

Long master thesis fall 2025 – spring 2027 for

MARINE LECOMTE

Can slow variations in heart rhythm cause net transport in perivascular space?

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The question to be investigated

Can nonlinear modulations of the heart rhythm, either through radiation stress or similar mechanisms, induce mean flows in peri/para-vascular spaces? Can this explain the transport of waste products from the brain? Construct the simplest possible model describing such a mechanism, taking inspiration from nonlinear models for ocean waves.

Radiation stress and induced flow in ocean waves

Modulation of ocean waves can induce mean flow by quadratic nonlinear interactions. This is related to radiation stress as described on [Wikipedia: Radiation stress](#):

In fluid dynamics, the radiation stress is the depth-integrated – and thereafter phase-averaged – excess momentum flux caused by the presence of the surface gravity waves, which is exerted on the mean flow.

A typical situation is shown in figure 1 where modulations in a train of surface waves cause both a lowering of the mean water level under a modulated group and an induced flux of mass and momentum. Another typical situation is shown in figure 2 where radiation stress is used as explanation for how beaches are formed.

Radiation stress is a rather complicated concept. It may be easier to derive the resulting induced flow directly by perturbation expansion up to quadratic nonlinear order. The induced flow shown in figure 1 will then typically have a velocity potential $\bar{\phi}$ given by a system of equations that look similar to the following

$$\frac{\partial \bar{\phi}}{\partial z} = \frac{1}{2} \frac{\partial |B|^2}{\partial x} \quad \text{at } z = \bar{\zeta}, \quad (1)$$

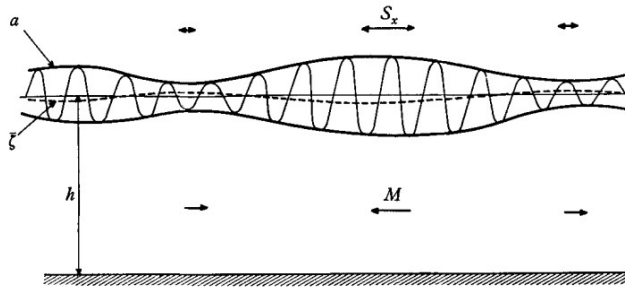


FIGURE 1. The effect of the radiation stress in depressing the mean level in a group of high waves.

Figure 1: Reproduction of figure 1 of Longuet-Higgins & Stewart (1962).

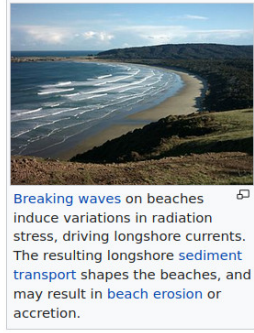


Figure 2:
Reproduction
of figure from
Wikipedia: Radiation stress.

$$\frac{\partial^2 \bar{\phi}}{\partial x^2} + \frac{\partial^2 \bar{\phi}}{\partial y^2} + \frac{\partial^2 \bar{\phi}}{\partial z^2} = 0 \quad \text{for} \quad -h < z < \bar{\zeta}, \quad (2)$$

$$\frac{\partial \bar{\phi}}{\partial z} = 0 \quad \text{at} \quad z = -h, \quad (3)$$

where B is the complex envelope (the bold solid curve) of the waves, and $\bar{\zeta}$ is the set-down of the mean surface (the dashed curve) shown in figure 1. Therefore we understand that the inhomogeneities in the amplitude of the waves, $\partial|B|/\partial x$, drive the induced flow.

Wave propagation in a viscous liquid in an elastic tube

In the textbook that is sometimes used in MEK3230, Kundu & Cohen (2008), chapter 17 is dedicated to “Biofluidmechanics”. A separate section is dedicated to wave propagation of a viscous liquid in an elastic cylindrical tube. Notice that figure 17.12 shows how an artery is built as two concentric tubes. All of these considerations are linear.

Mechanism for cleaning waste from the brain

Recently, theories have been proposed about how the brain is cleaned by means of convection in so-called peri/para-vascular spaces (Ilf et al., 2012). The convection is assumed to be driven by the pulse pressure in the same manner as the blood flow, however, this has been refused in several other studies, e.g. Daversin-Catty et al. (2020). A newer hypothesis is that it is driven by variations in the pulse pressure, e.g. van Veluw et al. (2020).

Recent master thesis

A recent master thesis by Ingvild Larsen (2021) entitled “Frie deformasjonsbølger i blodårer” <https://www.duo.uio.no/handle/10852/89867> dealt with the same topic, but the results were rather inconclusive.

Problem to be solved in this master thesis

In this thesis we start with the linear considerations in Kundu & Cohen (2008), and the preliminary investigations of Ingvild Larsen (2021), and extend the considerations to quadratic nonlinear effects. The goal is to describe pulsating propagation of a liquid in an elastic cylindrical tube, including the geometry of an artery consisting of two concentric tubes where the blood flows in the inner tube, and investigate if a different liquid in the outer tube can be convected as a consequence of the non-uniform pulsating flow in the inner tube.

References

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