

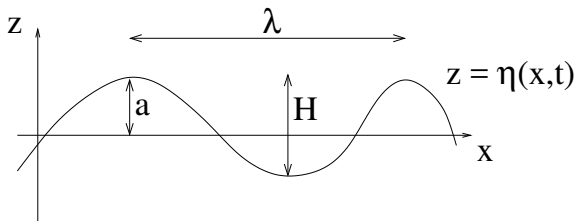
Ekstreme bølger over grunne og ekstreme bølgelaster på horisontal sylinder bak grunne

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16 August 2022

Bølger og statistikk



La η være en stokastisk prosess

Forventning $\mu = E[\eta]$, varians $\sigma^2 = E[(\eta - \mu)^2]$

Sjevhet $\gamma = \frac{E[(\eta - \mu)^3]}{\sigma^3}$, kurtosis $\kappa = \frac{E[(\eta - \mu)^4]}{\sigma^4}$

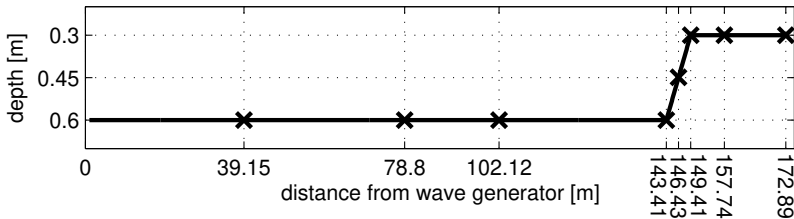
En Gaussisk variabel har skjevhet $\gamma = 0$ og kurtose $\kappa = 3$

Enkelt kriterium:

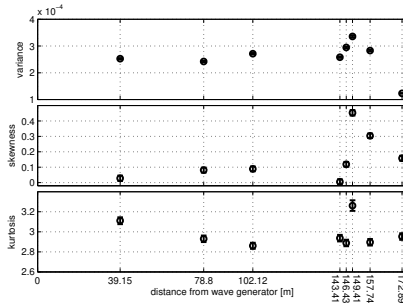
Høyere $\kappa \implies$ Høyere sannsynlighet for extreme/freaky bølger

Det startet med en konkurranse utlyst av MARIN 2010:

Hvem har den beste numeriske modellen for å simulere irregulære bølger inn på grunnere vann?



MARIN experiment: $k_p h = 0.81 \rightarrow 0.54$



PHYSICS OF FLUIDS 24, 097101 (2012)

Laboratory evidence of freak waves provoked by non-uniform bathymetry

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We show experimental evidence that as relatively long unidirectional waves propagate over a sloping bottom, from a deeper to a shallower domain, there can be a local maximum of kurtosis and skewness close to the shallower side of the slope. We also show evidence that the probability of large wave envelope has a local maximum near the shallower side of the slope. We therefore anticipate that the probability of freak waves can have a local maximum near the shallower side of a slope for relatively long unidirectional waves. © 2012 American Institute of Physics. [<http://dx.doi.org/10.1063/1.4748346>]

Lar seg reproducere med Boussinesq, men ikke med NLS?

PHYSICS OF FLUIDS 25, 122101 (2013)



Freak waves in weakly nonlinear unidirectional wave trains over a sloping bottom in shallow water

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Using a Boussinesq model with improved linear dispersion, we show numerical evidence that bottom non-uniformity can provoke significantly increased probability of freak waves as a wave field propagates into shallower water, in agreement with recent experimental results (K. Trulsen, H. Zeng, and O. Gramstad, "Laboratory evidence of freak waves provoked by non-uniform bathymetry," *Phys. Fluids* **24**, 097101 (2012)). Increased values of skewness, kurtosis, and probability of freak waves can be found on the shallower side of a bottom slope, with a maximum close to the end of the slope. The increased probability of freak waves is typically seen to endure some distance into the shallower domain, before it decreases and reaches a stable value depending on the depth. The maxima of the statistical parameters are observed both in the case where there is a region of constant depth after the slope, and in the case where the upshot slope is immediately followed by a downshot slope. In the case that waves propagate over a slope from shallower to deeper water, however, we do not find any increase in freak wave occurrence. © 2013 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4847055]

1. INTRODUCTION

Freak waves in waters of finite and shallow depth have been observed and have been reported to be responsible for a number of accidents.^{1–4} We shall here investigate if a bottom slope can provoke enhanced occurrence of freak waves in wave fields that propagate over non-uniform bathymetry. By freak waves we mean waves much larger than ambient waves, by a criterion similar to the classical one that the wave height is greater than twice the significant wave height H_s .⁵

We will consider the inhomogeneity of the medium, i.e., the non-uniform depth, as known *a priori*, and therefore we insist that the significant wave height H_s should be a function of space, and should not be computed by spatial averaging. The criterion for freak waves therefore becomes local.^{6,7} It has been anticipated that the freak wave probability in the coastal zone can be increased by bottom non-uniformity due to geometric focusing.⁸

Waves propagating over non-uniform depths experience a change of wavenumber, group velocity, amplitude, and height.^{9–11} The theory of linear refraction captures these changes subject to the dispersion relation $\omega^2 = gk \tanh kh$. Here, ω is the angular frequency, k the wavenumber, g the acceleration of gravity, and h the mean water depth. The nonrelativistic group velocity $v_g = (\partial \omega / \partial k) = (g/kh) \tanh kh + kh \cosh^2 kh$ is 1/2 at infinite depth, achieves a global maximum of approximately 0.6 at the depth where $k h = \cosh kh$, i.e., $kh \approx 1.2$, and decreases to zero as the depth goes to zero. In the shallow limit $kh \ll 1$, we have $v_g/v_g \approx kh$. In this limit, the amplitude of long-crested

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Natural Hazards
and Earth
System Sciences

Evolution of skewness and kurtosis of weakly nonlinear unidirectional waves over a sloping bottom

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Received: 28 October 2011 – Revised: 13 January 2012 – Accepted: 22 January 2012 – Published: 15 March 2012

Abstract. We consider the effect of slowly varying depth on the values of skewness and kurtosis of weakly nonlinear unidirectional waves propagating from deeper to shallower water. It is known that the equilibrium value of kurtosis decreases with decreasing depth for waves propagating on constant depth. Waves propagating over a sloping bottom must continually adjust toward a new equilibrium state. We demonstrate that weakly nonlinear waves may exhibit a considerable horizontal propagation distance in order to adjust to a new shallower environment, therefore the kurtosis can be notably different from the equilibrium value for each corresponding depth both on top of and beyond a bottom slope. A change of depth can provoke a wake-like spatially non-uniform distribution of kurtosis on the lee side of the slope. As an application, we anticipate that the probability of freak waves on or near the edge of the continental shelf may exhibit a rather complicated spatial structure for wave fields mixing from deep sea.

1 Introduction

As waves propagate from deeper to shallower water, their properties are transformed. Classical ray theory predicts slight shortening of wave length, with accompanying change of group velocity, amplitude and steepness, while the frequency is conserved for stationary conditions (Goda, 2010; Mei et al., 2005). The nonlinear stability of wave trains changes character. In water deeper than the critical depth $kh > 1.363$, a uniform wave train is modulationally unstable for narrow-band perturbations, while in water shallower than the critical depth $kh < 1.363$, long-crested waves become modulationally stable (Benjamin, 1967; Whitham, 1974), it being the wavenumber that corresponds to the depth h .

In deep water it is known that third-order nonlinearity can cause focusing of long-crested and narrow-banded waves, and can be responsible for the occurrence of freak waves

(Ostrov et al., 2001, 2005; Mori and Yasuda, 2002; Janssen, 2003; Mori and Janssen, 2006; Toffoli et al., 2008, 2009). As nonlinear modulation is reduced for long-crested waves on shallower depths, it may be anticipated that there is a corresponding reduction in the probability of freak waves (Mori and Janssen, 2006; Janssen and Ostrov, 2007; Toffoli et al., 2009; Janssen, 2009). When the depth is less than the critical depth $kh < 1.363$, the loss of modulational instability may lead to reduced probability of freak waves (Mori and Janssen, 2006).

Short-crested wave fields on deep water have near-Gaussian statistics despite the nonlinearity of the wave field (Ostrov et al., 2002; Serfaty-Juglar et al., 2005; Gramstad and Trulsen, 2007; Ostrov et al., 2009). In shallower water, however, Toffoli et al. (2009) suggested that when directional components are taken into account, the probability of freak waves is notably increased compared to long-crested waves. In this paper we do not consider the effect of short-crestedness.

In much shallower water the nonlinear dynamics are dominated by second-order nonlinearity. For this case Pelinovsky and Sergeeva (2006) studied the probability of freak waves with the Korteweg-de Vries equation, and found that the skewness and kurtosis was controlled by the Ursell number. In this paper we limit consideration to depths similar to or deeper than the critical value; third-order rather than second-order nonlinearity is important for nonlinear evolution. We do however account for second-order static nonlinear contributions in the reconstruction of the wave field.

A number of studies have been dedicated to nonlinear propagation of wave groups over a slope. Djordjević and Rodekopp (1978) discussed a solution for an envelope-kale soliton and the fusion of an envelope soliton moving over variable depth. Benlier et al. (2005) described how a wave packet propagating from deep to shallow water can be deformed in amplitude, and if the depth reaches the critical value, the packet can disperse. Fruesen and Grue (2007)

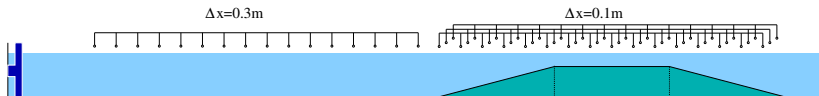
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Bølger over grunne

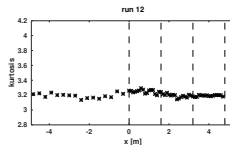
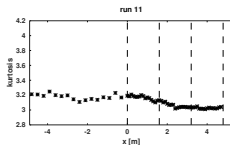
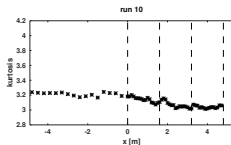
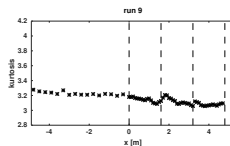
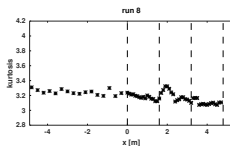
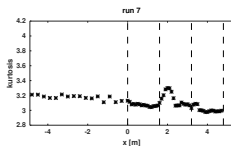
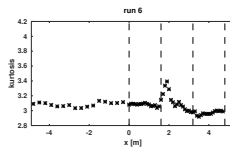
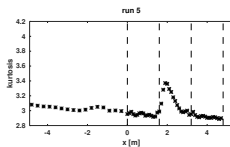
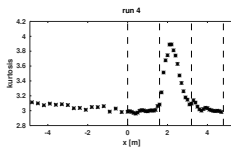
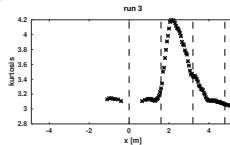
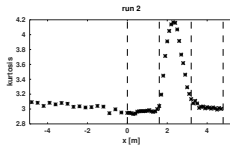
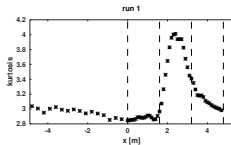
Deterministisk beskrivelse: Bølger inn på grunnere vann går langsommere og vokser i amplitude. Modulasjonsustabilitet blir vesentlig svakere.

Stokastisk beskrivelse: Sannsynligheten for freaky bølger over grunne kan bli vesentlig større enn for flat bunn.

Master oppgave til Anne Raustøl (2014):



Bølger over grunne — kurtose (Master oppgave til Anne Raustøl)

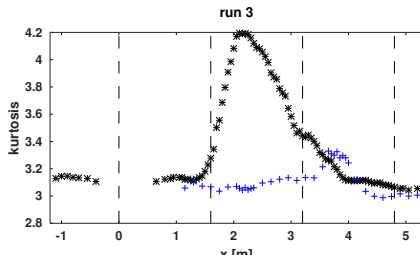
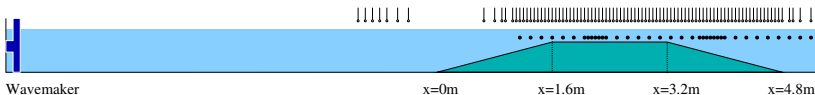


Overflatehevning og hastighetsfelt over grunne

Deterministisk beskrivelse: Overflatehevning og hastighetsfelt er “låst” til hverandre?

Stokastisk beskrivelse: Freake overflatebølger og freake hastigheter forekommer på forskjellige steder over grunna!

Master oppgave til Stian Jorde (2018):



Artikkel basert på arbeid til tre master studenter:

J. Fluid Mech. (2020), vol. 882, R2, doi:10.1017/jfm.2019.861

JFM RAPIDS
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Extreme wave statistics of long-crested irregular waves over a shoal

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We report laboratory experiments of long-crested irregular water surface waves propagating over a shoal. For a sufficiently shallow shoal we find that the surface elevation can have a local maximum of skewness and kurtosis above the shallower part of the shoal close to the edge on the incoming side, and a local minimum of skewness over the downward slope on the lee side of the shoal. We find that the horizontal fluid velocity can have a local maximum and minimum of skewness at the same locations as those for the surface elevation. However, the kurtosis of the

Det stemmer at dypere vann kompatibelt med NLS ikke skal gi slike fenomener:

Statistical properties of wave kinematics in long-crested irregular waves propagating over non-uniform bathymetry

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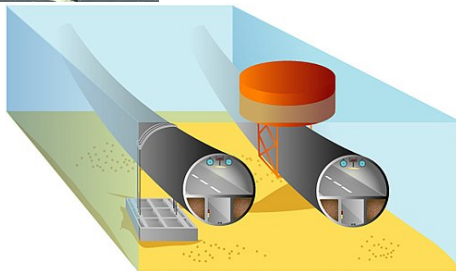
ABSTRACT

Experimental and numerical evidence have shown that nonuniform bathymetry may alter significantly the statistical properties of surface elevation in irregular wave fields. The probability of “rogue” waves is increased near the edge of the upslope as long-crested irregular waves propagate into shallower water. The present paper studies the statistics of wave kinematics in long-crested irregular waves propagating over a shoal with a Monte Carlo approach. High order spectral method is employed as wave propagation model, and variational Boussinesq model is employed to calculate wave kinematics. The statistics of horizontal fluid velocity can be different from statistics in surface elevation as the waves propagate over uneven bathymetry. We notice strongly non-Gaussian statistics when the depth changes abruptly in sufficiently shallow water. We find an increase in kurtosis in the horizontal velocity around the downslope area. Furthermore, the effects of the bottom slope with different incoming waves are discussed in terms of kurtosis and skewness. Finally, we investigate the evolution of kurtosis and skewness of the horizontal velocity over a sloping bottom in a deeper regime. The vertical variation of these statistical quantities is also presented.

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Submerged floating tunnel bridge — Archimedes bridge

= Rørbru = Flytetunnel



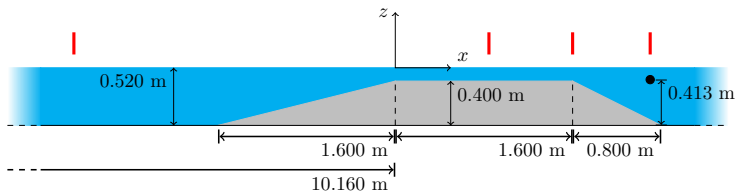
Aker Norwegian Contractors

Selmer



Krefter på horisontal sylinder bak grunne

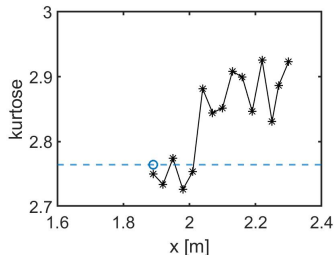
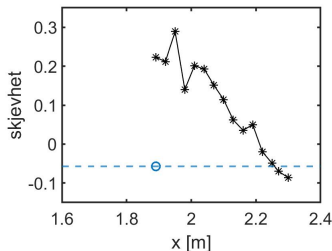
Master oppgave til Karen Samseth



Horizontal cylinder

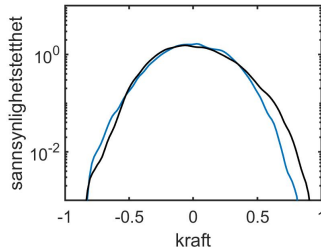
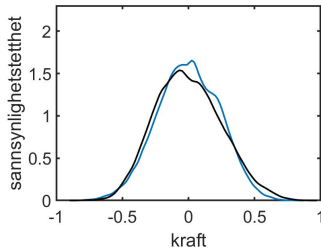


Statistikk avhenger av posisjon relativt til grunna



- Blå referanseverdi uten grunne
- Svart verdi med grunne ved ulike posisjoner relativt til grunna

Kreftene kan bli mer ekstreme som følge av grunna



- Blå referanseverdi uten grunne
- Svart verdi med grunne ved en posisjon relativt til grunna

En grunne kan forårsake:

- Flere freake overflatebølger
- Flere freake bølgehastigheter
- Flere freake krefter på horisontale rør

**Stor takk til Karen Samseth, Eva Charlotte Berner,
Christopher Lawrence, Odin Gramstad, Stian Jorde, Anne
Raustøl, Lisa Bæverfjord Rye, Huiming Zeng**

