



Matematiske harmonier og disharmonier

Faglig-Pedagogisk Dag 2005

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Grunntone: 1

Oktav: 2 (åttende tone)

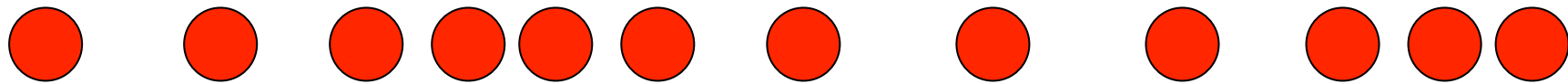
Kvint: $\frac{3}{2}$ (femte tone)

Kvart: $\frac{4}{3}$ (fjerde tone)

Kvint + Kvart = Oktav: $\frac{3}{2} \times \frac{4}{3} = 2$



A (= 440 Hz)



$\lambda = 0,75 \text{ m}$



Pytagoras forholdsskala:

24:27:30:32:36:40:45:48

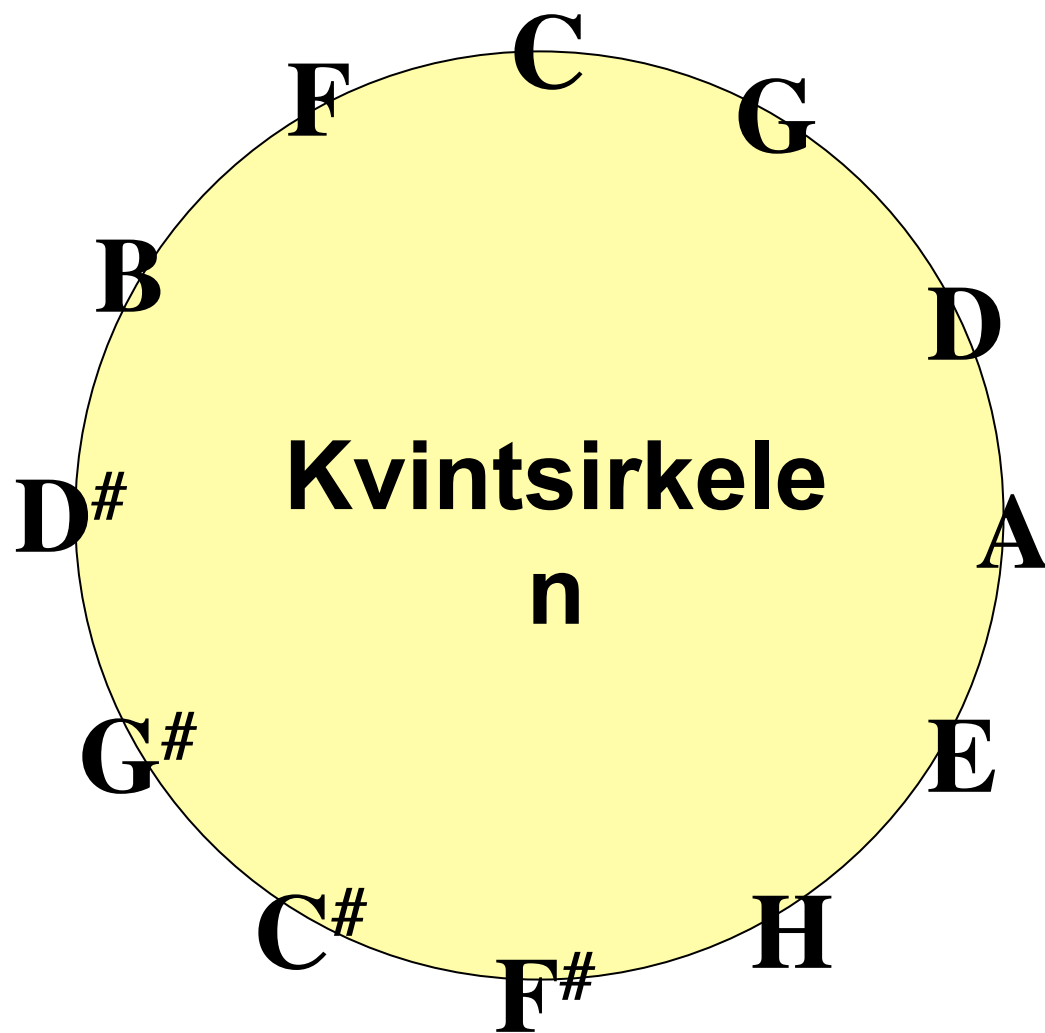
Gir frekvenser:

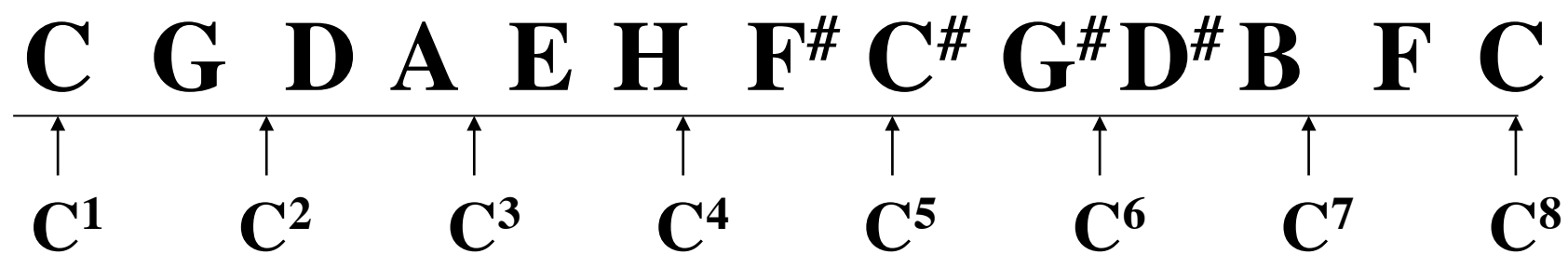
C: 264 Hz G: 396 Hz

D: 297 Hz A: 440 Hz

E: 330 Hz H: 495 Hz

F: 352 Hz C: 528 Hz







$$\left(\frac{3}{2}\right)^{12} \approx 129.746$$

$$2^7 = 128$$



$$\left(\frac{3}{2}\right)^N = 2^k$$

$$\frac{\ln(3)}{\ln(2)} = 1 + \frac{k}{N}$$



$$n = a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \dots}}} = [a_0, a_1, a_2, a_3, \dots]$$

hvor

$$a_0 < n < a_0 + 1$$

$$a_1 < b_0 < a_1 + 1 \quad b_0 = (n - a_0)^{-1}$$

$$a_2 < b_1 < a_2 + 1 \quad b_1 = (b_0 - a_1)^{-1}$$

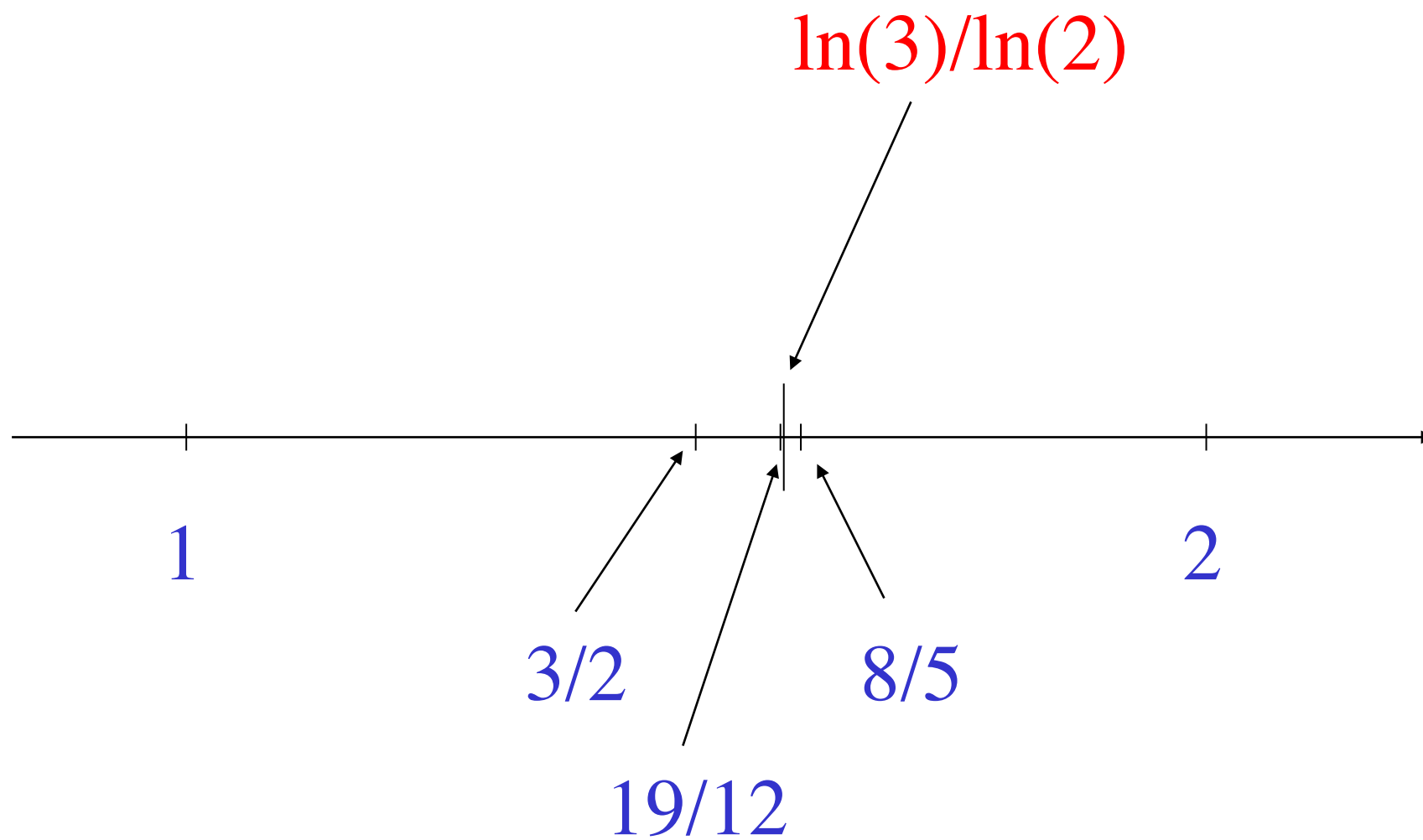
OSV.



$$\ln(3)/\ln(2) \approx 1,584962501$$

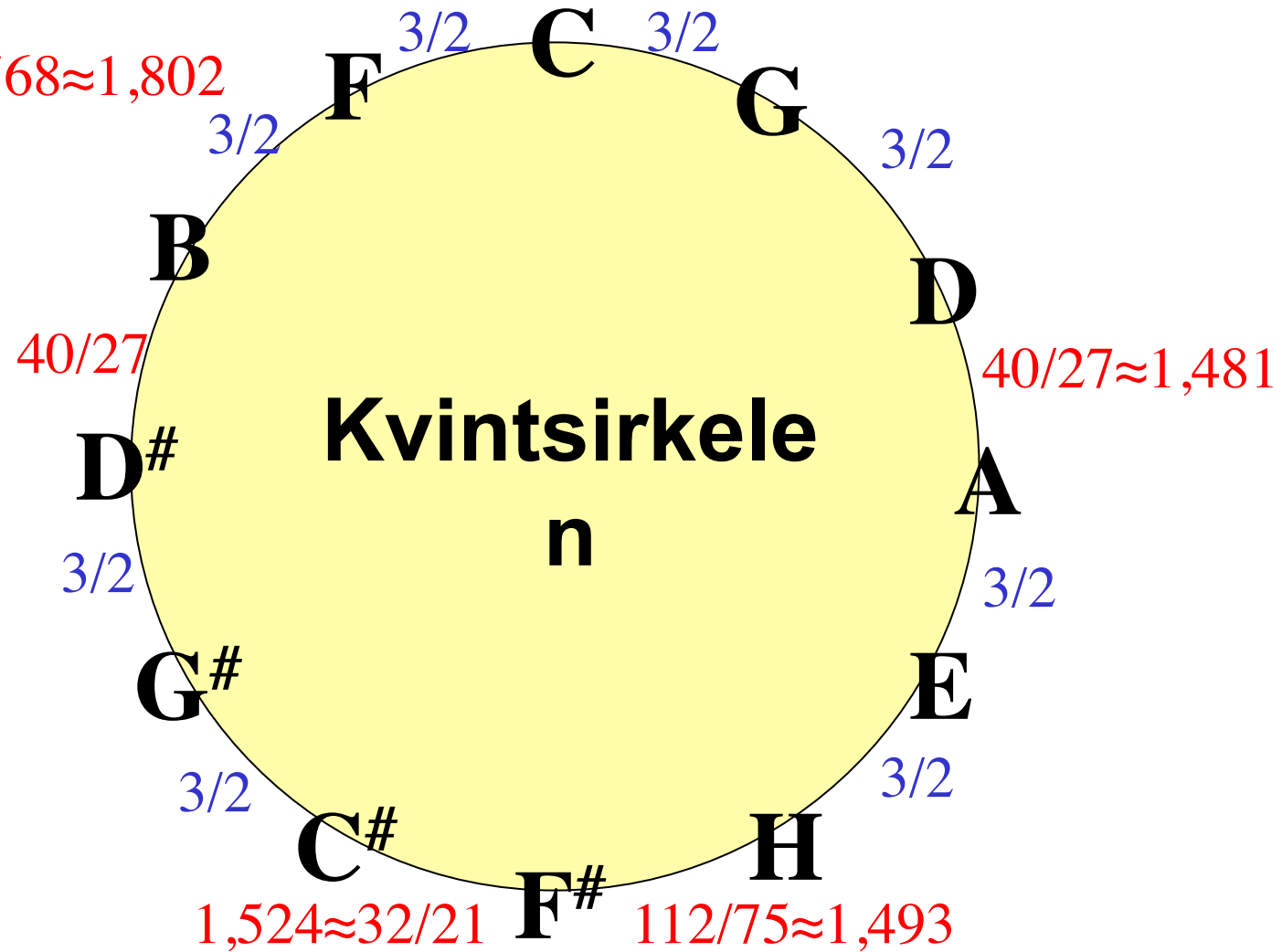
$$= 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \dots}}}$$

$$=[1, 1, 1, 2, 2, 3, 1, 5, 2, 23, \dots]$$





$$59049/32768 \approx 1,802$$





$$2^{(1/12)} \approx 1,059463$$

”Temperert halvtone”



24	27	30	32	36	40	45	48
24	26,94	30,24	32,04	35,96	40,36	45,31	48
C	D	E	F	G	A	H	C



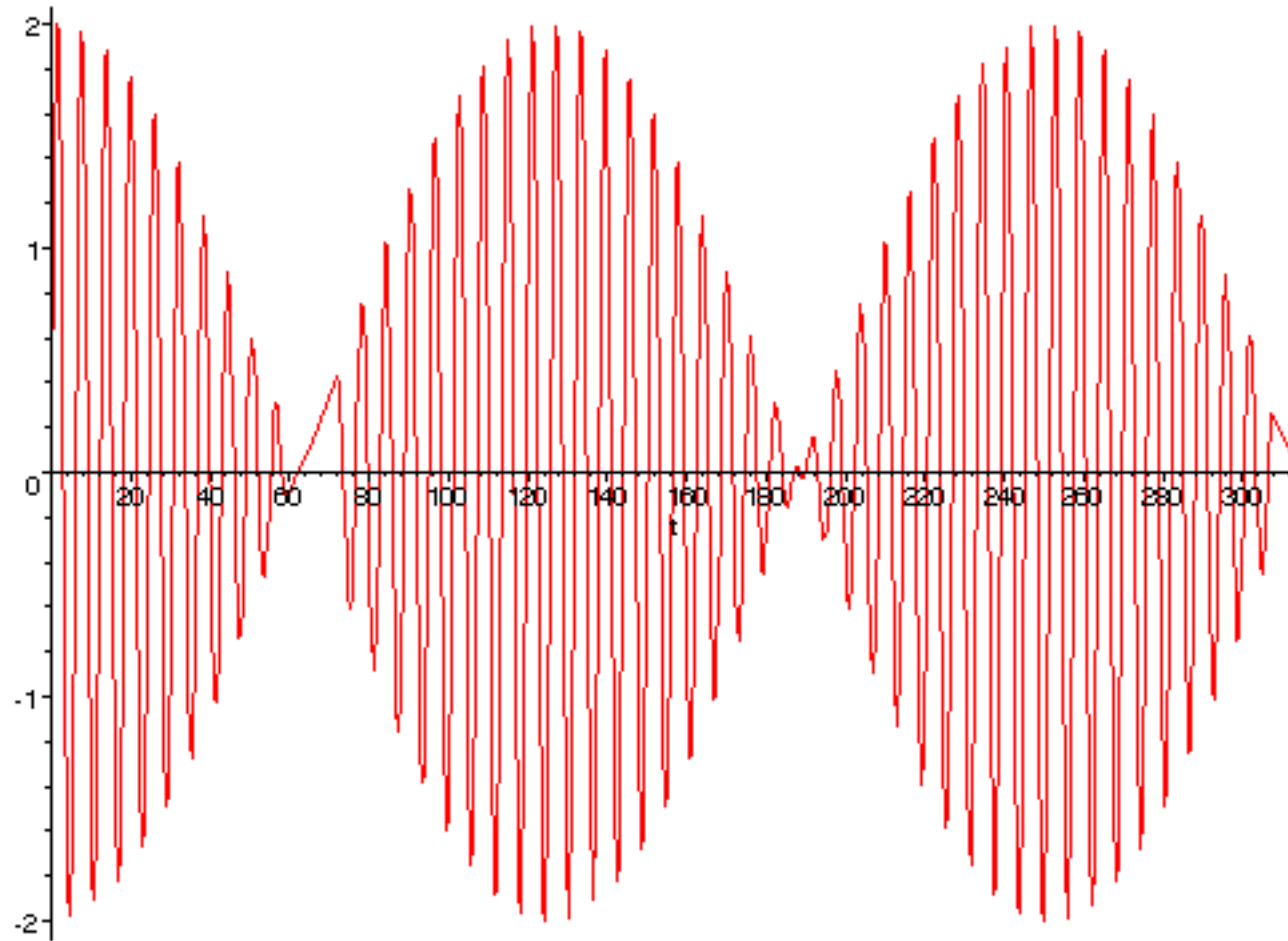
$$A \sin(a(t-t_0)) + B \sin(a(t-t_1)) = C \sin(a(t-t_2))$$

Addisjon av to faseforskøvede harmoniske bølger med samme frekvens resulterer i en ny harmonisk bølge med samme frekvens.

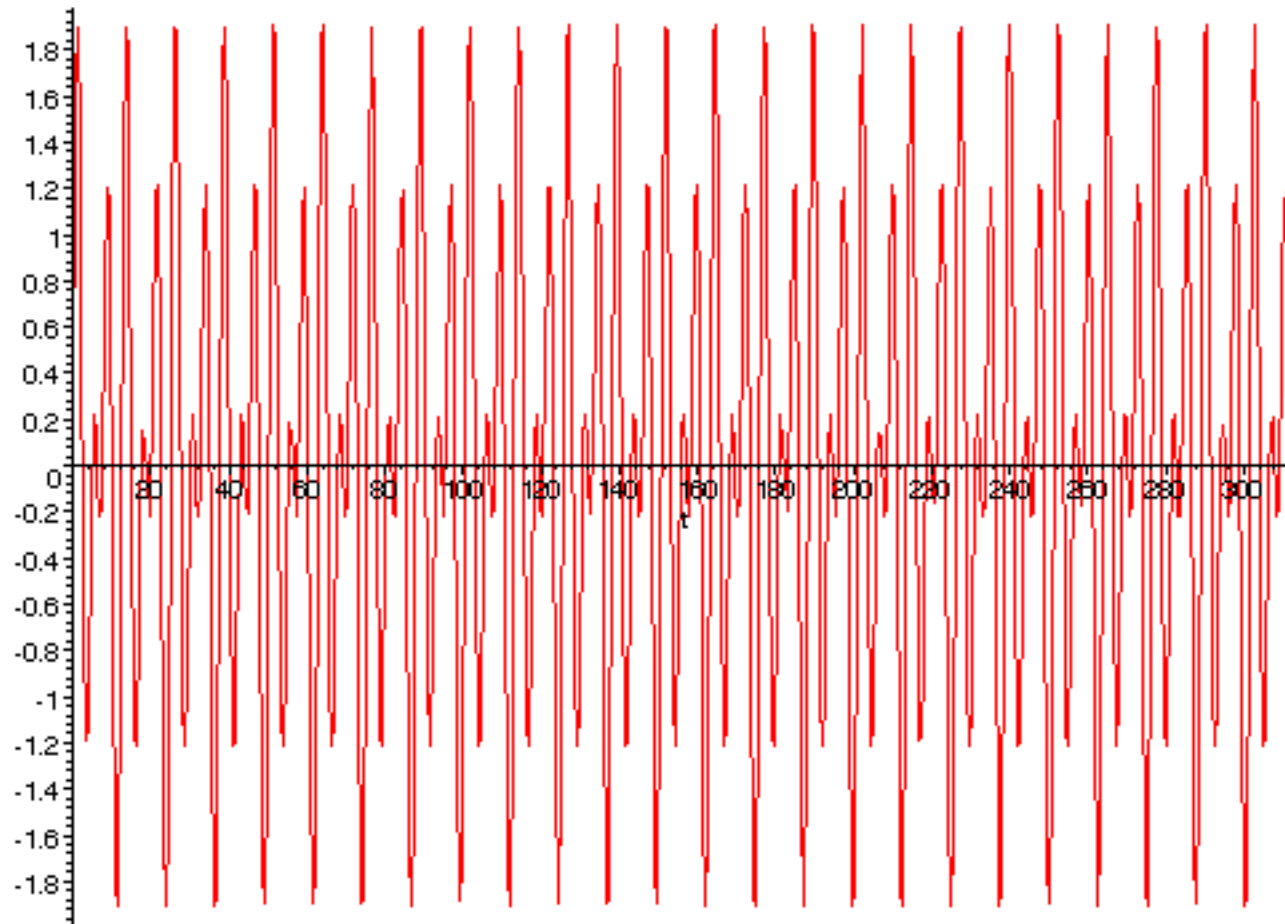


$$\sin(a t) + \sin(b t)$$

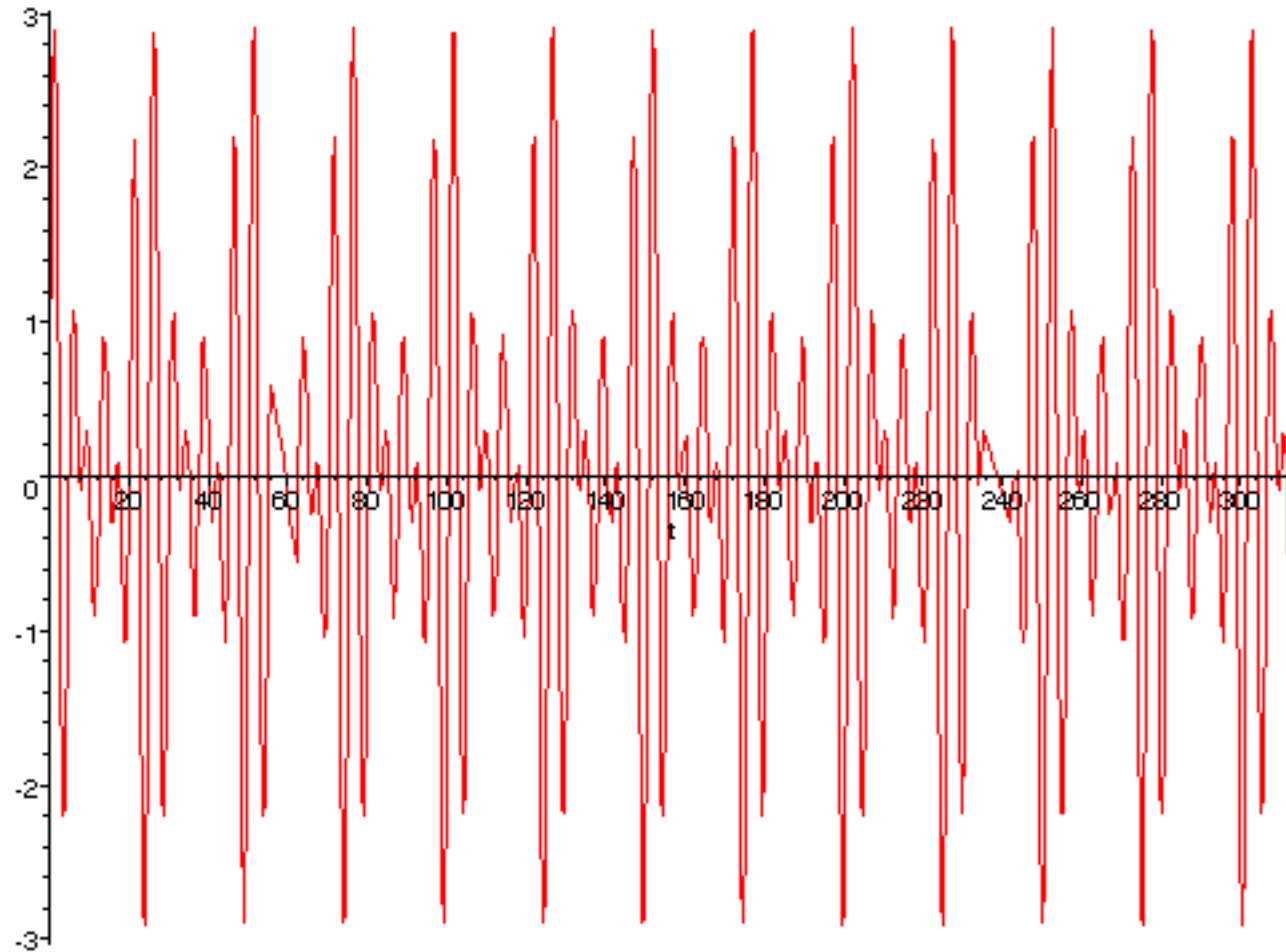
$$= 2 \sin\left(\frac{1}{2} (a + b) t\right) \cos\left(\frac{1}{2} (a - b) t\right)$$



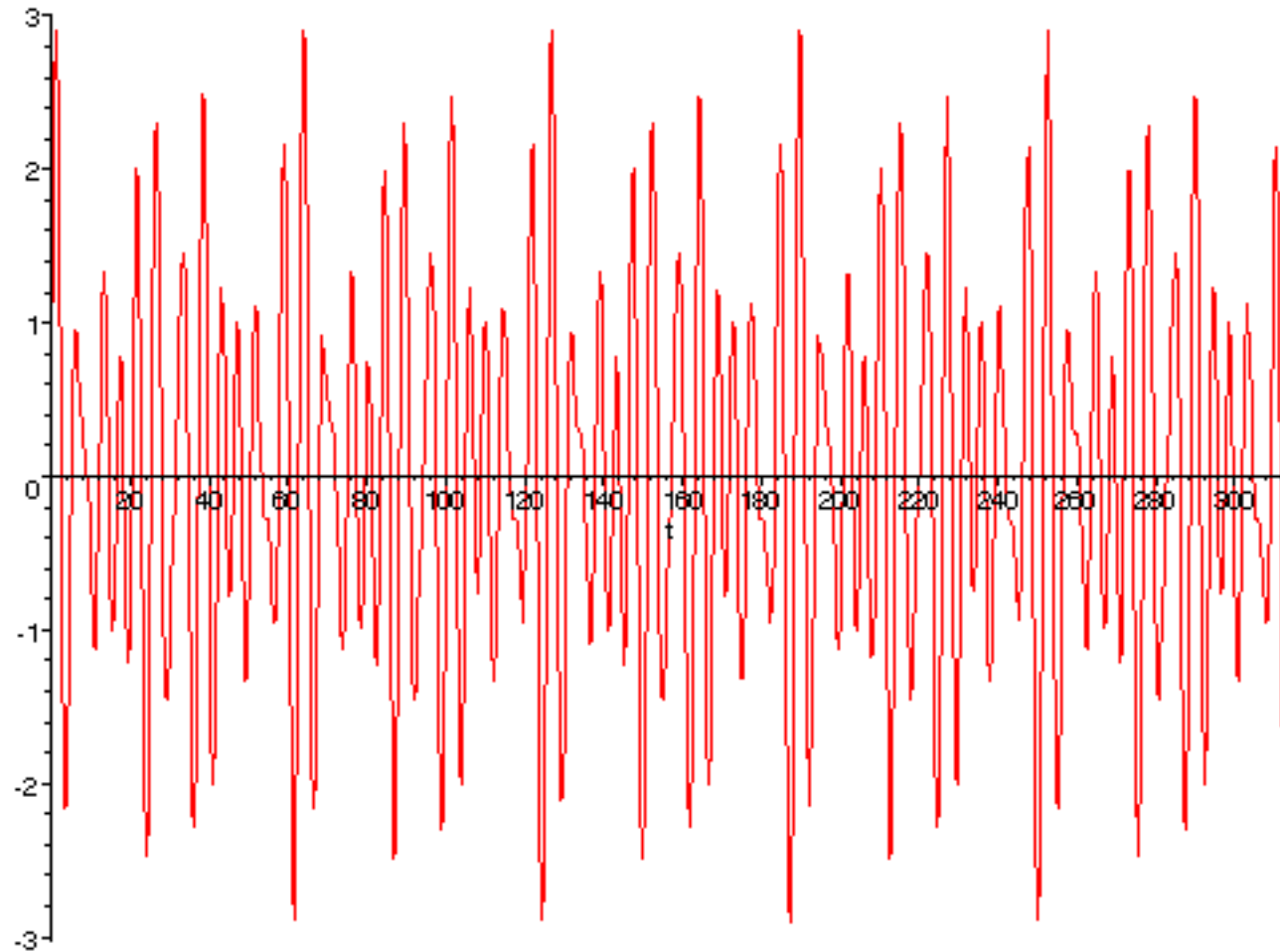
$$\sin(t) + \sin(1,05t)$$



$$\sin(t) + \sin(3/2 t)$$



$$\sin(t) + \sin(5/4 t) + \sin(3/2 t)$$



$$\sin(t) + \sin(6/5 t) + \sin(3/2 t)$$