

# Experimental investigation of the statistics of extreme waves and wave forces on top of and behind a shoal

Karsten Trulsen

and experimental work by

Karen Samseth, Eva Berner, Stian Jorde, Anne Raustøl, Lisa Rye

and numerical work by

Christopher Lawrence, Odin Gramstad, Geir Pedersen, Huiming Zeng



UiO : **Department of Mathematics**  
University of Oslo

Plymouth, 28 August 2024



## Invitation to Benchmark Workshop on Numerical Wave Modelling

Dear Colleague,

MARIN has been working on the modeling of wave-structure interaction for more than 75 years and is continuously aiming at improving its knowledge on waves. Therefore, we organize a workshop devoted to numerical modeling of waves. We invite you as an expert in this field to carry out some calculations on a few benchmark cases and to share your expertise in the workshop where we will compare the different methodologies, compare the calculated results to experimental data and discuss the possible differences.

The benchmark will consist of approximately 5 cases (laboratory measurements of long-crested waves) with variations in bottom topography and wave steepness. Some cases will be blind and for some cases the entire dataset will be provided to you. It is assumed that you can initialize your simulations with the measured motions of the wave generator or the measured wave elevation at a point in the basin. The workshop will then focus on the accuracy of the predicted waves at other points in the basin.

- Meet your colleagues to exchange and share ideas and insights
- Keep track of up-to-date developments in this field
- Establish networks



# The beginning of the adventure : 1 June 2010

Upon your agreement to participate in the workshop, we will provide you with the details of the laboratory measurements. Please let us know by **June 30<sup>th</sup>, 2010**, whether you will participate. Please send all correspondence to one of the MARIN organizers ([t.bunnik@marin.nl](mailto:t.bunnik@marin.nl), [c.schmittner@marin.nl](mailto:c.schmittner@marin.nl), or [j.hennig@marin.nl](mailto:j.hennig@marin.nl)).

- We expect the results of the calculations before **November 5<sup>th</sup> 2010**; MARIN will collect all data and will make the comparison with the laboratory data. The results of this comparison will be provided to you before the actual workshop.
- The workshop will focus on the details of the calculations and on the analysis of the different results. The date for this event will be **November 25<sup>th</sup> 2010**; the location will be at the MARIN office in Wageningen, the Netherlands.

This workshop is being organized for the following reasons:

- To better understand the physics of wave propagation
- To investigate statistics of extreme waves and wave groups
- To investigate basin effects
  - To design deterministic wave events in our basins a priori
  - To have an input to CFD models for wave-structure interaction
- To investigate low-frequency wave energy in coastal areas, which affect moored structures like LNG carriers.

An article in a magazine will be made giving an overview of the results of the calculations and the final conclusions.

We trust this workshop is on a very challenging topic, and we think the workshop can provide all of the participants with very valuable insights and ideas for future developments in this field. We look forward to your positive response to participate and contribute to this workshop!

Yours sincerely,

Prof. Dr. René Huijsmans, TU Delft

Dr. Tim Bunnik, MARIN

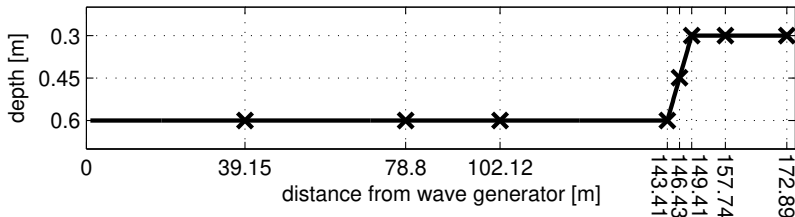
Dr. Christian Schmittner, MARIN

Dr. Janou Hennig, MARIN



[www.marin.nl](http://www.marin.nl)

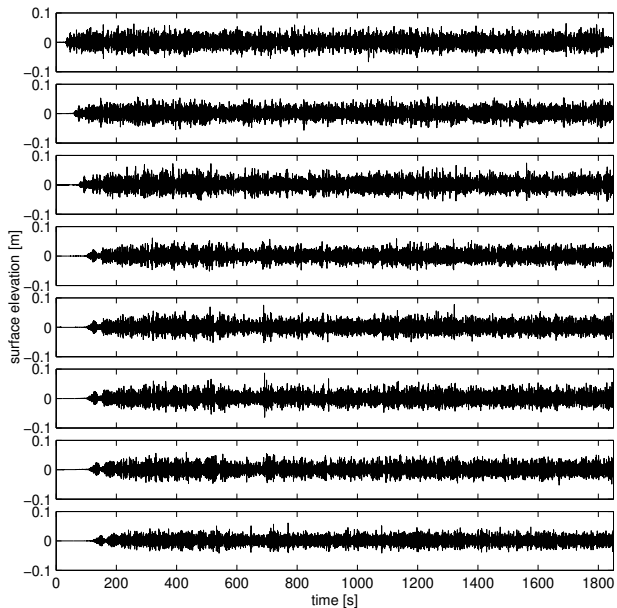
## Bottom profile and location of wave probes



Challenge: Initialize with time series at first location, predict time series at subsequent locations.

We did not participate in the Benchmark Workshop,  
but we found some very interesting behavior in the data ...

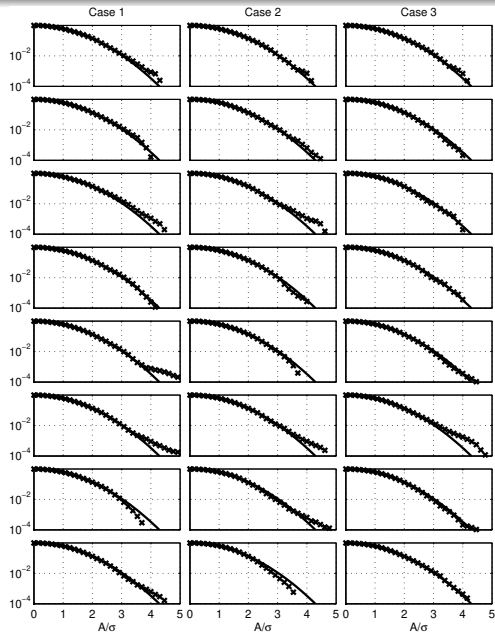
# Experiment at Marin 2010 — Time series



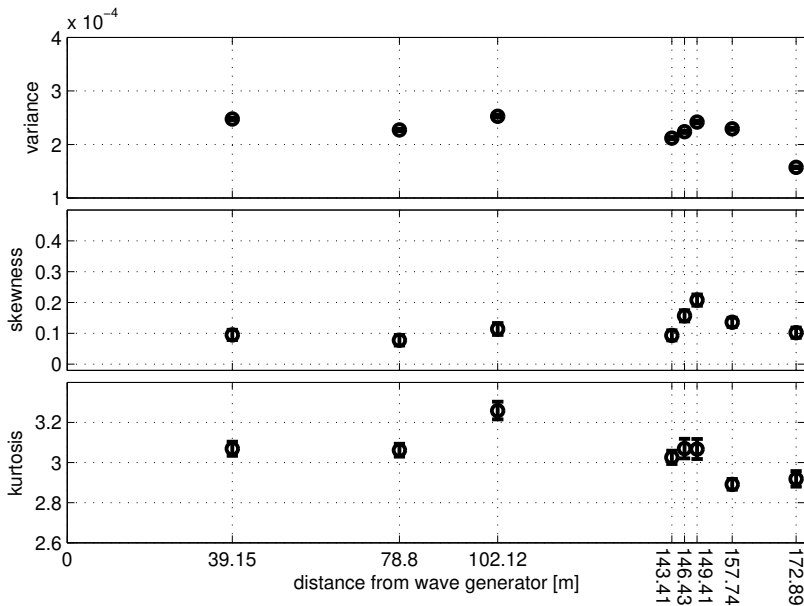
# Experiment at Marin 2010 — The three cases

| Case | $H_s$ [m] | $T_p$ [s] | deeper side |           |       | shallower side |           |       |
|------|-----------|-----------|-------------|-----------|-------|----------------|-----------|-------|
|      |           |           | $k_p h$     | $k_p a_c$ | $Ur$  | $k_p h$        | $k_p a_c$ | $Ur$  |
| 1    | 0.06      | 1.273     | 1.6         | 0.057     | 0.014 | 0.99           | 0.070     | 0.072 |
| 2    | 0.06      | 1.697     | 1.1         | 0.038     | 0.029 | 0.70           | 0.049     | 0.14  |
| 3    | 0.06      | 2.121     | 0.81        | 0.028     | 0.053 | 0.54           | 0.038     | 0.24  |

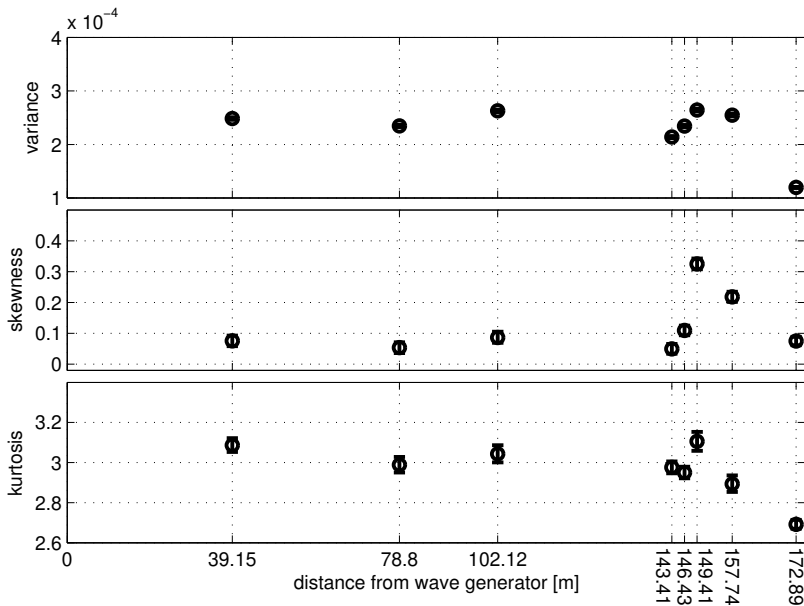
# Experiment at Marin 2010 — Exceedance probability



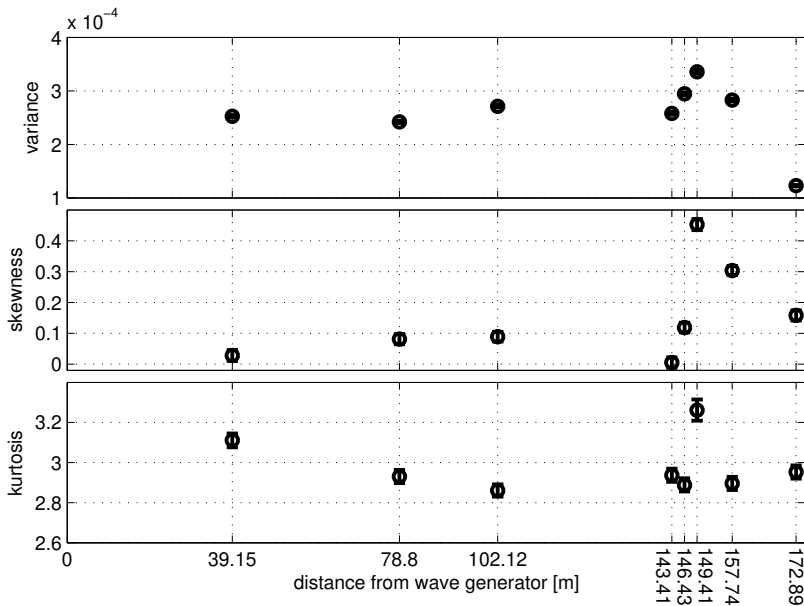
# Experiment at Marin 2010 — Statistics case 1



# (10) Experiment at Marin 2010 — Statistics case 2



# Experiment at Marin 2010 — Statistics case 3

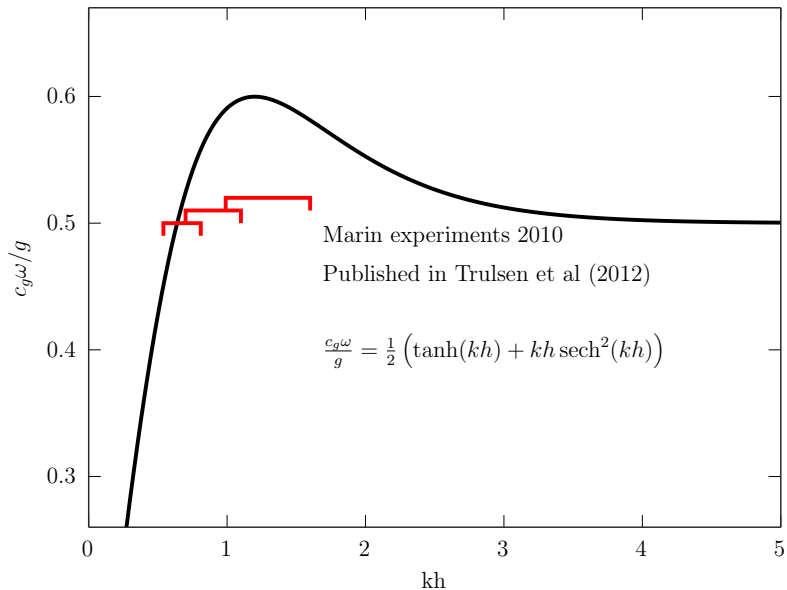


**First surprise! The skewness and kurtosis of surface elevation has a local maximum at the front end of the shoal — significantly more extreme than Gaussian statistics!**

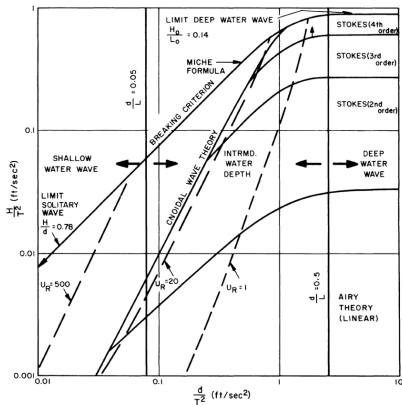
published in

Trulsen, Zeng & Gramstad (2012) “Laboratory evidence of freak waves provoked by non-uniform bathymetry” Phys. Fluids **24**, 097101.

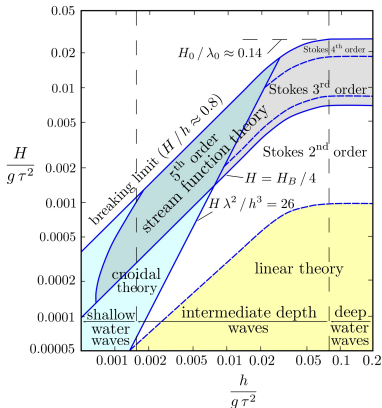
# Depends only on depth?



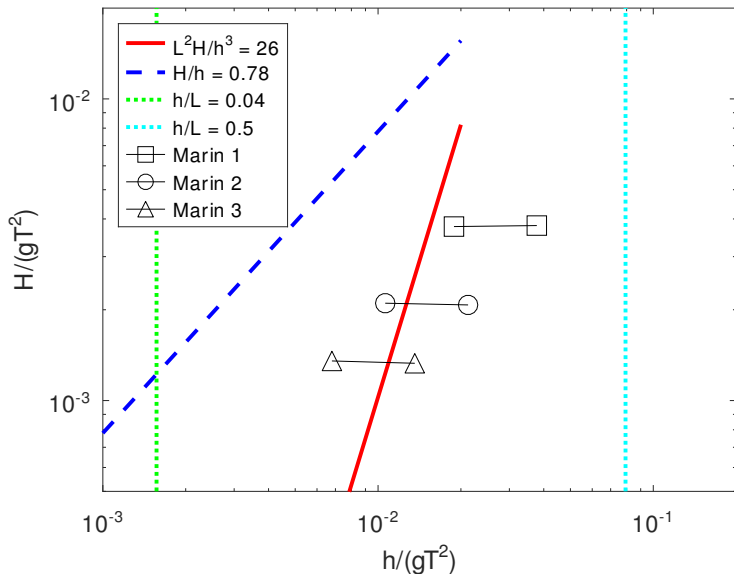
# Try the Le Méhauté diagram?



(stolen from the Internet)



# Depends on both depth and amplitude?



# Had this been discovered before? Almost, but not quite.

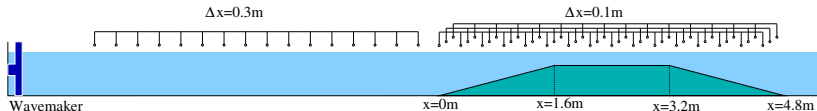
Field experimental observations that statistics deviates from Gaussian when waves enter shallower water:

- Bitner (1980)
- Cherneva et al. (2005)

Amplification of bound harmonics when waves enter shallower water:

- Grue (1992)
- Beji & Battjes (1993, 1994)

# We need more experiments!



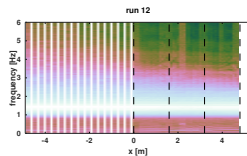
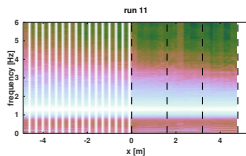
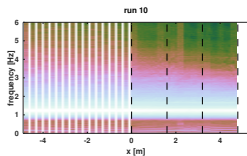
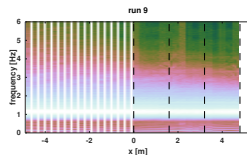
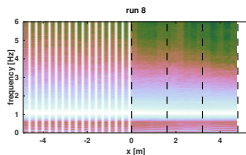
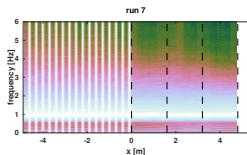
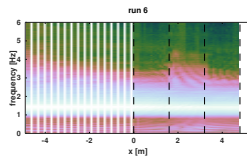
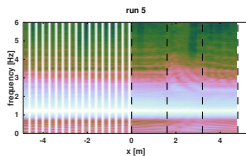
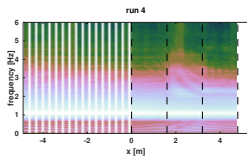
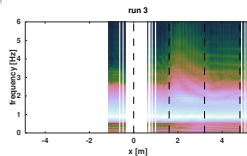
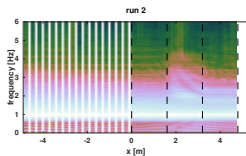
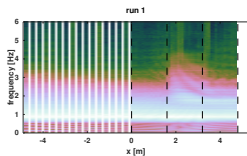
- 2013–2014: Anne Raustøl — master thesis  
“Freak waves over variable depth”
- 2013–2014: Lisa Bæverfjord Rye — master thesis  
“Freak waves in crossing sea with counterpropagating wave systems”



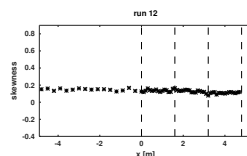
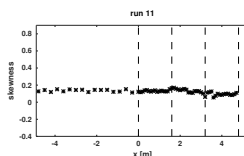
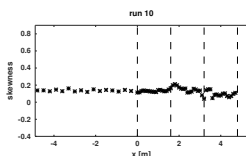
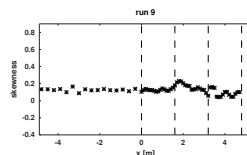
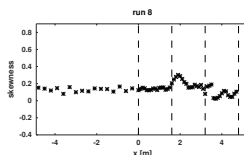
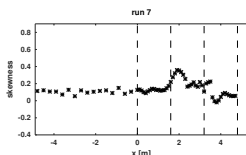
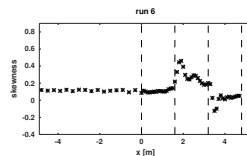
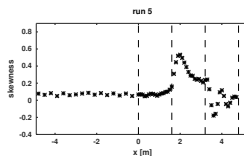
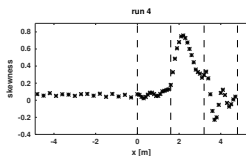
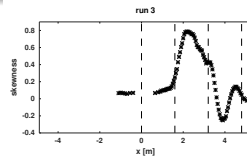
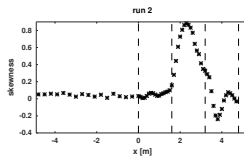
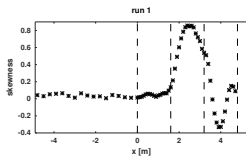
# Experiments of Anne Raustøl

| Run | $T_p$ [s] | deeper side |         |            | shallower side |         |            |
|-----|-----------|-------------|---------|------------|----------------|---------|------------|
|     |           | $h_1$ [cm]  | $k_p h$ | $H_s$ [cm] | $h_2$ [cm]     | $k_p h$ | $H_s$ [cm] |
| 1   | 1.1       | 50          | 1.8     | 1.4        | 8              | 0.54    | 1.5        |
| 2   | 1.0       | 50          | 2.1     | 1.6        | 8              | 0.60    | 1.6        |
| 3   | 1.1       | 53          | 2.5     | 2.5        | 11             | 0.65    | 2.5        |
| 4   | 0.9       | 50          | 2.5     | 1.7        | 8              | 0.68    | 1.6        |
| 5   | 0.8       | 50          | 3.2     | 1.7        | 8              | 0.78    | 1.5        |
| 6   | 0.7       | 50          | 4.1     | 1.7        | 8              | 0.91    | 1.4        |
| 7   | 1.0       | 60          | 2.5     | 3.4        | 18             | 0.97    | 3.1        |
| 8   | 0.9       | 60          | 3.0     | 3.2        | 18             | 1.1     | 2.9        |
| 9   | 0.85      | 60          | 3.4     | 2.7        | 18             | 1.2     | 2.5        |
| 10  | 0.8       | 60          | 3.8     | 2.6        | 18             | 1.3     | 2.3        |
| 11  | 0.75      | 60          | 4.3     | 2.4        | 18             | 1.4     | 2.1        |
| 12  | 0.7       | 60          | 4.9     | 2.1        | 18             | 1.6     | 1.8        |

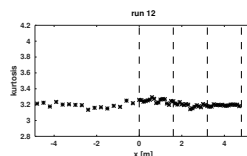
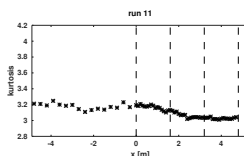
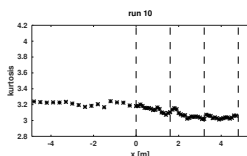
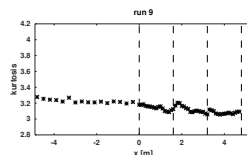
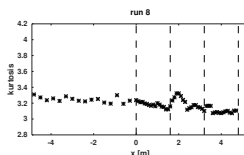
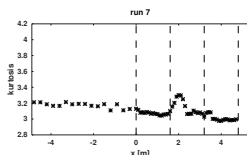
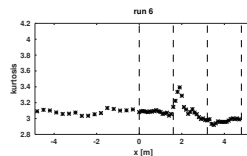
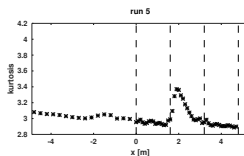
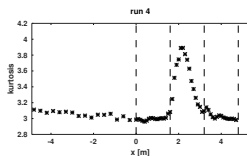
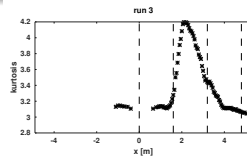
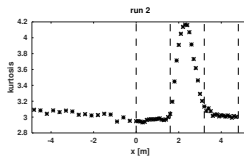
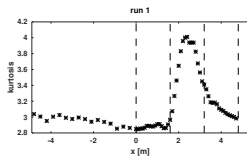
# Experiments of Anne Raustøl: spectrum



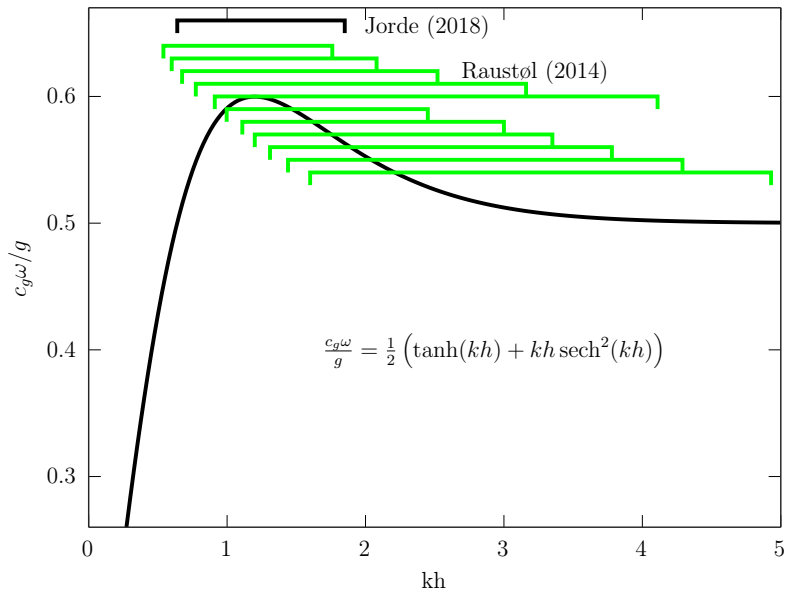
# (20) Experiments of Anne Raustøl : skewness



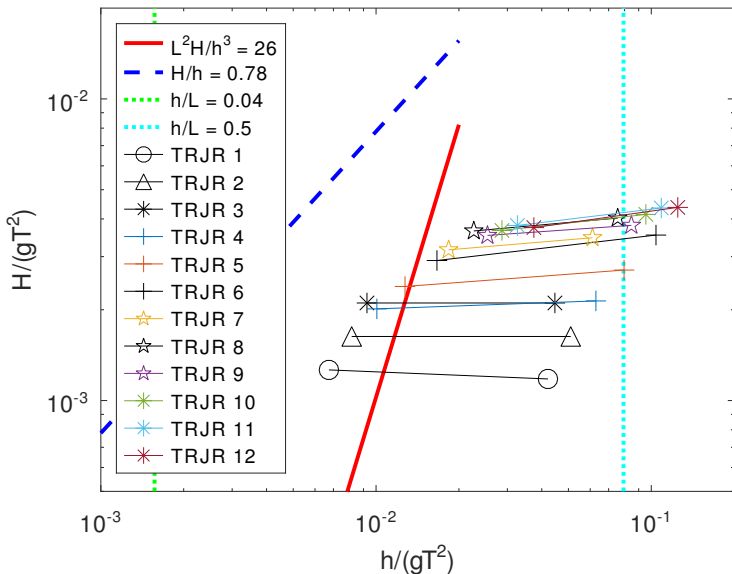
# Experiments of Anne Raustøl : kurtosis



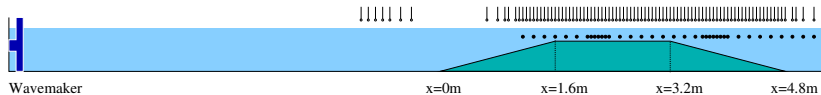
# Depends only on depth?



# Depends on both depth and amplitude?



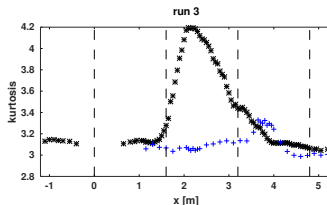
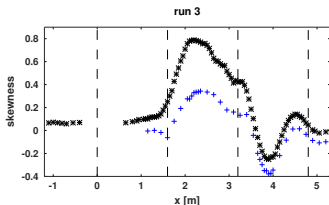
# How about wave kinematics?



- 2018–2018: Stian Jorde — master thesis  
“Kinematics of waves over a shoal”

# Experiments of Stian Jorde : skewness and kurtosis

## Comparison of surface elevation and horizontal velocity



Black : surface elevation  
Blue : horizontal velocity

**Second surprise! The kurtosis of surface elevation and horizontal velocity are completely different! The velocity field has maximum kurtosis behind the shoal!**

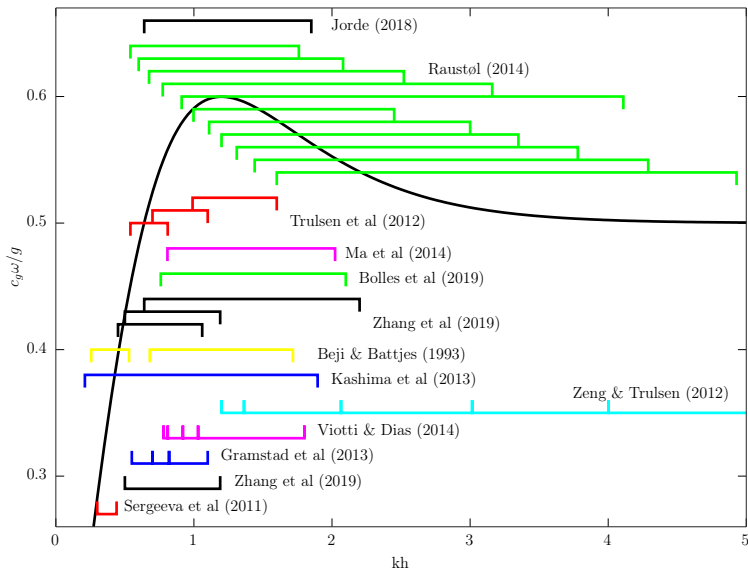
# Experiments of Anne Raustøl and Stian Jorde

Published in

Trulsen, Raustøl, Jorde & Rye (2020) “Extreme wave statistics of longcrested irregular waves over a shoal” J. Fluid Mech. **882**, R2.

Data published together with paper — downloaded many times and used by many in new publications

Many publications discussed this anomaly of surface elevation — here are some of the papers by 2020



# How well is this phenomenon captured numerically?

Reproduced by Korteweg–de Vries equation

- Sergeeva, Pelinovsky & Talipova (2011)

Reproduced by Boussinesq equation, e.g.

- Gramstad, Zeng, Trulsen & Pedersen (2013)

Reproduced by fully nonlinear methods etc. by many, many, e.g.

- Lawrence, Trulsen & Gramstad (2021)

**However, not reproduced by nonlinear Schrödinger equations for variable depth???**

e.g. variable depth NLS eq. by Djordjević & Redekopp (1978)?

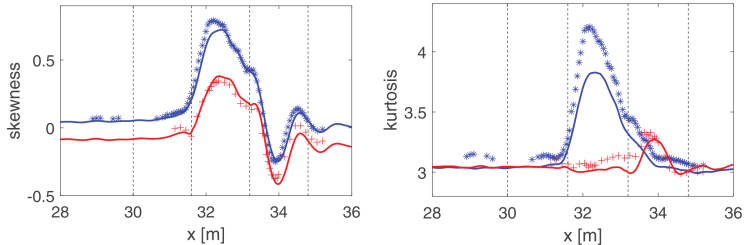
- Zeng & Trulsen (2012)

- 2008–2012: Huiming Zeng — Ph.D. thesis  
“Evolution of statistics of weakly nonlinear unidirectional waves over a sloping bottom”
- 2018–2021: Christopher Lawrence — Ph.D. thesis  
“Extreme wave statistics of surface gravity waves over bathymetry”



# <sup>(30)</sup> Numerical results of Christopher Lawrence 2021

He reproduced numerically the results in  
Trulsen, Raustøl, Jorde & Rye (2020)

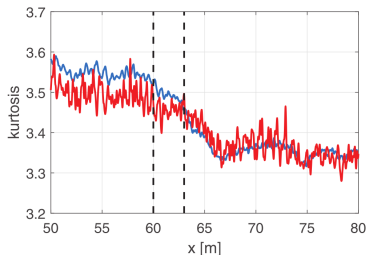
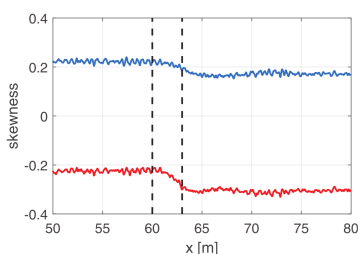


Red horizontal velocity  
Blue surface elevation  
Solid lines numerical simulation  
Symbols experiment of Stian Jorde

Published in Lawrence, Trulsen & Gramstad (2021)

# Numerical results of Christopher Lawrence 2021

He also reproduced the result of the NLS equation in Zeng & Trulsen (2012) with his more accurate numerical method



Red horizontal velocity

Blue surface elevation

The “opposite” result with nonlinear Schrödinger equation by Zeng & Trulsen (2012) appears to be correct!

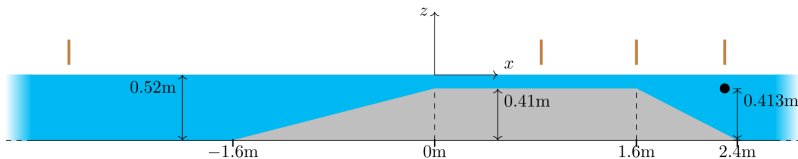
Published in Lawrence, Trulsen & Gramstad (2021)

# Conclusion from Christopher Lawrence

- Indeed, there appears to be two “regimes”:  
a deeper regime and a shallower regime.
- The extreme intermittent response in the statistics of surface elevation is produced as the wave field transitions from the deeper to the shallower regime as it enters the shoal, and back again as it leaves the shoal.
- The NLS framework is limited to the deeper regime?
- Fully nonlinear methods capture transition between regimes.
- Previously, we had seen that the Boussinesq and the Korteweg–de Vries frameworks also can capture the transition between the regimes.

# We wanted more experiments, especially for the kinematics

(But in the mean time the old symmetric shoal made of plywood had decayed so we made a new asymmetric one of Plexiglas)

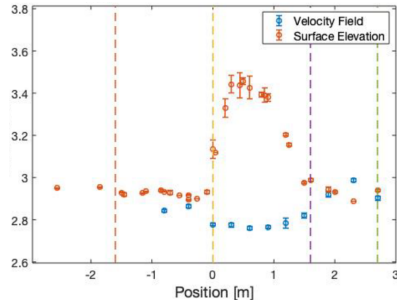
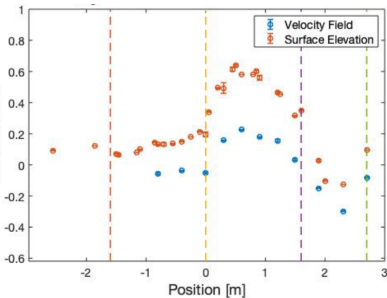
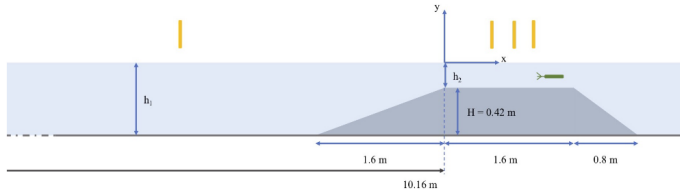


- 2020–2022: Eva Charlotte Berner — master student  
“A Statistical Analysis of Rouge Waves”
- 2021–2022 : Karen Samseth — master student  
“Freak forces on cylinder behind a shoal”

# Karen Samseth and Eva Charlotte Berner cleaning the tank



# Experiments of Eva Charlotte Berner : setup 1

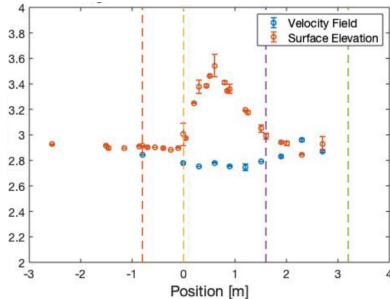
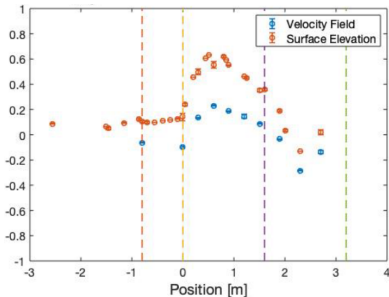
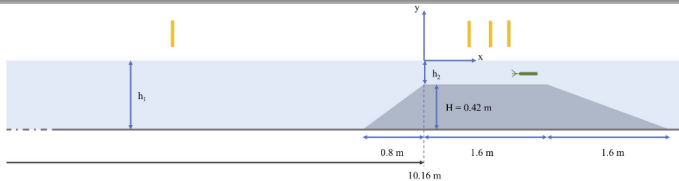


Black: surface elevation

Red: horizontal velocity

Dashed lines: end of sections of the shoal

# Experiments of Eva Charlotte Berner : setup 2

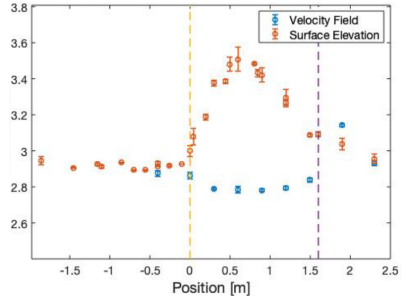
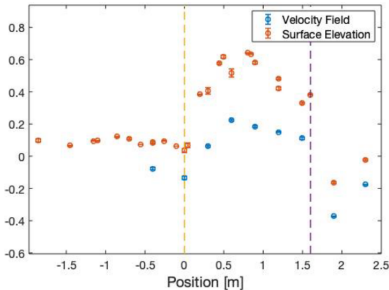


Black: surface elevation

Red: horizontal velocity

Dashed lines: end of sections of the shoal

# Experiments of Eva Charlotte Berner : setup 3



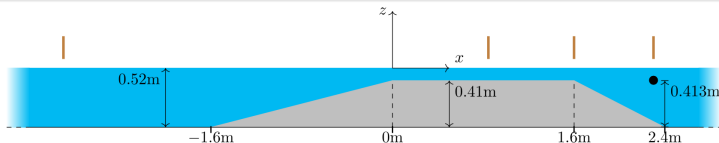
Black: surface elevation

Red: horizontal velocity

Dashed lines: end of sections of the shoal

- Additional experimental evidence confirming that the kurtosis of surface elevation and horizontal velocity are completely different
- The velocity field has maximum kurtosis behind the shoal
- With all three configurations

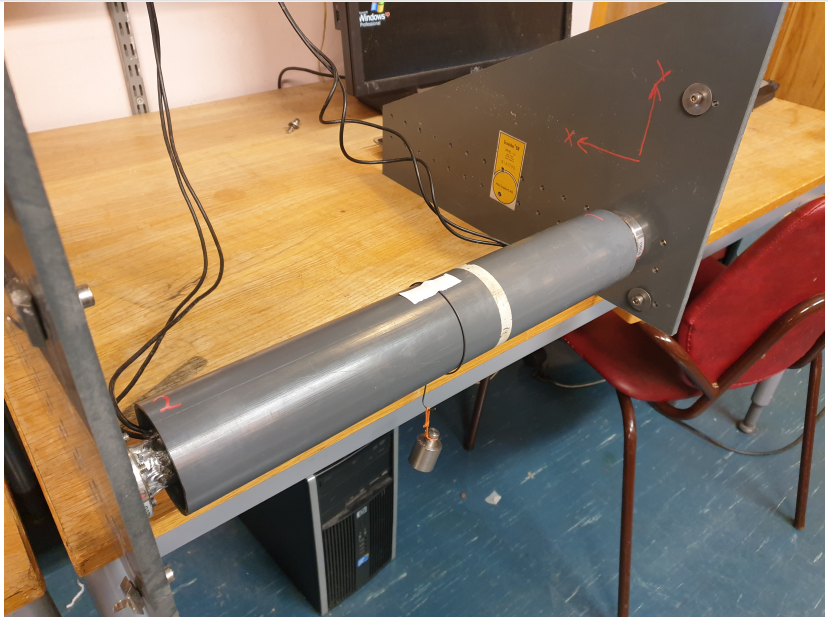
# Does the maximum of kurtosis of velocity have implications for the force experienced by a horizontal cylinder?



- 2021–2022 : Karen Samseth — master student  
“Freak forces on cylinder behind a shoal”
- 2023–2027 : Karen Samseth — Ph.D. student



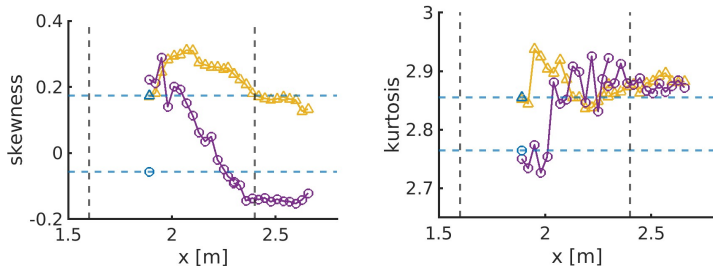
## (40) Horizontal cylinder



# Karen Samseth with horizontal cylinder in tank



# Skewness and kurtosis of forces over down slope

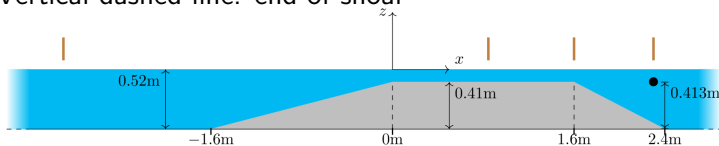


Purple circles: horizontal forces.

Yellow triangles: vertical forces

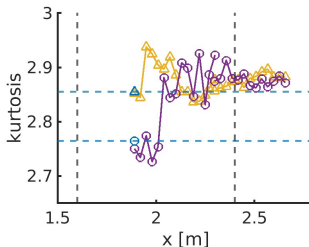
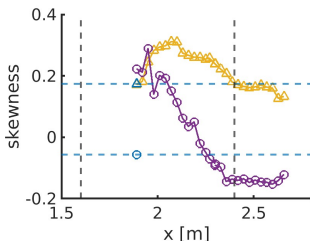
Blue: measurements without shoal

Vertical dashed line: end of shoal

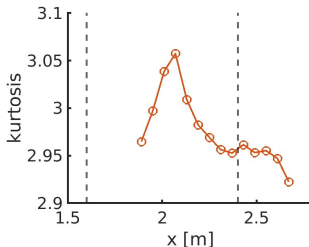
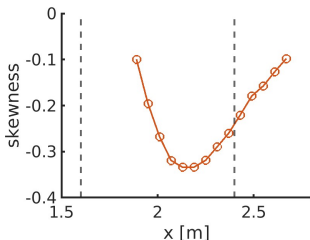


# Skewness and kurtosis of forces vs. velocity

Forces:



Horizontal velocity:



**Third surprise! Skewness and kurtosis of force are different from those of surface elevation and velocity.**

We would have liked to measure forces on the horizontal cylinder on top of the shoal

however, it does not fit,

the water is too shallow while the cylinder is too thick

# Conclusion of Karen Samseth

- Surface elevation: Maximum of skewness and kurtosis at front of plateau.
- Velocity: Maximum skewness at front of plateau, maximum kurtosis at lee side of shoal.
- Forces: Skewness and kurtosis are different from those of surface elevation and velocity.

# Overall conclusion for extreme wave statistics over a shoal — summary of all our work done since 2010

As an irregular wave field propagates over a shoal:

- Surface elevation can have local maximum of skewness and kurtosis at the front of the shoal
- Velocity field can have local maximum of skewness at the front of the shoal, while it can have local maximum of kurtosis behind the shoal
- Forces on a horizontal cylinder appear to have yet a different behavior behind the shoal
- All depending on the wave field transitioning into a sufficiently shallow regime on top of the shoal, and back again behind the shoal

# Explain why this happens

We suspect the explanation must have to do with a transition between two wave regimes, nicely captured by the Le Méhauté diagram

Le Méhauté diagram first use in this context by Zhang & Benoit (2021)?

Explanation by Li, Zheng, et al. (2021), Li, Draycott, Adcock, et al. (2021), Li, Draycott, Zheng, et al. (2021):  
New free waves generated at the front end of the shoal

Other explanations have also been proposed ...

# Many investigators have followed up on this topic

Number of citations of some papers:

- Trulsen, Zeng & Gramstad (2012) : 120
- Trulsen, Raustøl, Jorde & Rye (2020) : 92
- Zeng & Trulsen (2012) : 86
- Gramstad, Zeng, Trulsen & Pedersen (2013) : 75
- Lawrence, Trulsen & Gramstad (2021) : 31
- etc.

I was not able to describe all of those citing papers in this talk!

# Thanks to

Master students: Anne Raustøl, Lisa Rye, Stian Jorde, Karen Samseth, Eva Charlotte Berner

Ph.D. students: Christopher Lawrence, Huiming Zeng, Karen Samseth

Colleagues: Odin Gramstad, Geir Kleivstul Pedersen

Laboratory support : Olav Gundersen, Atle Jensen



UiO : **Department of Mathematics**  
University of Oslo