

Writing in mathematics

Recommended literature



BOK

A Primer of Mathematical Writing Being a Disquisition on Having Your Ideas Recorded, Typeset, Published, Read, and Appreciated

Forfatter: Krantz, Steven

ISBN: 978-1470436582

Utgave: 2nd ed

Utgivelsesår: 2017

Utgiver: American Mathematical Society

[Mer detaljer ▼](#)

Outline of Writing in mathematics

- The structure of a thesis
- The writing
- Presenting mathematics

The structure of a thesis

Writing a thesis in mathematics

- Find a topic / problem
 - Search for and select appropriate literature
 - Present a topic with known results
 - Present new results
 - Do and present simulations and experiments
 - Scientific methods
-
- Thesis
 - Oral presentation

The problem, broad terms

What others have done

What you will do
and how

The bulk of the thesis

Preliminaries, notation

Methodology

Results

Summary of
your work

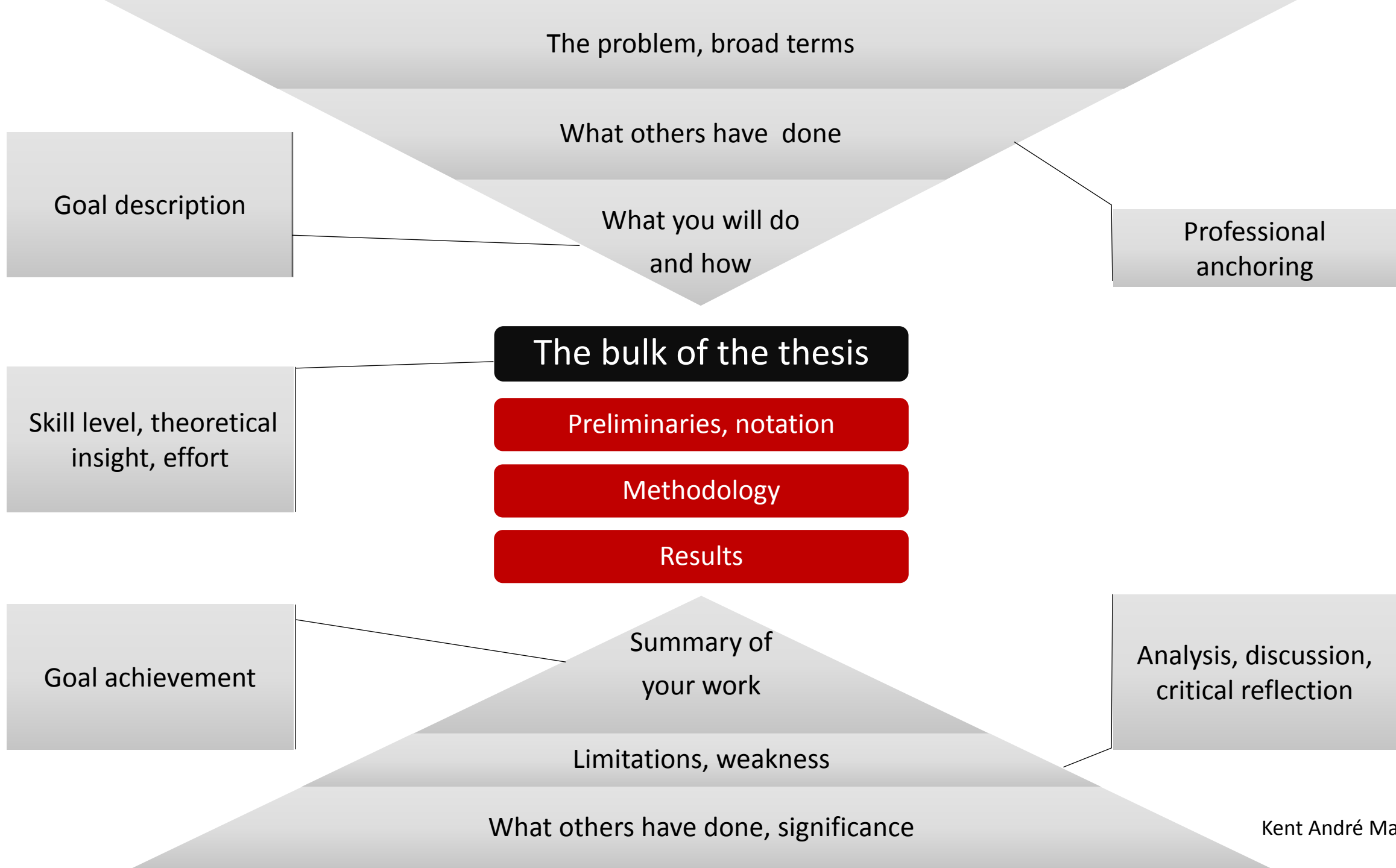
Limitations, weakness

What others have done, significance

Sensor guidance on censorship of master theses

The sensor assesses the extent to which the candidate has achieved these goals:

1. Professional anchoring
2. Theoretical insight
3. Goal description
4. Skill Level
5. The work
6. Analysis and discussion
7. Critical reflection
8. Own contribution / goal achievement
9. Structure
10. Language
11. Form



Structure your thesis (or article)

Mathematics

- Title
- Abstract
- Introduction
- Notation and definitions
- Results and proofs
- Simulations and pseudocode
- Applications and examples
- Conclusive chapter
- List of references

Experimental sciences

- IMRaD
- Title
- Abstract
- Introduction
- Method
- Results
- Analysis and Discussion
- List of references

Title

- Using Title Case is Normal
- Line break wisely

*On Real Matrices with
Positive Definite Symmetric Component*

- Different kinds of titles
 - Relatively short
 - A sentence
 - Descriptive
 - Provocative
 - Motivating
 - Engaging
 - Very rarely a newspaper headline or clickbait
 - Punchy

- Precise
- Searchable
 - So that other researchers will find it
 - Avoid special characters

**The Golden String, Zeckendorf
Representations, and the Sum of a Series**

Martin Griffiths

Abstract

- <200-300 words
- Summarizes the content
 - To read or not to read
 - Is read first!
- Lay claim to new results
- Put the topic and your results in context quickly
- Main results only, with conclusions
- Mention techniques used
- Avoid special characters
- Do not cite in the abstract
- Rewrite
- Avoid “In this article...”

Acknowledgements

- Where you thank everyone who has supported your work
- Funding (if applicable)
- Supervisor
- Anyone who read the manuscript or helped substantially
- Fellow students
- Family
- Use “I”
- I thank...
- Sometimes
 - Funny
 - Passive-aggressive
 - Mention odd happenings
- Remember that grandma and your future bosses and subordinates – basically everyone - will read this

Introduction

- Explain the problem
- Put the problem in context
 - Background and motivation
 - Why is this interesting?
 - What have others done?
- Present what you will do and how (thesis)
- Present your main results (article)
- Enough technical background to make it readable for an interested reader
- Present the outline
- Very important
- Everyone reads it
- Set the «feel» of the thesis
- Adjust the language and mathematics
- Storytelling
 - ABT (and-but-therefore)
- **Tip!** Write the introduction several times!
 - The first you write
 - The last you write
- You're NOT writing a textbook!

The bulk of the thesis

- Background and preliminaries
- Methods
- Theorems and proofs
- Simulations and pseudocode
- Applications and examples

Background and preliminaries

- Not a text book
- Often based on a standard text book in your field
- You need background to present your results coherently
- Don't go back to year 0

Methods (if applicable)

- Describe the methods you use
- History
- Background
- Advantages
- Limitations
- Applications
- What's new?

Theoretical results (if applicable)

- Build your result
- History
- What's new?
- The result itself
- Proof
- Corollaries

Applications and examples

- Applications and examples supporting your results
- Choose them wisely
- Argue for your choices!
- Complete overview
- Systematic overview
- Randomly
- Presented with tables and figures
- You're not writing a text book!

Conclusive chapter

- Summary of your work
- Broader perspective
- Significance and limitations
- Identify open problems
- Point to future research possibilities

Appendix

- Essential stuff that has no natural place in the thesis
- Code/Pseudocode
- Simulations
- Numerical results
- Complete tables
- Systematic/detailed overview

Simulations and pseudocode

- You have simulations and code written in a particular program
- Presented indirectly and referred to in the text
- Presented in form of pseudocode
- Complete code included in an appendix
- Store and make complete code available online (Zenodo or Github)
- Main aim: Make it possible to reconstruct your results now and in the future
 - Lost files
 - New software
 - Incompatibility

The writing

Writing in mathematics

- Mathematics happens NON-linearly, typically back and forth
- Mathematics is presented linearly

Precise, concise and focused

- Strong contrast to a novel or popular science
- Every word has a specific meaning
- Stay focused when you describe results
- Avoid describing your results with adjectives
 - **A result is never «wonderful» «interesting» «clever» «important»**

Familiarize yourself with the genre

- Read articles and theses
- Use MathSciNet
- Use Google Scholar
- duo.uio.no (Master theses / Ph.D. theses)
- arXiv.org
- Request from the library
 - Master theses from before 2007
 - Ph.D. theses

General advice

- Write for your audience
 - Yourself, sensor, supervisor, future employer, other students, researchers in your field
- Invite the reader into your text
- Make it easy for him/her
- Don't over- or underestimate the reader
- Make your intentions clear
- Build a clear structure with peaks
- Work on transitions
- Find your voice

The process

- Write immediately and all the time
- Write to create text and structure
- Write to learn
- Write to find results
- Write to clarify how your results are connected
- Write to correct yourself
- Write to get the details right
- Write to be efficient
- Expect to write everything at least 2-3 times
- The 30%-rule
 - Always let your supervisor in on your 30% draft
 - Expect feedback accordingly
- Academic writing centre

Choosing language

- English
 - Most science is in English
 - Easily shared with other researchers
 - Easy to find technical terminology
- Norwegian
 - Spend less time on grammar and spelling and more time on math
 - Better understanding
- Ask your supervisor!
- Use a dictionary
- Use a spell checker
- Read up on grammar
- Keep it simple

How to build a paragraph

- Claim
- Argument
 - Examples
 - Discussion
- Conclusion
- ABT
 - ...and...
 - ...but...
 - ...therefore...
- Storytelling
 - Once upon a time
 - Something bad happend
 - Something or someone changed everything
 - They lived happily ever after

Transitions

- Make logical transitions between sentences and paragraphs
- Many different types and subtleties
- Beware of precision level
- Keep a list of these in front of you when you write text
 - Combinations
 - Implications
 - Modifications
 - Emphasis
 - Consequences
- Heavily used in mathematics
 - In addition to
 - Furthermore
 - Moreover
 - Similarly
 - Additionally
 - First, second, third...
 - Thus
 - Therefore
 - Consequently
 - Hence
 - However
 - On the other hand
 - In particular
 - Yet
 - Although
 - Clearly
 - Actually
 - Indeed
 - Obviously

Who does what? – And it's not «cool»

- Write «we» (subject to debate...)
- Except in the acknowledgements, where «I» is used
- Be conscientious about who did what
 - Important to distinguish between your own and others' results
 - Never take credit for something someone else did
 - Take all credit for the things you did on your own (with help from supervisor)
 - «We define» vs. «Is defined as»
 - Who proved the theorem?
 - «We prove theorem X...»
 - «We present the proof of theorem X following Y...»
 - «We give a proof of theorem X...»
 - «We give a new proof of theorem X...»
- Avoid adjectives – be specific
 - Don't use adjectives to describe results
 - Be specific and describe how they can be used

Presenting mathematics

Notation and efficiency

<http://www.math.caltech.edu/~jmlb/abel.pdf>

Text problems

“Two men go into business together and have an unknown capital. Their gain is equal to the cube of the tenth part of their capital. If they had made three ducats less, they would have gained an amount exactly equal to their capital. What was their capital and their profit?”

Symbolic algebra

“What are the roots of the equation $\left(\frac{x}{10}\right)^3 - 3 = x$?”

François Viète (1540 – 1603)

Notation and definitions

- Introduce and explain most technical terms and symbols that are used, but not necessarily in separate environments
- Proximity – minimize the distance between notation/definition and first use
- Make sure that your text can be read by others (including yourself a few years from now)
- Some terms and notation must be considered to be well known
 - Depends on your audience
- Be **precise, concise and focused**
- Use standard notation

- In definitions: *if* means *if and only if*
- *Italicize* the word that is being defined

Common words and phrases

- Let
- Denote
- Write

Using symbols

Two golden rules

- Don't introduce unnecessary symbols
- Define all symbols before they are used

Three typical mistakes

- One thing is referred to by two different symbols
- One symbol means two things
- Breaking the golden rules

Alpha and Omega

A lot of symbols:

- Latin alphabet
- Greek alphabet
- Small or capital letters
- Caligraphic letters
- Upright letters
- Boldface letters
- Math symbols
 - In math environment
 - Looks almost like *italic*
 - Upright math symbols look unprofessional
 - Italic math operators look unprofessional
 - Detexify

Symbols can be decorated:

- Indexes, subscripts, superscripts
- Hats, curls, arrows, lines, stars, dots...
- Be consistent
- Avoid the Christmas tree

$$W_{\mathbf{a}} = q_{\mathbf{a}}^* H^0 \left(P_{\mathbf{a}}^{1,d}, \Omega_{P_{\mathbf{a}}^{1,d}}^1(\log L) \right) \subset H^0 \left(P^{r,d}, \Omega_{P^{r,d}}^1(\log L) \right).$$

Symbols and parenthesis

- Never start a sentence with a mathematical expression or symbol
- Follow conventions
 - Functions (f, g, h)
 - Sets (A, B)
 - Indices (i, j, k)
 - Small things (δ, ε)
 - Vectors (Bold/arrow)
 - Different conventions
 - See the textbook / main reference
- Avoid special characters in title and abstract
- Keep an eye on the parentheses
 - Correct number
 - Correct nesting $\{[(())]\}$
 - Correct size
 - Correct place
 - Readability
 - Line breaks
- Look out for the indices
 - Subscript and superscript
 - In LaTeX need $\{ \}$

Some useful things to remember

- “The” or “A”?
- Stay consistent
 - Synonyms in notation
 - Square root or exponent
 - Vectors
- If you work with complex numbers, don’t count with “*i*”
- Math operators are upright
 - $\tan(x)$, $\ln(x)$, $\gcd(m,n)$
- “*k*th term” (note the italic *k*, no superscript “th” and no ‘ or – before th)
- Different ellipses: ... or ...?
 - Baseline between elements in lists
 - Vertically centered between operators
- Multiplication sometimes needs a vertically centered dot
- “Wicked which”
 - That defines and restricts
 - Which informs and does not restrict
- False “if”
 - “If” should be followed by “then”.

Words and abbreviations

- Write it out
 - It is, can not, do not
- Use abbreviations correctly
 - i.e. means complete list of examples or explanation
 - e.g. means incomplete list of examples
 - etc. means and so on
 - Never combine with e.g.
 - cf. means confer (compare)
 - See means find more information

Sentences

- Always write in full sentences
- A mathematical expression is a part of a sentence
 - Never start with a mathematical expression
- A citation is a part of a sentence
- Beware of punctuation
 - In mathematics
 - Commas
- Build your sentence with logical order
 - The important part first (depends on context)
 - Objects and properties must match

In the projective plane we show that a rational cuspidal curve can have at most four cusps

We show that four is the maximum number of cusps on a rational cuspidal plane curve

We show that a rational cuspidal curve in the projective plane can have at most four cusps

More trouble than you might think...

Let P be the escaped wombat population (in thousands) t years after 1990 and suppose that

$$P = 0.5(1.12)^t.$$

The wombat population in 1992 is approximately 672. We can see this by setting $t = 2$ and observing that

$$P = 0.5(1.12)^2 = 0.6272 \text{ thousand wombats.}$$

If we want to predict when the wombat population will reach 2000, we set $P = 2$ and solve for t using logarithms.

$$\begin{aligned} 2 &= 0.5(1.12)^t \\ \log 2 &= \log 0.5 + t \log 1.12 \\ t &= \frac{\log 2 - \log 0.5}{\log 1.12} \approx 12.23 \text{ years.} \end{aligned}$$

The wombat population will reach 2000 in the year 2002.

Let P be the escaped wombat population (in thousands) t years after 1990 and suppose that

$$P = 0.5(1.12)^t.$$

By substituting 2 for t in the above equation, we can see that in 1992, the wombat population is approximately 672.

$$0.5(1.12)^2 = 0.6272 \text{ thousand wombats.}$$

Let t_{2000} be the year when the wombat population reaches 2000. Then,

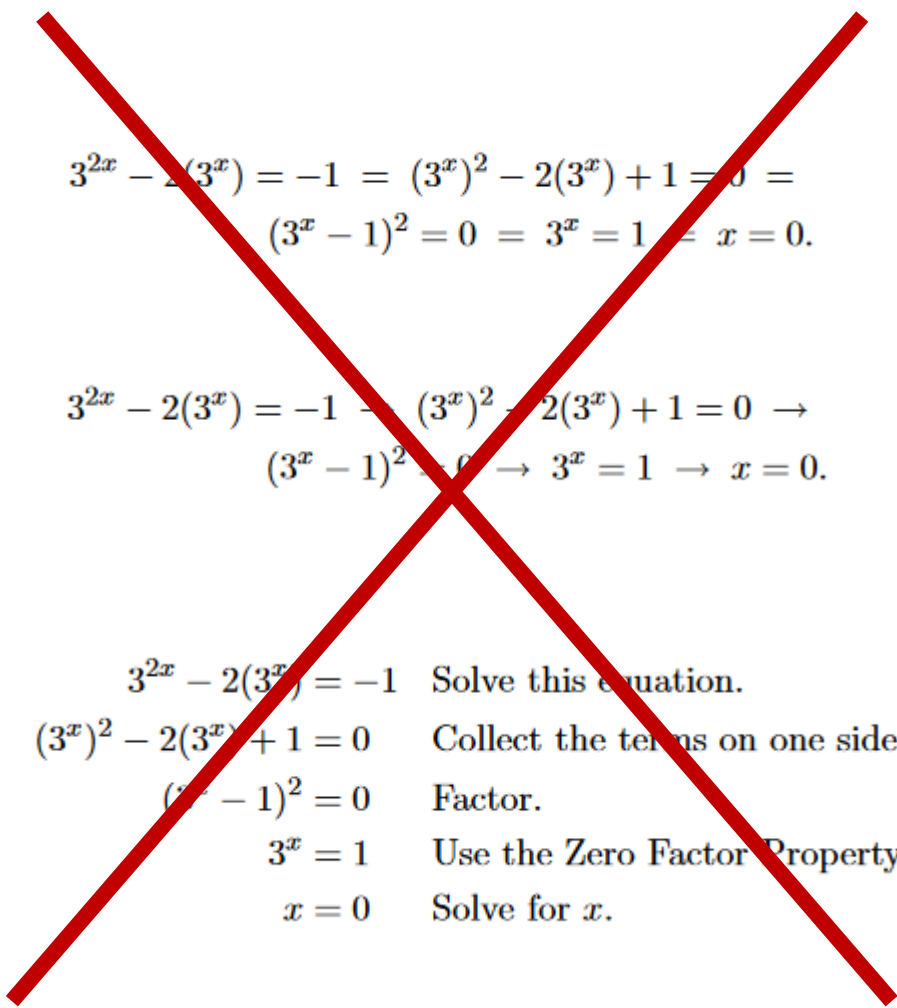
$$\begin{aligned} 2 &= 0.5(1.12)^{t_{2000}} \\ \log 2 &= \log 0.5 + t_{2000} \log 1.12 \\ t_{2000} &= \frac{\log 2 - \log 0.5}{\log 1.12} \approx 12.23 \text{ years.} \end{aligned}$$

The wombat population will reach 2000 in the year 2002.

Displaying equations

- In-line or displayed equations?
- Depends on height
 - Tall expressions should be displayed
- Depends on length
- If too long
 - In-line: Try to keep mathematical expressions in one line only
 - Displayed: Break line before binary operation
- Don't abuse signs and symbols

Displaying equations


$$3^{2x} - 2(3^x) = -1 = (3^x)^2 - 2(3^x) + 1 = 0 = \\ (3^x - 1)^2 = 0 = 3^x = 1 = x = 0.$$

$$3^{2x} - 2(3^x) = -1 \rightarrow (3^x)^2 - 2(3^x) + 1 = 0 \rightarrow \\ (3^x - 1)^2 = 0 \rightarrow 3^x = 1 \rightarrow x = 0.$$

$$\begin{array}{ll} 3^{2x} - 2(3^x) = -1 & \text{Solve this equation.} \\ (3^x)^2 - 2(3^x) + 1 = 0 & \text{Collect the terms on one side.} \\ (3^x - 1)^2 = 0 & \text{Factor.} \\ 3^x = 1 & \text{Use the Zero Factor Property.} \\ x = 0 & \text{Solve for } x. \end{array}$$

$$\begin{aligned} 3^{2x} - 2(3^x) &= -1 \\ (3^x)^2 - 2(3^x) + 1 &= 0 \\ (3^x - 1)^2 &= 0 \\ 3^x &= 1 \\ x &= 0. \end{aligned}$$

We want to solve for x in the equation

$$3^{2x} - 2(3^x) = -1.$$

We can rewrite this equation in terms of 3^x :

$$(3^x)^2 - 2(3^x) + 1 = 0.$$

After factoring, this becomes

$$(3^x - 1)^2 = 0$$

and it follows that $3^x = 1$, or $x = 0$.

Results and proof

- Axiom (Building block)
 - Definition (Meaning of word/symbol)
 - Theorem (Strong result)
 - Proposition (Smaller result)
 - Lemma (Intermediate or technical result)
 - Corollary (Follows from a Theorem, Lemma or Proposition)
 - Conjecture (Not proven)
 - Remark
 - Proof
 - Introductory and transitional text
- Prepare notation and concepts
 - Assumptions
 - State the result in an appropriate environment
 - Continuous numbering
 - Important for later reference!

5 minute paper

- State Pythagoras' theorem

Stating a result

- Conditional statement
 - Premises or hypothesis
 - All of them should be present in the theorem for later references
 - Notation should be presented before the theorem
 - A conclusion
 - Corollaries and conjectures are placed in different environments
- If A, then B
 - B is a necessary consequence of A
 - An object subject to certain premises has these properties
 - Keep it strictly to business

Writing a proof

Type of proof

- Direct ($A \Rightarrow B$)
- Contrapositive ($\text{Not } B \Rightarrow \text{Not } A$)
- Induction (First + Hypothesis and Step)
- Contradiction (Assume that B holds)
- Computational
- Construction
- Finite proof
- Obvious
- Short
- Long
- Technical
- Cited

- Logical structure
 - Outline the steps first
 - Step by step
 - Tell the reader where you are in the proof
 - First, second, finally
 - Sum up main
 - Point the reader to the key ingredient in each step
- Explain calculations with words
- Ends with a symbol
 - Q.E.D. (quod erat demonstrandum)
 - Square

Useful phrases in a proof

From Higham's Handbook of Writing for the Mathematical Sciences

Structure

The aim/idea is to...

Our first goal is to show that...

The trick of the proof is to find...

The essential observation is that...

Placeholders

First, we establish that...

Our task is now to...

Our problem reduces to...

It remains to show that...

Finally, we have to show that...

We are almost ready to invoke...

Omitted parts

Some tedious manipulation yields...

The result follows after straightforward computations

An obvious induction gives...

It is easy to see that...

NB! Easy must be easy for your reader, too!

5 minute paper - review

- State Pythagoras' theorem
- Explain the logic behind your text (not the theorem) to your partner!
 - With what did you start and why?
 - Did you need to go back and add details to make the result clear?
 - Is your notation consistent?
 - Is there any unexplained notation?
 - Is there any superfluous notation or objects?

From draft to final

Proof reading

- Use a spell checker
- Not picked up by the spell checker
 - to or too or two
 - flaw or flow
 - plain or plane
- Mathematics
 - Formulas
 - Indexes
- Table of contents
- The reference list
- Check figures
- Ensure flow between paragraphs and sections
- Beware of words repeating themselves
- Extra words and mistakes after rewrites
- Punctuation
- Notation
 - No unnecessary symbols
 - All symbols are defined
 - One-to-one and onto
- Typography
- Option FINAL in documentclass

Done. Now what?

- Tidy up in your folders and notes
- «Final»
 - MasterSurname.tex
 - A tidy bib.bib
 - All figures in one folder
 - MasterSurname.pdf
 - Program code
 - Data
- «Portfolio»
 - MasterSurname.pdf

Further reading

- Halmos: How to write Mathematics (22 pages)
- Higham: Handbook of Writing for the Mathematical Sciences (304 pages)
- Kleiman: Writing a Math Phase Two Paper (12 pages)
- Lee: A Guide to Writing Mathematics (17 pages)
- Vivaldi: Mathematical Writing (213 pages)