in rewritten form, without citing!
- Self-plagiarism is still plagiarism
- Make sure the motivation, introduction and presentation of the results and proofs are independent of your sources'
- Read the source, put it away and write what you want to include in your own words
- Proper data and file-management

Key principles

- Who made it?
- Why do you include it?
- How do you use it?
- Where can the reader find it?
 - Citations
 - Reference list
- Your own contribution is the important part!
- Must be stated explicitly!

Fru Trump-talen: – Patetisk plagiat

– Dette er berre patetisk, seier retorikkekspert Kjell Terje Ringdal om Melania Trump sin tale i natt. Store delar av talen var lik ein tale Michelle Obama heldt for åtte år sidan.



Eivind Molde

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Journalist

Publisert 19. juli 2016 kl. 13:57

Oppdatert 19. juli 2016 kl. 14:05



Artikkelen er
mer enn to år
gammel.