

The group line in wavenumber-frequency spectra of water waves

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The front page depicts a section of the root system of the exceptional Lie group E_8 , projected into the plane. Lie groups were invented by the Norwegian mathematician Sophus Lie (1842–1899) to express symmetries in differential equations and today they play a central role in various parts of mathematics.

Abstract

For a long time, the radar community has been wondering what the causes for this feature called the group line are. The group line is a feature that appears in the frequency area close to zero in the (k, ω) spectrum. In this thesis, we aim to do hydrodynamic experiments of long-crested, breaking and non-breaking waves and non-linear simulations of long-crested, non-breaking waves in order to find out more about this feature and its properties. We will primarily be using the JONSWAP spectrum and the Pierson-Moskowitz spectrum to make the waves in the experiments and the simulations. Including the non-breaking runs done, the results show the group line in all of our results. This indicates that wave breaking alone is not the cause of the group line. In addition, we recognise that there are two group lines in some cases and that they are seemingly dependent on the steepness of the waves. This double group line appears when the waves seem to almost be breaking which makes us inclined to believe that wave breaking might be the factor that changes the slope of the group line.

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Contents

Abstract	i
Acknowledgement	iii
Contents	v
1 Introduction	1
2 Theory	5
2.1 Properties of waves	5
2.2 Discrete Fourier transform	6
2.3 The wavenumber - frequency spectrum	7
2.4 JONSWAP/Pierson-Moskowitz spectrum	8
2.5 Aliasing	9
2.6 Compensate for damping	11
2.7 Finding relevant times	12
2.8 Simulations	14
3 Experiments	17
3.1 Preparation	17
Comparing to earlier work	17
Parameters	20
Finding relevant times	22
Checking setup	23
Probes	23
Input vs output distribution of frequencies	23
3.2 Experiments	25
JONSWAP/Pierson-Moskowitz experiments	27
Gaussian experiments	28

4	Results	33
4.1	Compensate for damping	33
4.2	The exponent of the damping	35
4.3	Slopes and steepness	40
4.4	Comparing to simulations	46
4.5	Comparing to other work	50
4.6	NFFT	55
4.7	Robustness	58
4.8	What can produce the group line?	60
4.9	Bandwidth	62
4.10	Length of the group line	64
5	Conclusion	67
	Bibliography	69

CHAPTER 1

Introduction

When plotting the dispersion relation $\omega^2 = gk$ in the (k, ω) -plane, the graph will follow a curve bending towards the k -axis. This feature is known as the dispersion relation for deep water waves. Doing 2D spatio-temporal Fourier transform of gravity waves and plotting the power spectrum, one would expect that the intensity will follow this dispersion relation. However, if the energy in the wavefield is localized around some characteristic place, (k_c, ω_c) , and we look at non-linear interaction between two waves described by (k_1, ω_1) and (k_2, ω_2) close to the characteristic wave, then there can either be sum-interaction or difference-interaction. The second-order, non-linear sum-interaction is the cause of higher harmonics, while it is the second-order, non-linear difference-interaction that gives rise to energy in the lower frequencies (Toffoli et al., 2006) and appears as a cloud. This cloud is what we are interested in, and is known as the “group line” among people working on radar imaging of the ocean surface.

The question asked among researchers today is what is it that generates this group line. It is in the radar community this feature is mainly known, and some of the most prominent theories today are breaking waves and the mechanism of the scattering of the radar beam. There have been multiple articles (Kammerer & Hackett, 2019; Plant & Farquharson, 2012; Platzer et al., 2019a, 2019b; Smith et al., 1996) that have been trying to figure out the reason for this cloud-shaped phenomenon. In addition, we can also see this feature in Frasier and McIntosh (1996), Krogstad and Trulsen (2010), Olsen (2019), Plant (2012), Støle-Hentschel et al. (2018), Støle-Hentschel et al. (2018) and Taklo et al. (2015).

Among these, many of them are using radar data with a grazing incidence angle, and where there likely have been wave breaking and possibly shadowing. Taklo et al. (2015) did experiments in the lab and used ultrasound probes, so no shadowing could have occurred, but there were

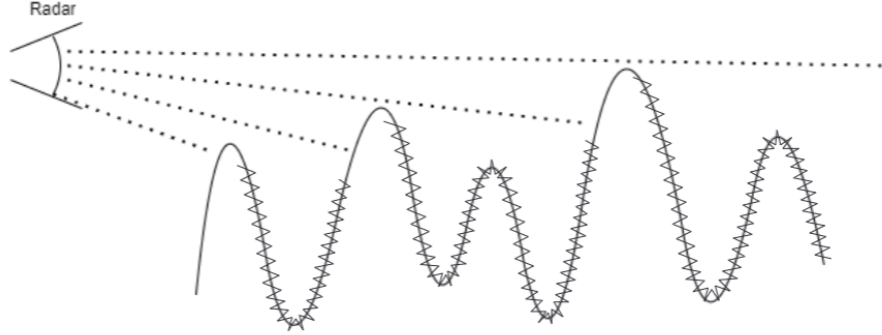


Figure 1.1: An example of shadowing. The radar is not able to see parts of the surface elevation, as represented by the zig-zag. This is because of the grazing incident angle of the radar.

observed some breaking at times. Olsen (2019) did experiments with an elastic sheet using ultrasound probes, so there should be no shadowing and no breaking in her case. Krogstad and Trulsen (2010) did simulations of non-breaking waves and should therefore not have any shadowing either. Støle–Hentschel et al. (2018) had two counter-propagating, irregular wave fields caused by a mirror at the end of the tank.

We believe that there are at least three possibilities of causation. The first one is shadowing that comes from when taking the radar image. A lot of the radar images are taken with a grazing incidence, that is, not from above, but as seen in Figure 1.1. This means that smaller waves might not be detected because of larger waves in front. This means that shadowing will be a prominent factor when thinking about the measurements. The second and third possibilities are breaking waves and non-linear interaction, respectively. While there is no analytical work indicating that the group line should exist, there has also been breaking waves in the cases where the group line has been presented. This gives us the idea that breaking waves might be a cause for the group line.

Some might be inclined to believe that the group line is an indication of currents and that the effects induced by the current, such as Stokes drift is what we are observing. However, the surface elevation has Eulerian properties, while Stokes drift has Lagrangian properties. That is not to say that they have nothing in common, but there is nothing that points to them having anything to do with each other. However, because there has been little to no research about this, there is nothing that says they do not

have anything to do with each other either. Whether or not currents have anything to do with the group line would be an interesting topic to further explore in later theses. Still, in this thesis, we will focus on measuring the surface elevation and not consider the velocity field in the water.

This thesis will look at irregular, long-crested, breaking gravity waves and irregular, long-crested, non-breaking gravity waves. The result of running a two-dimensional spatio-temporal Fourier transform on the surface elevation of experiments will be compared to a two-dimensional spatio-temporal Fourier transform on simulations. In addition, we will be comparing the radar images of Plant and Farquharson (2012) and Platzner et al. (2019b) and simulations of short-crested, non-breaking waves. The goal is to find out as much as possible about this group line and what affects its properties.

The thesis has the following chapters:

Chapter 2 – Theory

In this chapter, we will put forth the theoretical groundwork used in this thesis. We will also present the different power spectra used and some of the challenges that come when doing experiments or simulating the real world.

Chapter 3 – Experiments

In this chapter, we will present the preparation done before the experiment. We will explain the experimental setup, the details behind the experiment, and how the experiments were done.

Chapter 4 – Results

In this chapter, we will give the results of the experiments and look into them. We discuss the results and compare them to simulations and work done by other authors. The difficulties from Chapter 2 are assessed, and we also look into different properties of the group line, such as the slope and its length.

Chapter 5 – Conclusion

In the last chapter, we will summarize the essential parts of the thesis as well as the results and discoveries done.

CHAPTER 2

Theory

2.1 Properties of waves

Equation (2.1) represents a monochromatic wave in space and time where k and ω are constant.

$$\eta(x, t) = \text{Re}(A \exp\{i(kx - \omega t)\}) \quad (2.1)$$

Here k is the wavenumber, ω is the angular frequency, and A is the amplitude of the wave. The wave will be propagating in positive x -direction as represented by the minus sign. By the principle of linear superposition, the ocean surface can be described as a sum of these monochromatic waves (Trulsen, 2020a).

$$\eta(x, t) = \sum_n \text{Re}(A_n \exp\{i(k_n x - \omega_n t)\}) \quad (2.2)$$

Here k_n and ω_n will be coupled by the dispersion relation $\omega = \omega(k)$.

The dispersion relation refers to the fact that waves of different wavelengths propagate at different phase speeds. The phase speed is given by $c = \omega(k)/k$. For gravity waves, the linear dispersion relation (Gjevik et al., 1989) is given by

$$\omega^2 = kg \tanh(kh) \quad (2.3)$$

where, g is the acceleration due to gravity and h is the water depth. For deep water, $kh \gg 1$, equation (2.3) simplifies to

$$\omega^2 = kg \quad (2.4)$$

2.2. Discrete Fourier transform

In the case of breaking waves, we will also introduce the steepness, ϵ (Trulsen, 2021). Assuming that there is a characteristic wavenumber, k_c , and a characteristic amplitude a_c , the steepness is given by

$$\epsilon = a_c k_c \quad (2.5)$$

The JONSWAP spectrum and the Pierson-Moskowitz spectrum are both quite narrow with a prominent top, this means that most of the frequencies will lie around the peak. To account for this, we will be using the wavenumber that corresponds to the peak angular frequency, k_p as our characteristic wavenumber. The characteristic amplitude can be defined in a variety of different ways. We have defined it as

$$a_c = \sqrt{2}\sigma \quad (2.6)$$

where σ is the standard deviation of the surface elevation.

2.2 Discrete Fourier transform

The Fourier transform is a transform that inputs a function in space or time and outputs a function dependent on wavenumber or temporal frequencies. If we have a function of both space and time, we can project it onto the wavenumber-frequency plane by using a superposition of the basis functions in the Fourier space, $\exp\left\{\frac{2\pi i n j}{N}\right\}$. So assuming we have a function a sequence of N elements, $\{f_j : j = 0, \dots, N-1\}$, we can now rewrite this as

$$f_j = \sum_{n=0}^{N-1} \tilde{f}_n \exp\left\{\frac{2\pi i n j}{N}\right\} \quad (2.7)$$

Using the inner product $\langle f, g^* \rangle = \sum_{j=0}^{N-1} f_j g_j^*$, where g^* is the complex conjugate, we have that the basis functions are orthogonal(Trulsen, 2020b). With this we can find $\tilde{f}_n$.

$$\tilde{f}_n = \frac{1}{N} \sum_{j=0}^{N-1} f_j \exp\left\{-\frac{2\pi i n j}{N}\right\} \quad (2.8)$$

This is the discrete Fourier transform(DFT). The $1/N$ comes from the fact that $\langle e^{\frac{2\pi i n j}{N}}, e^{\frac{2\pi i n j}{N}} \rangle = N$. On the computer, the function `numpy.fft.fft` can be used in Python or the `fft` in MATLAB. These functions compute the DFT by implementing a faster algorithm known as Fast Fourier Transform(FFT) and is a lot faster than directly using the equations above when dealing with large amounts of data.

2.3. The wavenumber - frequency spectrum

Because of the cyclic properties of the DFT(Trulsen, 2020b), we have to suppose that f_j and $\tilde{f}_n$ is continued periodically with $f_j = f_{j+N}$ and $\tilde{f}_n = \tilde{f}_{n+N}$. This means that we can start the sequence at any integer and still get the same result. In addition, we can shift the data in order to start the data where we find suitable. This shifting can be done on the computer by the function `numpy.fft.fftshift` in Python or `fftshift` in MATLAB.

In the experiments in this thesis, we have that $0 \leq x < L$ giving us $x_j = j\Delta x = j\frac{L}{N}$. In addition we have $k_n = n\Delta k = n\frac{2\pi}{N}$. Using this, we can rewrite equations (2.7) and (2.8) to

$$f_j = \sum_{n=0}^{N-1} \tilde{f}_n \exp\{ik_n x_j\} \quad (2.9)$$

and

$$\tilde{f}_n = \frac{1}{N} \sum_{j=0}^{N-1} f_j \exp\{-ik_n x_j\} \quad (2.10)$$

2.3 The wavenumber - frequency spectrum

In order to show the spectral energy density, we will need to find the power spectrum. The power spectrum can be found by taking the Fourier transform of the autocorrelation function(Trulsen, 2020c).

The autocorrelation function is given by equation (2.11) where $E[\bullet]$ is the expected value of $\bullet$.

$$R(\xi, \tau) = E[\eta(x + \xi, t + \tau)\eta^*(x, t)] \quad (2.11)$$

The power spectrum is given by equation (2.12).

$$S(k, \omega) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R(\xi, \tau) \exp\{-i(k\xi - \omega\tau)\} d\xi d\tau \quad (2.12)$$

By plotting the power spectrum we can see where the energy lies. In order to better showcase the spectral energy, we have opted to use decibel(dB) relative to the peak.

$$S_{dB}(k, \omega) = 10 \log_{10} \left\{ \frac{S(k, \omega)}{S_{max}} \right\} \quad (2.13)$$

Where S_{max} is the maximum value of $S(k, \omega)$. What we expect then is intensity following the dispersion relation given by equation (2.4). Should

2.4. JONSWAP/Pierson-Moskowitz spectrum

the group line be present, we will see intensity for low frequencies both in time and space below the dispersion relation. The second harmonic is also something we would expect to see at the top right or the bottom left of the plot, due to the symmetry. However, our point of interest will be within the interval $[-k_p, k_p]$.

We will also be scaling the spectrum with k_p and ω_p .

2.4 JONSWAP/Pierson-Moskowitz spectrum

An already made program generated the wavefields used in the experiments and the numerical simulations. The phases were randomly picked from a uniform distribution on the interval $[0, 2\pi)$, and the amplitudes were chosen from the JONSWAP or the Pierson-Moskowitz spectrum. We will later also use the Gaussian spectrum, but this will be further talked about in Section 3.2.

The Pierson-Moskowitz spectrum(Pierson & Moskowitz, 1964) is given by

$$S_{PM}(\omega) = \alpha g^2 \omega^{-5} \exp \left[-\frac{5}{4} \left(\frac{\omega_p}{\omega} \right)^4 \right] \quad (2.14)$$

where α is a constant that indicates the wind speed on the ocean. In this case, it has been used as an amplitude factor and is the only factor changed to make steeper or less steep waves.

The JONSWAP spectrum has its origin in the Pierson-Moskowitz spectrum. In this thesis, we will be using both spectrums. This is because the Pierson-Moskowitz spectrum has a broader bandwidth than JONSWAP, and part of the thesis is to see if the difference in bandwidth impacts the group line.

The JONSWAP spectrum(Hasselmann et al., 1973) is given by

$$S_{JONSWAP}(\omega) = \alpha g^2 \omega^{-5} \exp \left[-\frac{5}{4} \left(\frac{\omega_p}{\omega} \right)^4 \right] \gamma^a \quad (2.15)$$

$$a = \exp \left[-\left(\frac{\omega - \omega_p}{\sqrt{2}\omega_p\sigma} \right)^2 \right] \quad (2.16)$$

$$\sigma = \begin{cases} 0.07 & \text{if } \omega \leq \omega_p \\ 0.09 & \text{if } \omega > \omega_p \end{cases} \quad (2.17)$$

Where α is the same amplitude factor as for the Pierson-Moskowitz spectrum. γ is a constant, γ^a is the "peak enhancement factor" (Hasselmann et al., 1973) and ω_p is the peak angular frequency given by the peak period, $\omega_p = \frac{2\pi}{T_p}$. The peak period is one of the parameters to be given when generating the time series.

The only difference between the Pierson-Moskowitz and the JONSWAP spectrum is the constant γ . In order to change between the two different spectrums, we will use $\gamma = 3.3$ for the JONSWAP runs and $\gamma = 1.0$ for the Pierson-Moskowitz runs.

2.5 Aliasing

The sea surface can be represented by a continuous function. What we measure in the experiments and simulations, however, are discrete points in time and space. Dependent on the refinement in the grid we have, we might be victims to aliasing when plotting the power spectrum.

If the surface elevation can be written as a function $f(x)$ on $0 \leq x < L$, we can express it through a Fourier series. That is, a sum of linearly independent basis functions, similar to that of the discrete Fourier transform (Trulsen, 2020b).

$$f(x) \sim \sum_{n=-\infty}^{\infty} \hat{f}_n \exp\{ik_n x\} \quad (2.18)$$

Because x is continuous, the Fourier series of the function $f(x)$ will also be continuous as it is a sum of continuous functions. When doing experiments, we will be looking at the same function $f(x)$, but now it is made up of discrete points $f_j = f(x_j)$, and we must therefore use DFT to represent it, as opposed to a continuous sum.

$$f_j = \sum_{n=0}^{N-1} \tilde{f}_n \exp\{ik_n x_j\} \quad (2.19)$$

Ideally, we would like to have $\hat{f}_n = \tilde{f}_n$. If we evaluate the Fourier series at the discrete points x_j and take the inner product with $\exp\left\{\frac{2\pi i m j}{N}\right\}$, we will get the formula for aliasing (Trulsen, 2020b).

$$\tilde{f}_n = \sum_{l=-\infty}^{\infty} \hat{f}_{n+lN} \quad (2.20)$$

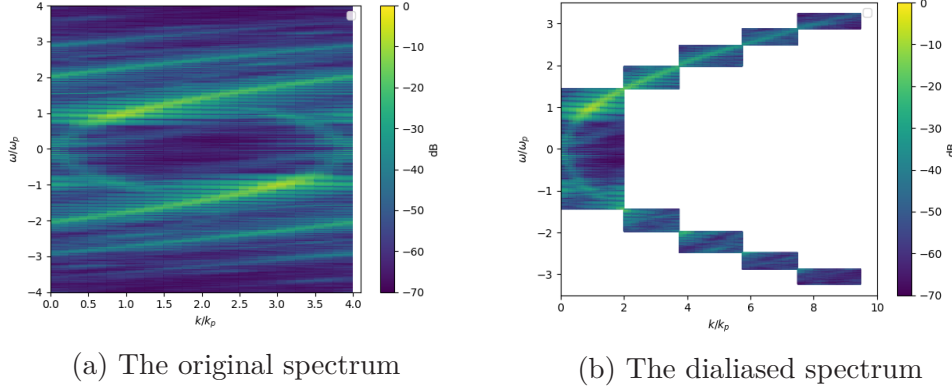


Figure 2.1: Here we have dealiased the spectrum of Anne Raustøl's data.

Here we have used the identity

$$\sum_{j=0}^{N-1} e^{\frac{2\pi i j n}{N}} = N \sum_{l=-\infty}^{\infty} \delta_{n, lN} \quad (2.21)$$

where $\delta_{n,m}$ is the Kronecker delta. What we can get from this is that if $\hat{f}_n$ is 0 for $|n| \geq \frac{N}{2}$, then $\hat{f}_n$ and $\tilde{f}_n$ is equal on that interval. This is more commonly known written as $|k_n| \geq k_{\frac{N}{2}}$ where the wavenumber $k_{\frac{N}{2}}$ is known as the Nyquist wavenumber and the criterion is called the Nyquist sampling criterion (Trulsen, 2020b). What it says is that the number of samples points N must be large enough such that $\hat{f}_n$ is sufficiently small enough outside the interval. This must be satisfied in order for $\hat{f}_n$ and $\tilde{f}_n$ to be almost equal on the interval.

In the experiments, however, it is not always possible to have enough points in space due to constraints such as time, capacity and limited resources. This means that we could expect aliasing to happen in space when plotting our power spectrum. What this means is that we would see intensity at the correct place in time but not space. Støle–Hentschel et al. (2018) has overcome this by dealiasing the power spectrum. By knowing what we are looking at, we can cut parts of the spectrum and shuffle them around to get the spectrum we are looking for. An example of this can be seen in Figure 2.1. Not only is it easier to see what we are after, but we have also managed to extend the axis by over two times the original axis. This was done as part of the preparation before beginning with the experiments, and we will come back to this later in Chapter 3.



Figure 2.2: This is how we expect the energy to dissipate in the tank.

2.6 Compensate for damping

When the waves propagate down the tank, we expect the energy to be higher for the waves at the beginning of the tank and lower when the waves have reached the end of the tank. This dissipation of energy is primarily due to the breaking of waves and leads to damping of the waves. It is possible to visualize this by plotting the characteristic amplitude for each point in space. What we expect is the energy to decline as the waves propagate the tank, something like Figure 2.2.

We expect the energy to be exponentially decreasing. In order to compensate for this decreasing energy, we would like to divide the surface elevation with the energy at each point. It is possible to find the function of the damping by trying to curve fit the function 2.22. Here a , b and c are the fitting parameters.

$$f(x) = a \exp\{-bx\} + c \quad (2.22)$$

Because of the way we have defined the characteristic amplitude, see equation (2.6), we have solved this by taking the Fourier transform of the time series at each sample point in space and then integrate over the frequency spectrum.

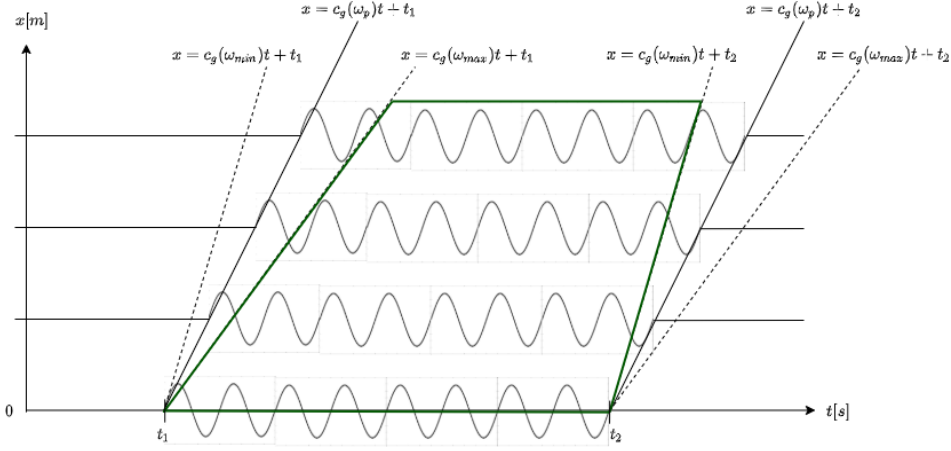


Figure 2.3: The green area contains the waves we are interested in measuring as they should not include start-up or end waves.

$$\int S(\omega) d\omega = \sigma^2 = \frac{a_c^2}{2} \quad (2.23)$$

2.7 Finding relevant times

One of the challenges we face when doing experiments is that experiments have a start and an end, whereas the ocean does not. Because of this, start-up- and end effects are a problem. Because the wave paddle is starting and stopping very abruptly, the start and end waves will behave differently from the rest of the wave train. And it is this behaviour we do not want to be included in our time series.

Had the change from no waves to a lot of waves been more subtle, this would not have been such a big problem, but because of the abruptness, this can pose quite a problem when doing the analysis later. In our case, we are interested in the speed of which the energy is propagating, that is, the group velocity. Because of the sudden start of waves incoming, the energy will be distributed to a wider range of frequencies, and there will be multiple group velocities. So, in order to compensate for this, we want to only look at the part of the time series where there are no start-up- or end effects.

In Figure 2.3 we can see how we will estimate the time range. At first, the tank has no waves. Then, at some time t_1 , we start the wavemaker, and waves will be propagating down the tank. Along the x -axis, we have the points where the probes will be located. In the figure, only the first and last probe is indicated. The waves inside the green shape are the waves we

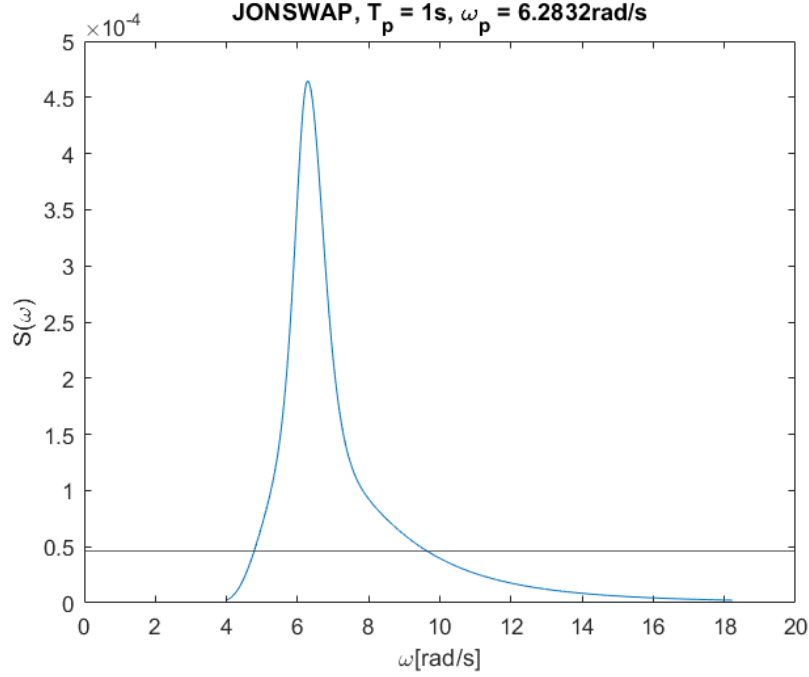


Figure 2.4: The line shows the threshold.

want to pick out. Since the relevant time period is different for the different probes, we will use the last probe as a base for how long our time series has to be in order to get enough data.

In Figure 2.3, we set c_g , the group velocity, to be a function of frequency and we use the dispersion relation, see equation (2.4), to make it a function of ω .

$$\begin{aligned} c_g &= \frac{\partial \omega(k)}{\partial k} \\ &= \frac{\partial \sqrt{gk}}{\partial k} \\ &= \frac{1}{2} \sqrt{\frac{g}{k}} \\ &= \frac{1}{2} \frac{g}{\omega} \end{aligned}$$

To get ω_{min} and ω_{max} , we will have to find it from the relative spectrums. Because the tail for the JONSWAP spectrum is so thin and stretched, we have disregarded the bottom 10% in the spectrum as most of the frequencies will come from around the peak angular frequency. ω_{min} and ω_{max} have

then been chosen to be the left-most and right-most frequency above the threshold line as seen in Figure 2.4. While some of the higher frequencies are cut from the tail, it should not impose a problem picking the time range we are interested in.

2.8 Simulations

In this thesis we will be simulating both long-crested, non-breaking waves. Equations (2.24) and (2.25) are the same equations as equation (7) and (9) from Toffoli et al. (2006).

$$\eta^{(1)}(\mathbf{x}, t) = \sum_{i=1}^N \sum_{l=1}^M a_{il} \cos [k_i (x \cos \theta_l + y \sin \theta_l) - \omega_i t + \varepsilon_{il}] \quad (2.24)$$

$$\eta^{(2)}(\mathbf{x}, t) = \frac{1}{4} \sum_{i,j=1}^N \sum_{l,m=1}^M a_{il} a_{jm} \left[K_{ijlm}^- \cos (\varphi_{il} - \varphi_{jm}) + K_{ijlm}^+ \cos (\varphi_{il} + \varphi_{jm}) \right] \quad (2.25)$$

where $\varphi_{il} = k_i (x \cos \theta_l + y \sin \theta_l) - \omega_i t + \varepsilon_{il}$, $\mathbf{x} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$ and

$$K_{ijlm}^+ = \left[D_{ijlm}^+ - (k_i k_j \cos (\theta_l - \theta_m) - R_i R_j) \right] (R_i R_j)^{-1/2} + (R_i + R_j)$$

$$K_{ijlm}^- = \left[D_{ijlm}^- - (k_i k_j \cos (\theta_l - \theta_m) + R_i R_j) \right] (R_i R_j)^{-1/2} + (R_i + R_j)$$

$$D_{ijlm}^+ = \frac{(\sqrt{R_i} + \sqrt{R_j}) \{ \sqrt{R_i} (k_j^2 - R_j^2) + \sqrt{R_j} (k_i^2 - R_i^2) \}}{(\sqrt{R_i} + \sqrt{R_j})^2 - k_{ijlm}^+ \tanh (k_{ijlm}^+ h)}$$

$$+ \frac{2 (\sqrt{R_i} + \sqrt{R_j})^2 (k_i k_j \cos (\theta_l - \theta_m) - R_i R_j)}{(\sqrt{R_i} + \sqrt{R_j})^2 - k_{ijlm}^+ \tanh (k_{ijlm}^+ h)}$$

$$D_{ijlm}^- = \frac{(\sqrt{R_i} - \sqrt{R_j}) \{ \sqrt{R_j} (k_i^2 - R_i^2) - \sqrt{R_i} (k_j^2 - R_j^2) \}}{(\sqrt{R_i} - \sqrt{R_j})^2 - k_{ijlm}^- \tanh (k_{ijlm}^- h)}$$

$$+ \frac{2 (\sqrt{R_i} - \sqrt{R_j})^2 (k_i k_j \cos (\theta_l - \theta_m) + R_i R_j)}{(\sqrt{R_i} - \sqrt{R_j})^2 - k_{ijlm}^- \tanh (k_{ijlm}^- h)}$$

$$\begin{aligned}
 k_{ijlm}^- &= \sqrt{k_i^2 + k_j^2 - 2k_i k_j \cos(\theta_l - \theta_m)} \\
 k_{ijlm}^+ &= \sqrt{k_i^2 + k_j^2 + 2k_i k_j \cos(\theta_l - \theta_m)} \\
 R_i &= \omega_i^2 / g
 \end{aligned}$$

Here, k_i is given by equation (2.3) and the amplitudes, a_{ij} , are given by $a_{ij} = a(\omega_i, \theta_j) = \sqrt{2E(\omega_i, \theta_j) \Delta\omega \Delta\theta}$ where $E(\omega_i, \theta_j)$ in this case is the spectrum.

In the case of the long-crested waves, the waves are propagating in the same direction which gives us $\theta = 0$. This will simplify the equations a lot.

$$\eta^{(1)}(\mathbf{x}, t) = \sum_{i=1}^N a_i \cos[k_i x - \omega_i t + \varepsilon_i] \quad (2.26)$$

$$\eta^{(2)}(\mathbf{x}, t) = \frac{1}{4} \sum_{i,j=1}^N a_i a_j \left[K_{ij}^- \cos(\varphi_i - \varphi_j) + K_{ij}^+ \cos(\varphi_i + \varphi_j) \right] \quad (2.27)$$

where $\varphi_i = k_i x - \omega_i t + \varepsilon_i$ and

$$\begin{aligned}
 K_{ij}^+ &= \left[D_{ij}^+ - (k_i k_j - R_i R_j) \right] (R_i R_j)^{-1/2} + (R_i + R_j) \\
 K_{ij}^- &= \left[D_{ij}^- - (k_i k_j + R_i R_j) \right] (R_i R_j)^{-1/2} + (R_i + R_j) \\
 D_{ij}^+ &= \frac{(\sqrt{R_i} + \sqrt{R_j}) \left\{ \sqrt{R_i} (k_j^2 - R_j^2) + \sqrt{R_j} (k_i^2 - R_i^2) \right\}}{(\sqrt{R_i} + \sqrt{R_j})^2 - k_{ij}^+ \tanh(k_{ij}^+ h)} \\
 &\quad + \frac{2(\sqrt{R_i} + \sqrt{R_j})^2 (k_i k_j - R_i R_j)}{(\sqrt{R_i} + \sqrt{R_j})^2 - k_{ij}^+ \tanh(k_{ij}^+ h)} \\
 D_{ij}^- &= \frac{(\sqrt{R_i} - \sqrt{R_j}) \left\{ \sqrt{R_j} (k_i^2 - R_i^2) - \sqrt{R_i} (k_j^2 - R_j^2) \right\}}{(\sqrt{R_i} - \sqrt{R_j})^2 - k_{ij}^- \tanh(k_{ij}^- h)} \\
 &\quad + \frac{2(\sqrt{R_i} - \sqrt{R_j})^2 (k_i k_j + R_i R_j)}{(\sqrt{R_i} - \sqrt{R_j})^2 - k_{ij}^- \tanh(k_{ij}^- h)}
 \end{aligned}$$

$$k_{ij}^- = \sqrt{k_i^2 + k_j^2 - 2k_i k_j}$$

$$k_{ij}^+ = \sqrt{k_i^2 + k_j^2 + 2k_i k_j}$$

$$R_i = \omega_i^2 / g$$

CHAPTER 3

Experiments

3.1 Preparation

Comparing to earlier work

Before starting the experiments, some processing on Tore Magnus Taklo's work (Taklo et al., 2015) and Anne Raustøl's work (Raustøl, 2014) was done to test the programs made for the post-analysis. Their raw data was acquired in order to do so.

Tore Magnus Taklo published the paper *Measurements of the dispersion relation for random surface gravity waves*. In his paper, he carries out experimental and numerical analysis to confirm Krogstad and Trulsen (2010) using various bandwidths for the Gaussian spectrum and the JONSWAP spectrum.

In his thesis, there were multiple occurrences of intensity parallel to the dispersion relation. The reason for this was not understood, but it was suggested that inaccuracy in the measurements was the reason. We are mentioning this because we also got similar results at first, as seen in Figure 3.1. However, this has not been an issue during work on this thesis as a solution for the parallel lines has been found.

When plotting each of the times series in the same plot, we can see whether or not there are any problems with them. From Figure 3.2 we have plotted part of the data from Taklo et al. (2015). Explanations for the issues can be inaccuracies when doing the measurements. When moving the rail along with the tank or keeping the order of the 384 probes, errors can happen, and together they can accumulate to a greater error. In order to not make the same mistakes, we have done all the experiments with this in mind. We are also checking the time series after each experiment.

Anne Raustøl's master thesis is called *Freaky bølger over variabelt*

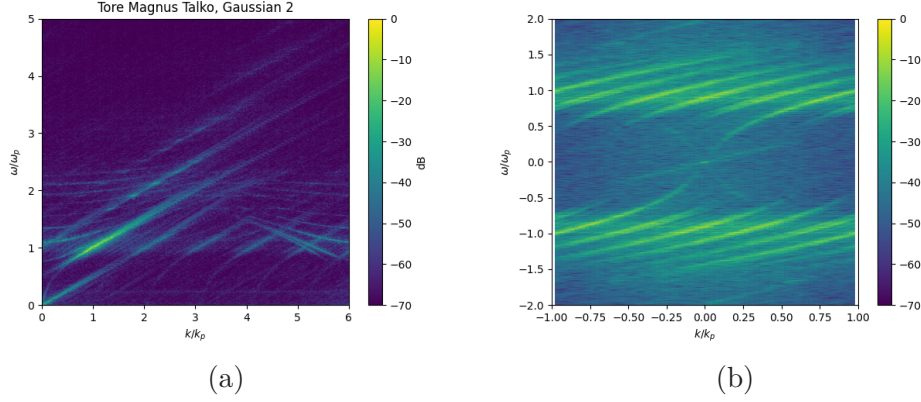


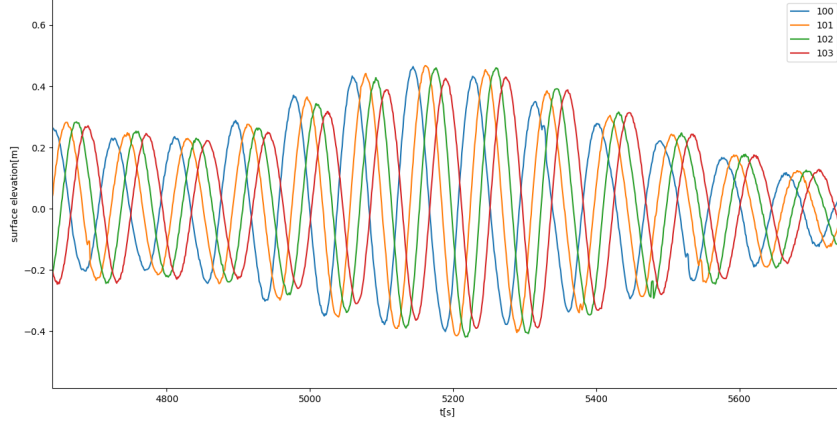
Figure 3.1: On the left, (a), we have one of Tore Magnus Talko's (Taklo et al., 2015) results where we can see the parallel lines. On the right, (b), we have one of the early results from our experiments where there are similar parallel lines.

dyp (Freak waves over variable depth) where she computed the skewness and the kurtosis of waves going over a gradually changing step. The wavenumber-frequency spectrum has not been computed on her data before, which made this both interesting as well as practice.

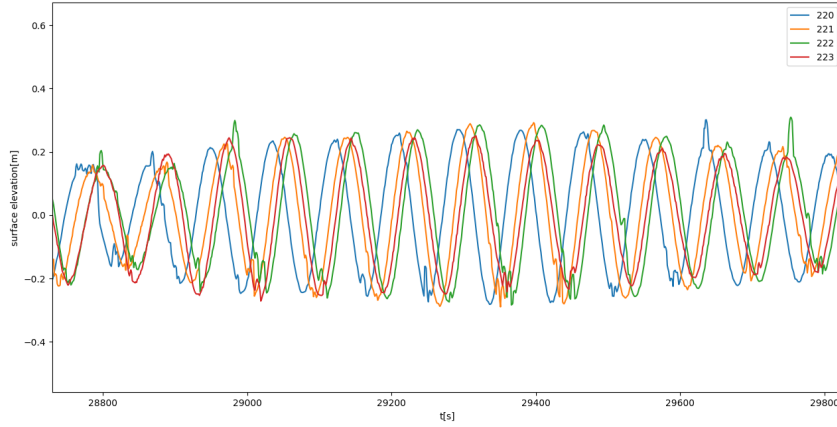
When Anne Raustøl did her experiments, she used only 12 probes for each run. This is too few probes and will give a very coarse refinement in space. As a result, the spectrum is being aliased. In Figure 3.3 we have plotted the spectrum for both positive and negative values for k and ω . The first and third quadrant corresponds to waves going to the right, while the second and the fourth quadrant corresponds to waves going to the left. Because of the aliasing, it might seem like there are a lot more left going waves than there actually are. However, because we are aware of the aliasing, we can see that the intensity from the left is the extension of the dispersion relation going out on the right.

In Section 2.5 we mentioned that this might be a problem when there are not enough sampling points in space (or time). Using that we know how the spectrum is supposed to look, we can pick the spectrum apart and piece it together to get the actual spectrum. When doing this, we can see that we have expanded the k -axis. We now see that there are almost no left going waves. This is expected as the right going waves should be dominating compared to the reflected waves.

When analysing the data of Raustøl (2014) and Taklo et al. (2015), we used parts of their programs as a basis for the programs made for this thesis.



(a)



(b)

Figure 3.2: This is filtered data from Tore Magnus Taklo's experiments. We see probes from the same wave field but at different places in space, and the label is the number of the probe out of the 382 probes in the synthetic array he made. On the top, (a), we see the time series as we expect compared to the location relative to each other. On the bottom, (b), we see the time series further back in the tank, but we can tell that probe 223 is not behaving as expected compared to the others.

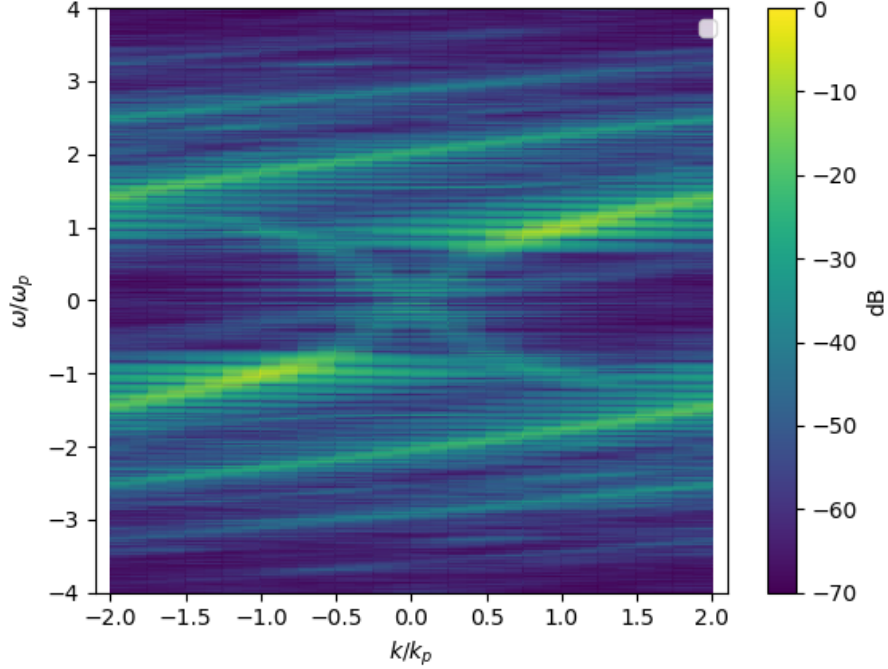


Figure 3.3: Wavenumber–frequency spectrum of Anne Raustøl’s data.

We have written in the code where their MATLAB code has been used and translated to Python.

Parameters

When deciding on the distance between the probes in the synthetic array, we wanted a length of $2k_p$ on our axis. This way, we could either focus on the group line or the dispersion relation as these are the two crucial parts we are looking for in the spectrum. By knowing the peak period, used as a parameter in the time series generator, we could calculate Δx .

We have that $L = N\Delta x$, $\Delta k = \frac{2\pi}{L}$ and $k_n = n\Delta k$, where n is the n ’th probe, N is the number of probes in our synthetic array and L is the length in x -direction. We have already said we want the axis to be $2k_p$ long, so we would need $2k_p = k_N$. Using this, we can find an approximation to the distance between the probes.

$$2k_p = k_N$$

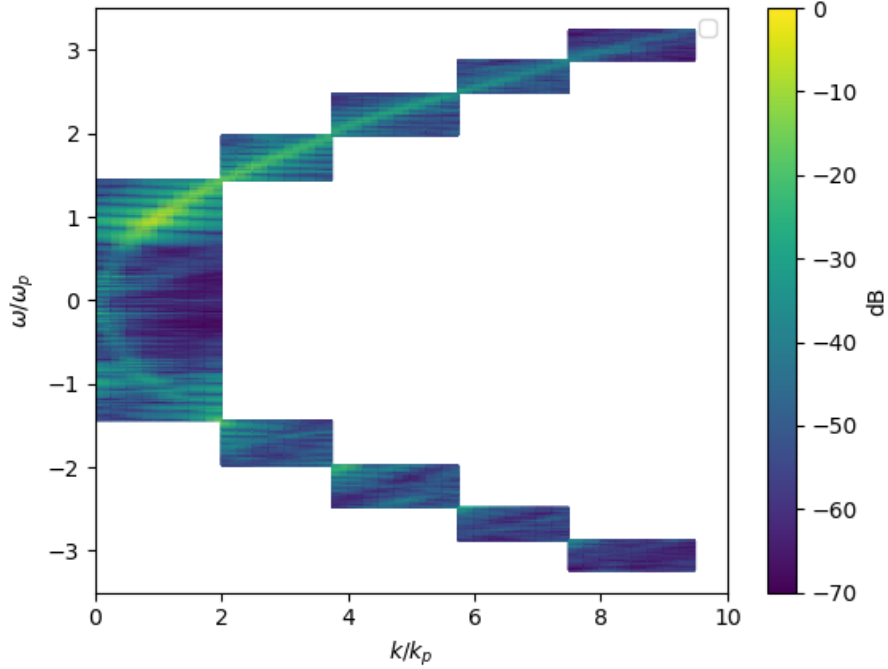


Figure 3.4: Here we have de-aliased Figure 3.3.

$$\begin{aligned}
 &= N\Delta k \\
 &= N\frac{2\pi}{L} \\
 2k_p &= N\frac{2\pi}{N\Delta x} \\
 \implies \Delta x &= \frac{\pi}{k_p} \tag{3.1}
 \end{aligned}$$

Here, k_p is decided by equation (2.4) and ω_p is given by $\omega_p = \frac{2\pi}{t_p}$. As we can see from the equation (3.1), the number of probes does not matter, and as such, we have used as many probes that we could get in the last 12 meters of the tank where the waves have fully propagated.

During this thesis, we have tried different values for T_p . In the end, $T_p = 1.0\text{s}$ was chosen. We set the number of probes to 48. Other numbers based on the equation (3.1) has also been tried before 48 was picked. In equation (2.15) we changed α in order to change the steepness of the waves. We picked the α to include both non-breaking and breaking waves. Later

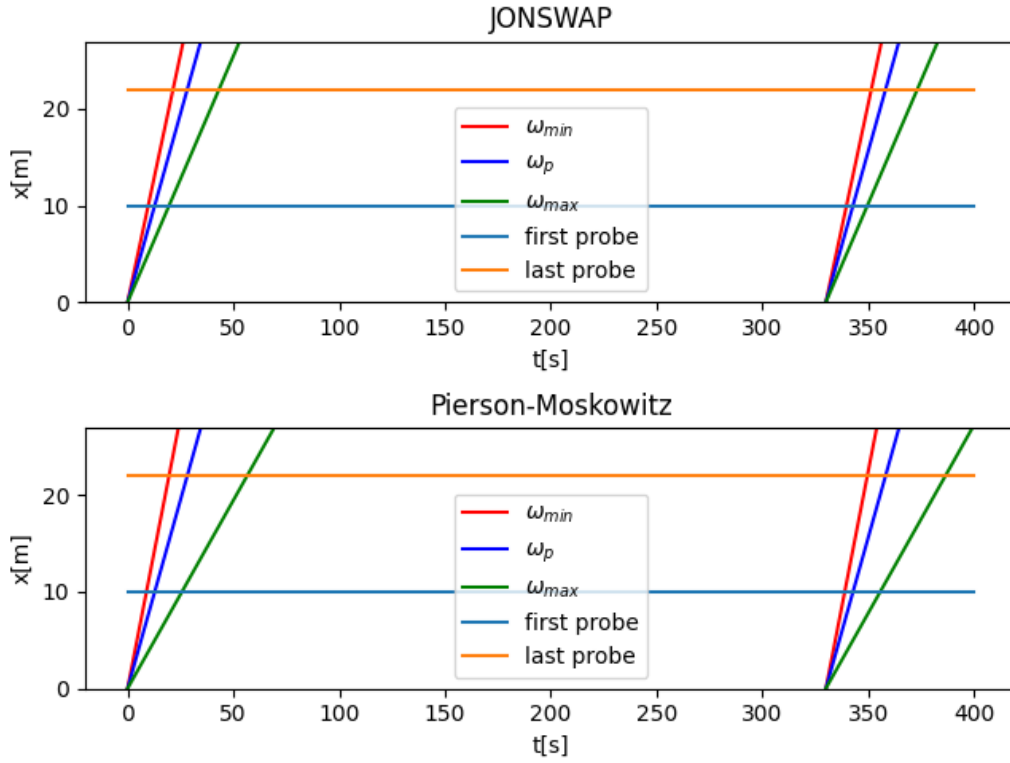


Figure 3.5: The two horizontal lines indicate where the first and the last probe are located. The other lines indicate the fastest and slowest wave based on the spectra in Figure 2.4 and Figure 3.9. They are rooted at two different times. $t = 0$ s is when the paddle starts and $t = 330$ s is when the paddle stops.

experiments have α chosen to match steepness from the first round.

Finding relevant times

In Section 2.7 we explain why it's important that we don't want start-up or end effects in the part of the time series we are using when calculating the spectrum. Using Figure 2.4 and Figure 3.9 we can find ω_{min} and ω_{max} , and with this, we can see how long we should wait before start recording.

In Figure 3.5, we have the same as in Figure 2.3, but with parameters. Since the Pierson-Moskowitz spectrum has a broader bandwidth, especially when the bottom 10% is cut, we would expect a bigger difference between ω_{min} and ω_{max} , which we can see. From Figure 3.5 we can see that we would need at least a 50 s wait before we start recording with the probes for the JONSWAP waves. In the case of the Pierson-Moskowitz, we should wait

at least 60 s to avoid start-up and end effects. In order to have everything as similar as possible, we will wait 70 s for both the JONSWAP and the Pierson-Moskowitz runs before we start recording.

Checking setup

Probes

For the experiments, *UltraLab ULS Advanced Special Device* has been used with a total of four probes. The probes have been placed normal to still water and measure the surface elevation. This is measured by sending a pulse from the probe, which is then reflected by the surface and the probe then receives the signal. The distance is then calculated by

$$d = \frac{c_s t}{2}$$

where c_s is the speed of sound. The accuracy of the measurements is ± 1 mm.

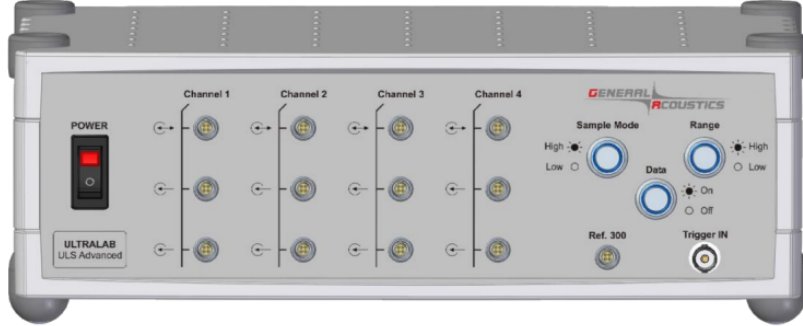
From Figure 3.6 we can see that there are four independent channels. These are synchronised and can each hold up to three sensors. The first sensor can send and receive signals while the two others can only receive. Because there are three sensors, it is possible to have different setups for the probes. Because of limited probes, we have opted to only use the first sensor for sending and receiving signals. The sample mode can be set to either low(125 Hz) or high(250 Hz), where the last is recommended if the wave speed is relatively high (*UltraLab ULS Advanced – User Manual*, 2013; *Ultralab ULS Advanced Special Device – Supplementary Manual*, 2014).

The range the probes can measure can also be toggled between high and low. Here, low range means that the probes will register within 30 - 250 mm while high range is within 65 - 350 mm.

In the experiments done for this thesis, both the sample mode and the range of the probes are set to low.

Input vs output distribution of frequencies

The array that the paddle maker takes in is an array of different voltages. The machinery in the paddle reads the array, and through the transfer function, comes out as waves. Comparing this to the time series we get out, and except the shape, their values are nothing alike. Looking at this, we would like to know if the input and the output are the same. Sending in an array to produce waves with some wanted parameters and getting out waves with entirely different properties is not wanted. Therefore, it was thought important enough to check.

Figure 3.6: The *UltraLab ULS Advanced*.

Considering that only the values are different and not the shape, simply running FFT on the input data should give us the same output as running FFT on the time series. Instead of using the time series from all the probes, only the first probe for each different time series has been used. One should also note that the waves are propagating, and there are nonlinear interactions, meaning frequency shifts when measuring different places. So, slight differences between the input frequencies and the frequencies from the output time series would be expected.

As one can see from Figure 3.7, the top figure shows the frequency spectrum after running FFT on the time series, while the bottom figure is after running FFT on the input array. The orange line is the theoretical peak frequency, which is the peak frequency we expect after giving the wanted parameters to the wave generator. The green line is the peak frequency when looking at the output only. In an ideal world, the orange and green would align.

When comparing the output from different α , we see that none of the peak frequencies are the same as the theoretical. However, there are minimal differences. As mentioned, we should expect some differences because of the nonlinear interactions, and the waves have propagated about 10 meters before reaching the first probe.

However, the input spectrum does not look like the output spectrum at all. This could mean that the transfer function, that we know nothing about, is also changing the frequency spectrum. When running the waves, it seems that an increase of 1 V would result in a wave about 10 cm higher. However, how the frequencies are changing in the transfer function is not something we can comment on, only that there has to be a function there.

3.2. Experiments

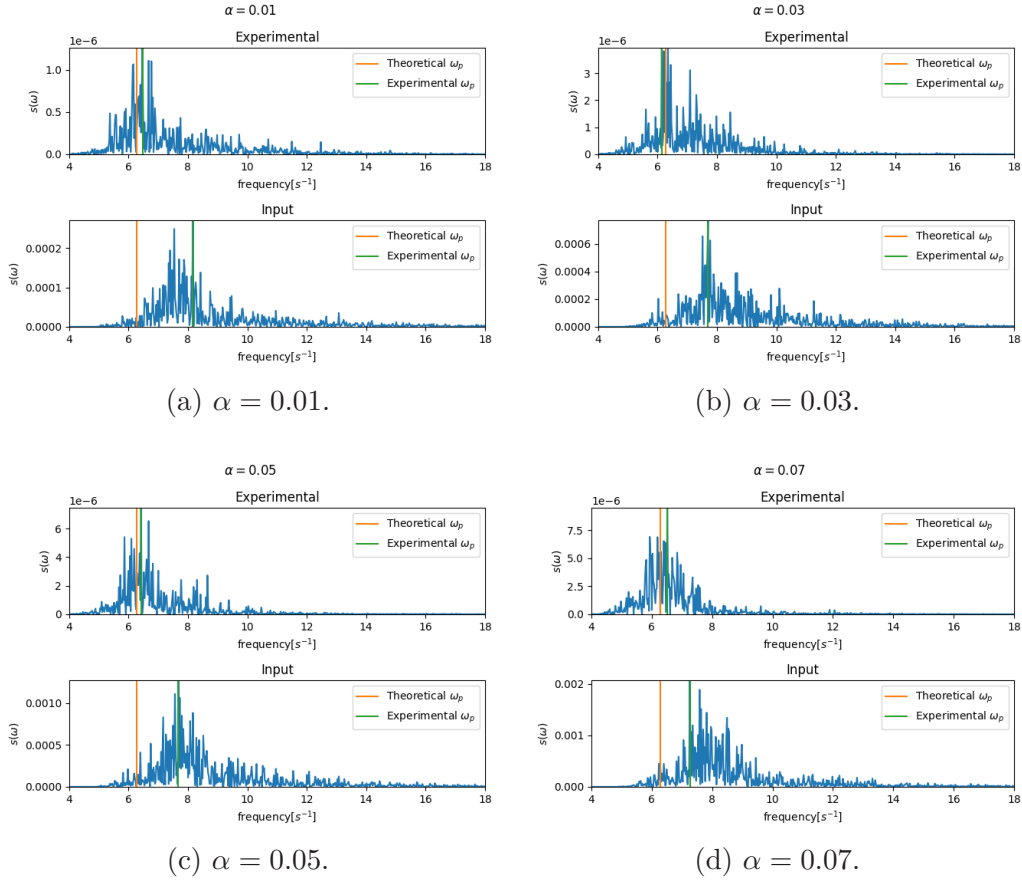


Figure 3.7

When looking at them both, the important part is that the output waves align with what we expect when giving the parameters to the time series generator. And from the results above, it seems quite accurate. There are only small differences between the theoretical peak frequency and the peak frequency from the output.

3.2 Experiments

The experiments took place in the basement of Niels Henrik Abel's building at the University of Oslo. The wave tank is measured to be 24.6m long and 0.5m wide. Four probes were mounted on a railway, and they were positioned at a distance of 1m from each other. Each run lasted 4.5 min, from Section 3.1 we have that the sampling rate is 125 Hz, giving us $\Delta t = 0.008$ s.

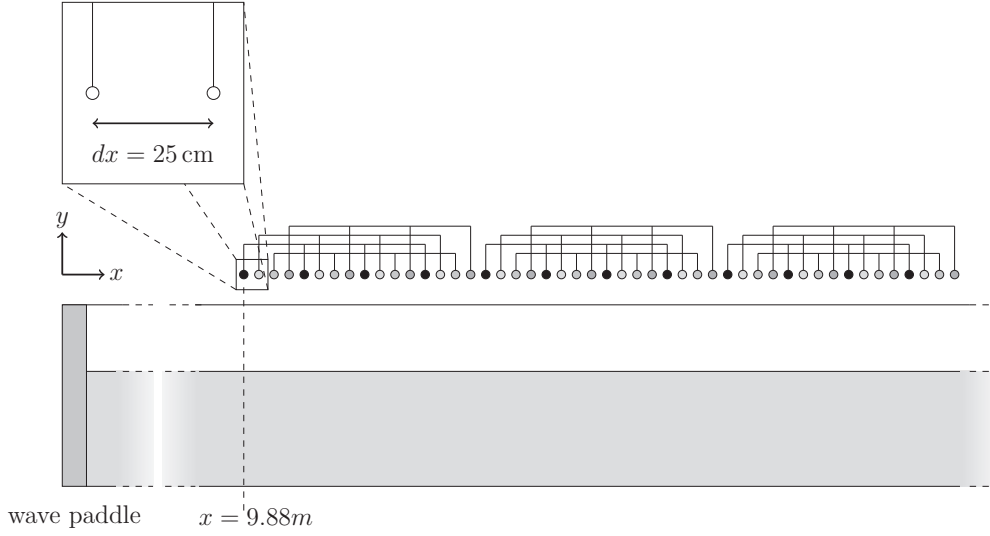


Figure 3.8: The experimental setup

We ran the experiments multiple times for each wavefield with the railway in different positions to produce the synthetic array. The probes were connected to a computer that shows the surface elevation in real-time. There is a wave maker at one end of the tank with a paddle, and at the other end, there is a dampening beach. The first probe is positioned at ~ 10 m and the last probe in the synthetic array is placed 11.75 m away from the first probe in the synthetic array, giving us a distance between the probes of $\Delta x = 25$ cm. The setup can be seen in Figure 3.8.

For each run, we let the wave paddle run for 30 s before we started recording, and the recording stopped when the wave paddle stopped. From Section 3.1 we have that the first 70 s should not be included in order not to have start-up or effects in our time series. Because this analysis was done after the experiments, we have cut the first 40 seconds from the time series. This should ensure us that there are no start-up or end-effects included.

The wavemaker is connected to a computer stationed beside the wave tank. The computer uses the program *LabView*, which makes the wave paddle move according to a generated time series taken as input.

To record the data, it is possible to use both the *LabView* program as well as the computer the probes are connected to. However, the data is not automatically stored as this has to be done manually on the computer connected to the probes for each time you run the wave field.

γ	α	Steepness
1.0	0.005	0.0317
	0.01	0.0424
	0.02	0.0593
	0.03	0.0707
	0.04	0.0782
	0.05	0.0865
	0.06	0.0924
	0.07	0.0981
	0.08	0.1037
	0.09	0.1042
	0.1	0.1051
3.3	0.005	0.0366
	0.01	0.0504
	0.02	0.0730
	0.03	0.0845
	0.04	0.0986
	0.05	0.1048
	0.06	0.1125
	0.07	0.1252

Table 3.1: Table of the different α used and the corresponding steepness of the waves.

JONSWAP/Pierson-Moskowitz experiments

The experiments were done for the two spectra JONSWAP and Pierson-Moskowitz. They are differentiated by the constant γ in equation (2.15). We have $\gamma = 3.3$ for JONSWAP and $\gamma = 1$ for Pierson-Moskowitz. Different steepness has been used for each of the two different spectra to have both breaking and non-breaking waves. This is represented by α in equation (2.15). Because the JONSWAP wavefields were the first to be run, the values of α of the Pierson-Moskowitz runs were chosen to overlap with the steepness we had for the JONSWAP runs. This can be seen in Table 3.1. The steepness is given by equation (2.5), and the standard deviation of the surface elevation, σ , is calculated based on all the 48 probes in the synthetic array.

We have used two different spectra with different bandwidths to see if this affects the shape or form of the group line. The JONSWAP spectrum is given by Figure 2.4 and the Pierson-Moskowitz spectrum can be seen in Figure 3.9.

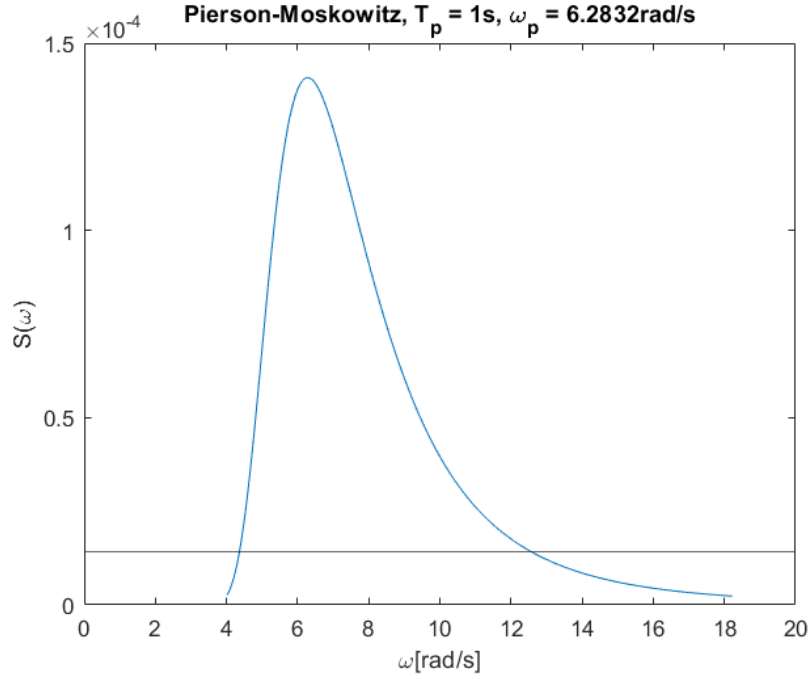


Figure 3.9: The Pierson-Moskowitz spectrum.

Gaussian experiments

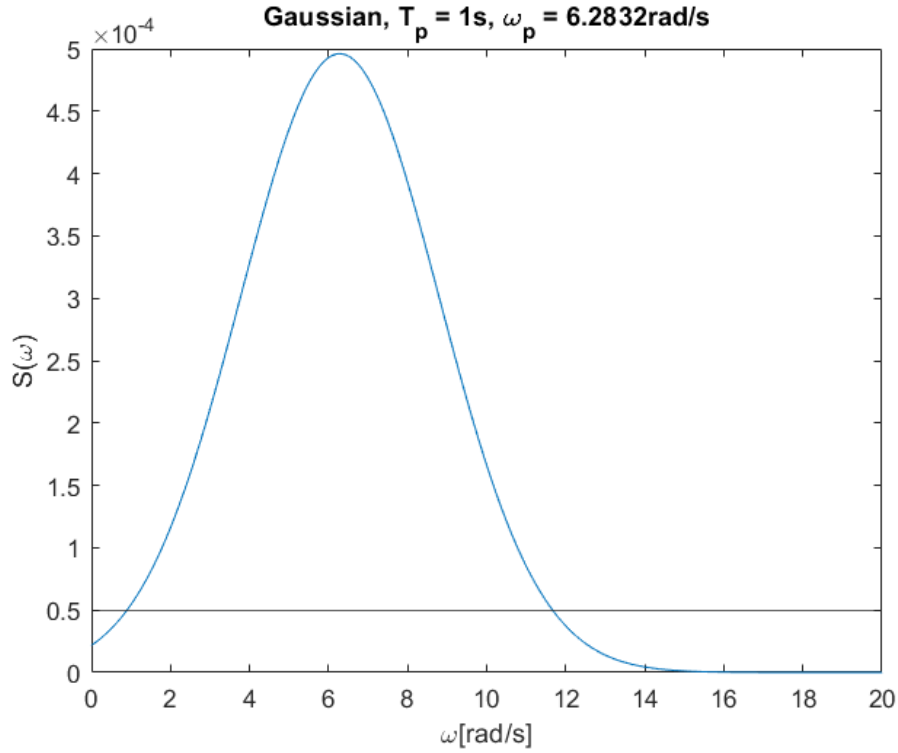
As mentioned in Section 3.2 we would like to compare spectra with different bandwidths to see if this affects the group line. Therefore, it should also be included that we tried this with the Gaussian spectrum where we could easily change the bandwidth. The equation we used for the Gaussian spectrum is given by equation (3.2).

$$S_G(\omega) = \frac{\alpha}{16\sigma\omega_p\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\omega - \omega_p}{\sigma_g\omega_p}\right)^2\right] \quad (3.2)$$

Here $\omega > 0$ where ω_p is the peak frequency, α is an amplitude factor and σ_g is the bandwidth.

The Gaussian experiments were run the same way as the JONSWAP experiments with $\Delta x = 25$ cm and with a total of 48 probes in the synthetic array. The JONSWAP spectrum has a quite narrow bandwidth, and as such, we would like a much wider bandwidth to see if there is a difference. In the experiments two different values of σ_g was used, $\sigma_g = 0.4$ and $\sigma_g = 0.6$.

While the Gaussian spectrum does not have a tail, it is symmetric around the peak frequency ω_p . In order to make the Gaussian runs as similar to

Figure 3.10: The Gaussian spectrum with $\sigma = 0.4$.

σ_g	α	Steepness
0.4	0.01	0.0421
	0.05	0.0864
	0.07	0.0975
	0.09	0.1091
0.6	0.005	0.0285
	0.03	0.0648
	0.05	0.07896
	0.1	0.1003

Table 3.2: An overview of the Gaussian runs

the JONSWAP runs as possible, only the frequencies above the bottom 10% have been considered. However, this does not have an as significant impact on the Gaussian spectrum as it has on the JONSWAP spectrum.

The runs done with the Gaussian spectrum can be seen in Table 3.2 where the steepness is given by equation (2.5) and the standard deviation

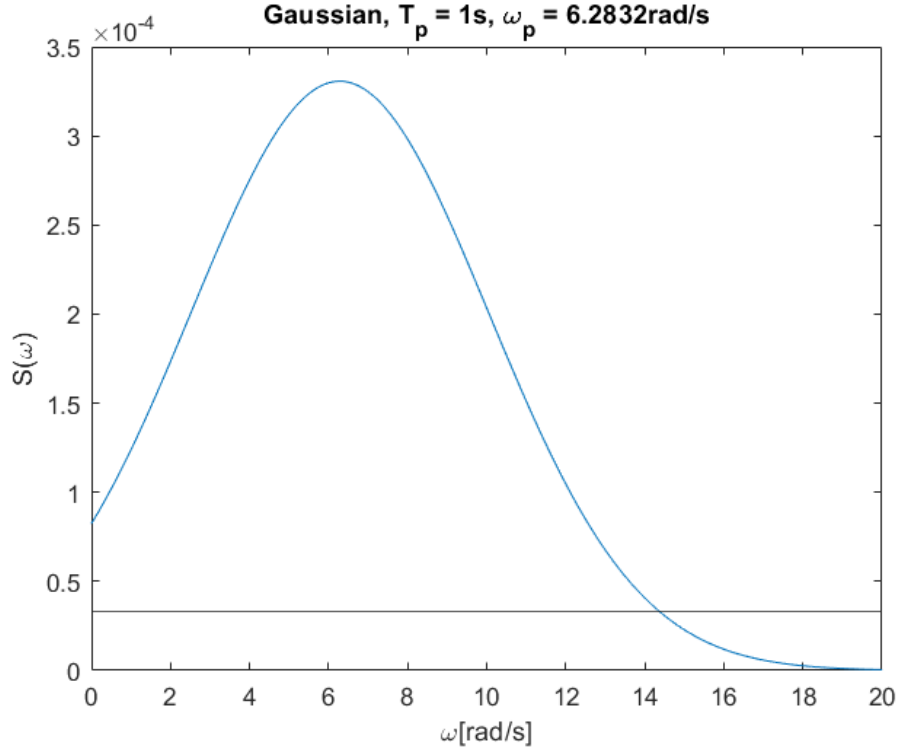


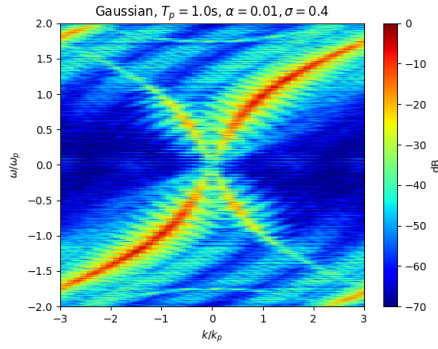
Figure 3.11: The Gaussian spectrum with $\sigma = 0.6$.

of the surface elevation is calculated based on all of the 48 probes in the synthetic array. The different α are chosen in order to overlap with the JONSWAP runs. The results of the runs can be seen in Figure 3.12 and Figure 3.13.

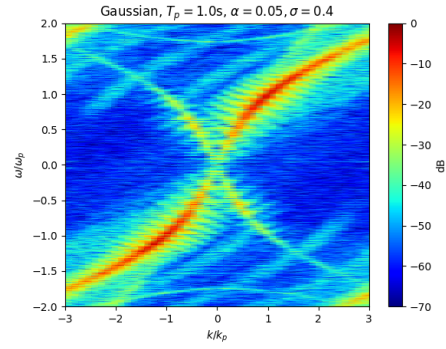
What we notice from the results is that we can only see the group line in the cases where the waves are not very steep such as Figure 3.13a or Figure 3.12a. The results from Figure 3.13 were first done, and it was suggested that trying with a narrower bandwidth might solve the problem. However, as we can see from Figure 3.12, that seems not the case. In the cases of steeper waves, it is not possible to say if there is a group line there or not because of the noise.

As we were interested in the effect the bandwidth had on the group line, results that do not yield this phenomenon is not wanted. Hence, we aborted the Gaussian spectrum and returned to the JONSWAP spectrum. In order to change the bandwidth, we opted to use a different γ and thus, the Pierson-Moskowitz spectrum.

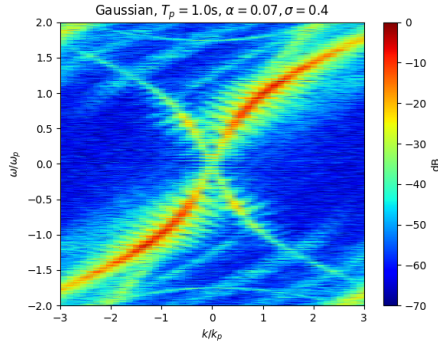
3.2. Experiments



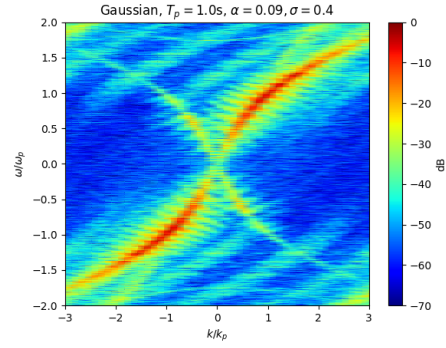
(a) $\alpha = 0.01$.



(b) $\alpha = 0.05$.

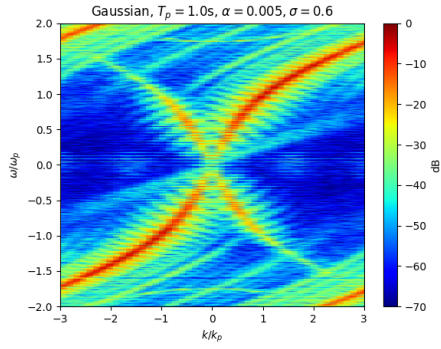


(c) $\alpha = 0.07$.

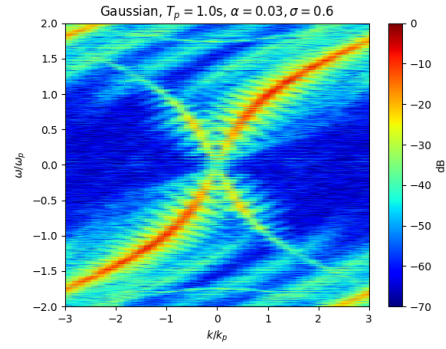


(d) $\alpha = 0.09$.

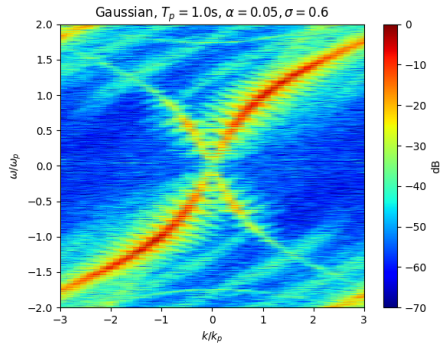
Figure 3.12: The spectrum for the Gaussian runs where $\sigma_g = 0.4$.



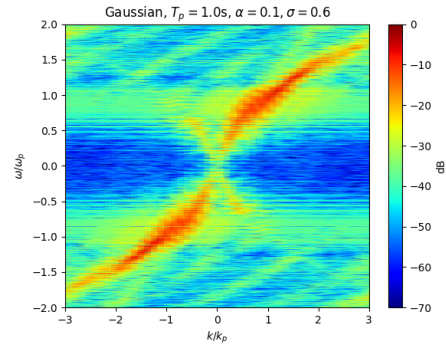
(a) $\alpha = 0.005$.



(b) $\alpha = 0.03$.



(c) $\alpha = 0.05$.



(d) $\alpha = 0.1$.

Figure 3.13: Gaussian runs for $\sigma_g = 0.6$.

CHAPTER 4

Results

After recording the time series for the runs using both the JONSWAP spectrum and the Pierson-Moskowitz spectrum and running 2D spatio-temporal Fourier transform on the data, we then get the results in Figure 4.1-4.4.

From the different figures, we can see that in the runs where the waves were larger and there were more breaking, the group line is becoming thinner and smaller. However, we also notice more noise than the runs with smaller waves and less breaking. So the group line might still be there, but the noise drowns it out. Compared to articles like Plant and Farquharson (2012) and Platzner et al. (2019b), our group line seems to be thinner and more like a line than a cloud. The spectra are also symmetrical around the origin which is due to the waves being real. We can still see traces of the parallel lines that Taklo et al. (2015), but they are not as prominent as in Figure 3.1b. Compared to the lines that Taklo et al. (2015) had, ours are also not consistent in the distance between the parallel lines, something we have in Figure 3.1b. This could mean that other types of errors have occurred like the placement of the probes and the moving of the probes.

We can see the group lines for the runs where there was a lot of breaking and even for the runs with no breaking. This suggests that breaking waves are not the only thing that affects the group line. We can also see that the group line is visible in Støle-Hentschel et al. (2018) and in Anne Raustøl's data (Raustøl, 2014), and in both cases, there were no traces of breaking waves.

4.1 Compensate for damping

In Section 2.6 we were looking at how the energy in the tank will dissipate and that there will be less energy the further down the tank we look. To

4.1. Compensate for damping

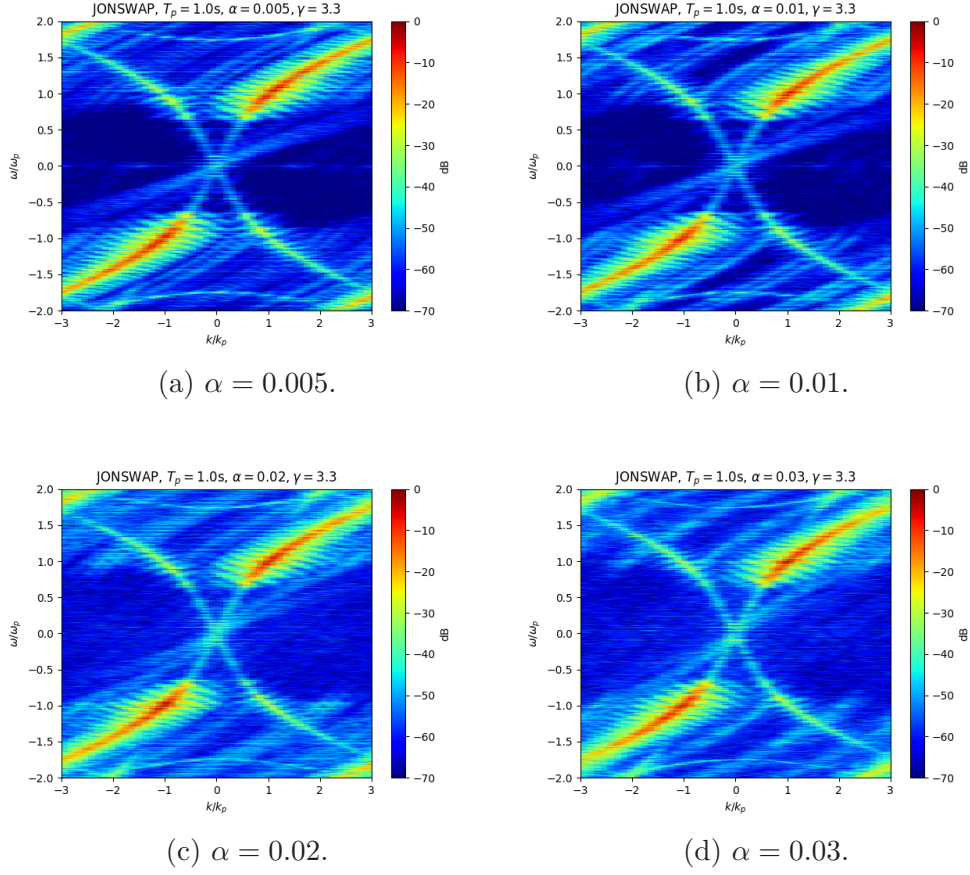


Figure 4.1: The spectra of the JONSWAP runs for lower steepness.

compensate for this, we would try to curve fit the function from (2.22), using the fitting parameters a , b and c . In Figure 4.5-4.8 we can see the results of the curve fit. By dividing the surface elevation by the root of the energy at each point, we can normalise the energy at each point in space, so the energy is constant throughout the tank.

We can see from the figures that the energy is indeed decreasing when we move along the tank. As expected, the energy is generally higher for steeper waves, as there is more energy in bigger waves. We also see that the difference between the energy in the first and last probe is more significant for the steeper waves. This also makes sense as there is more breaking and more energy to dissipate.

The goal of this exercise is to see if the damping in the tank has any impact on the shape or form of the group line. However, when looking at the differences in the spectrum with and without compensating for the damping,

4.2. The exponent of the damping

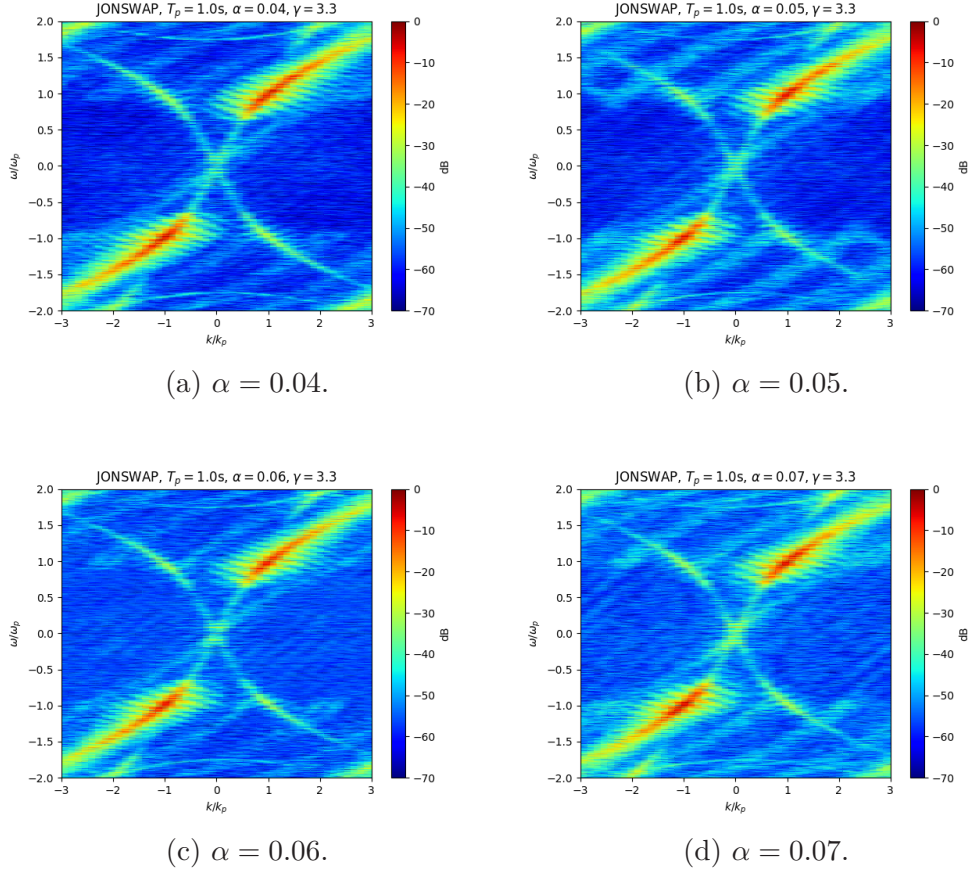


Figure 4.2: The spectra of the JONSWAP runs for higher steepness.

we see no difference at all. In Figure 4.9 and 4.10, we can see there is no difference when trying to compensate for damping.

4.2 The exponent of the damping

In the cases where there is a lot of breaking, we would expect the energy at the first and last probe to differ by a large margin as a lot of the energy could dissipate due to the breaking. This means that the exponent should be quite big in absolute magnitude. On the other hand, in the cases where there are no breaking, we would expect there to be a lot less energy dissipation along the tank. And as such, the exponent should be very small.

From this, we would expect there to be a constant exponent for the steepnesses where there is no breaking. Then when the breaking starts, the exponent should do a small jump as there would be more energy dissipation.

4.2. The exponent of the damping

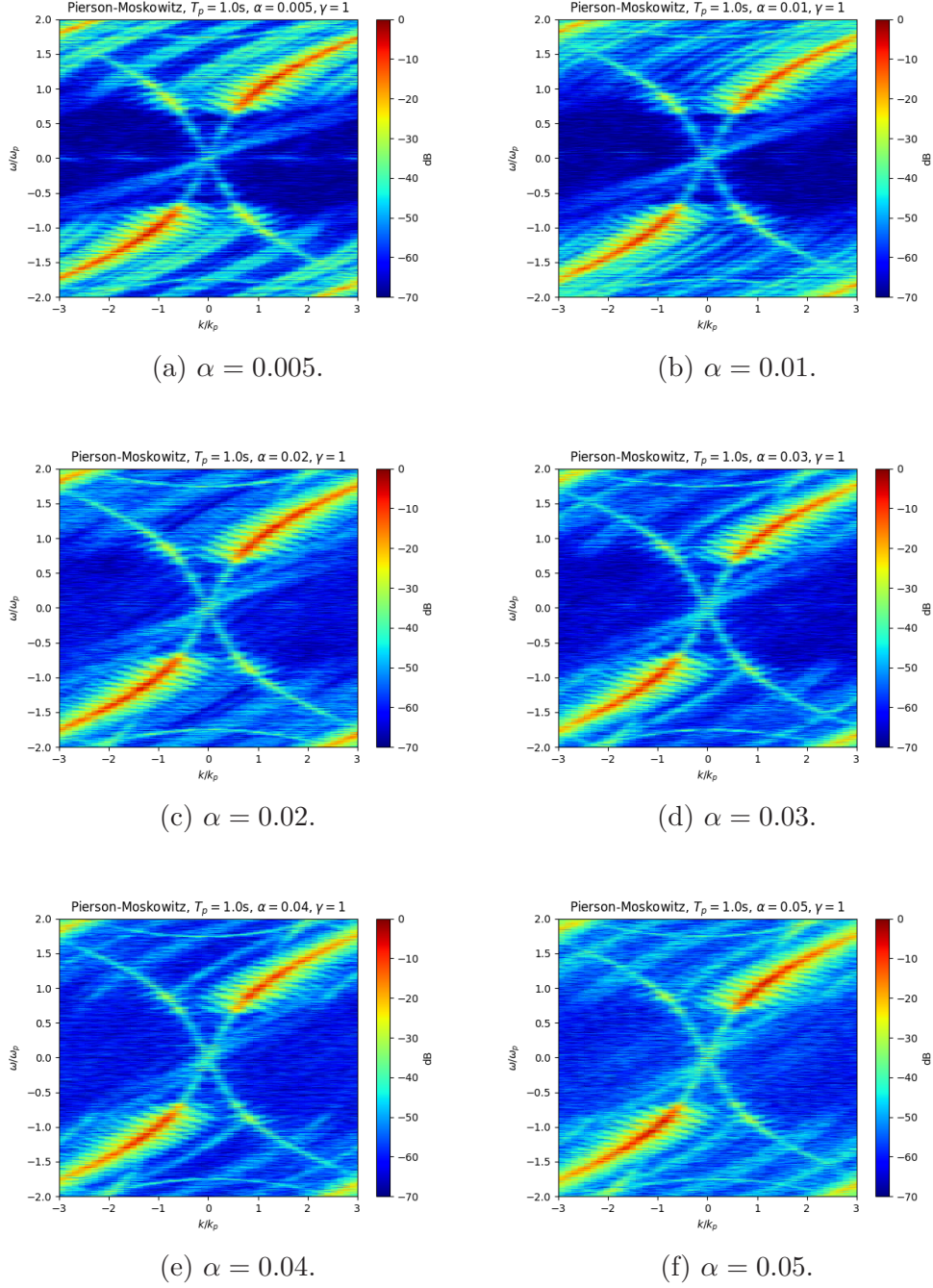


Figure 4.3: The spectra of the Pierson-Moskowitz runs for lower steepness.

4.2. The exponent of the damping

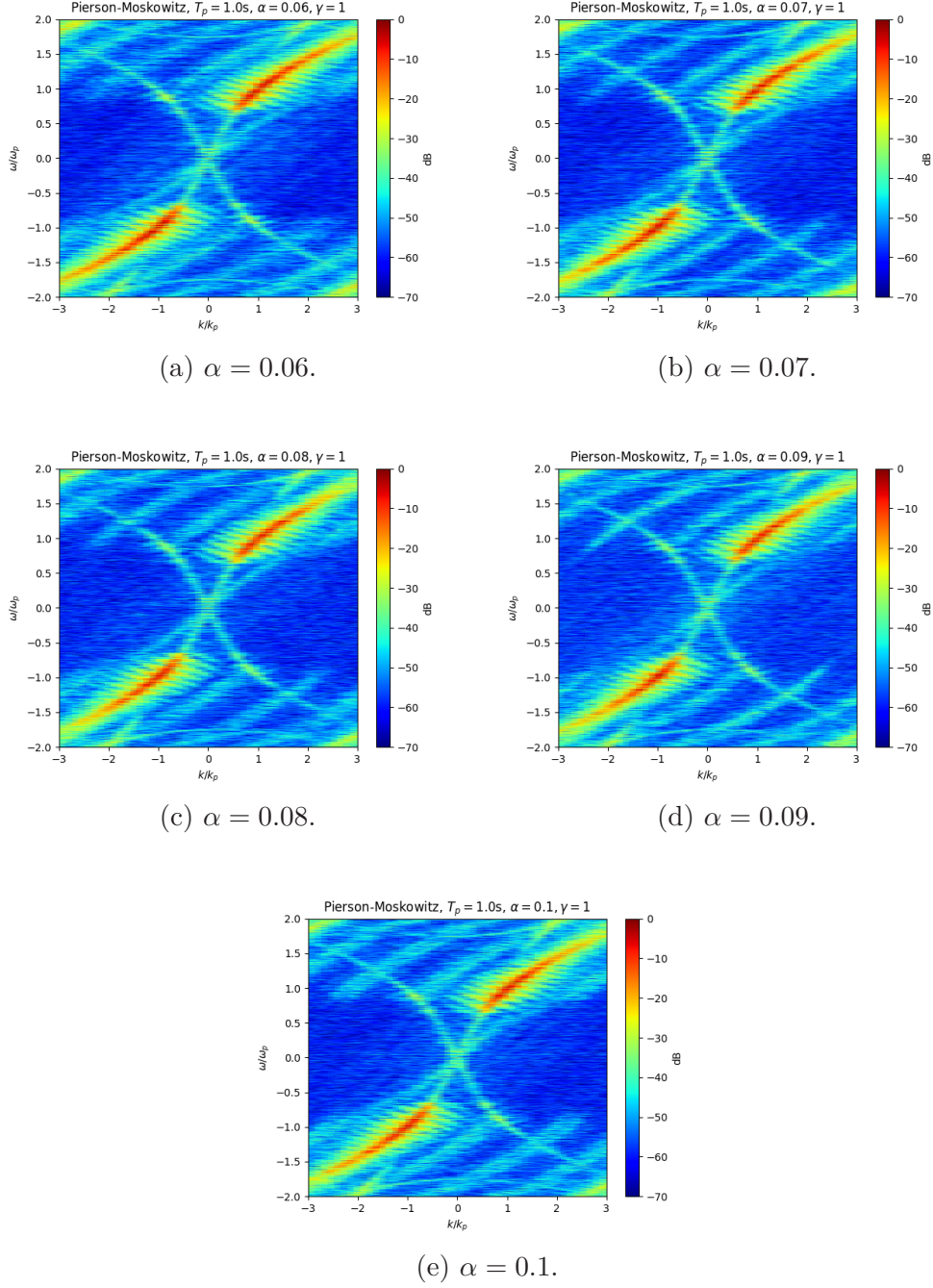
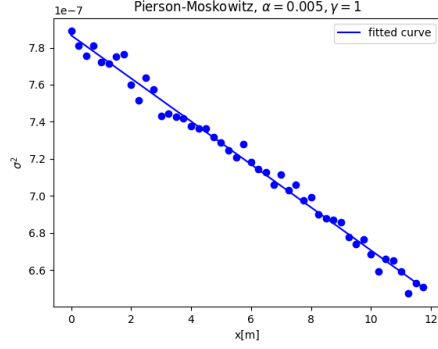
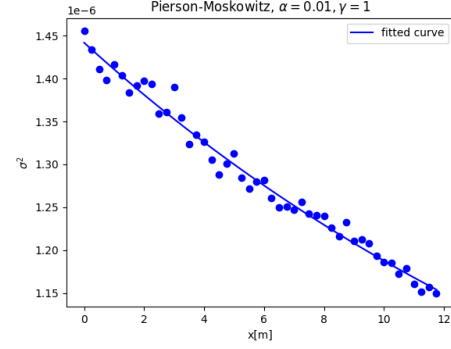


Figure 4.4: The spectra of the Pierson-Moskowitz runs for higher steepness.

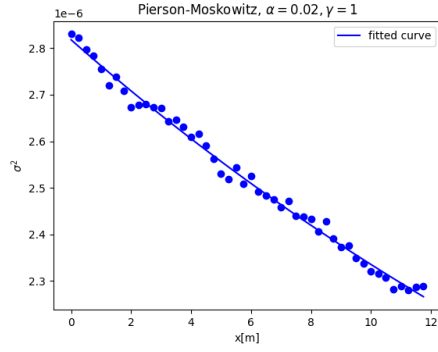
4.2. The exponent of the damping



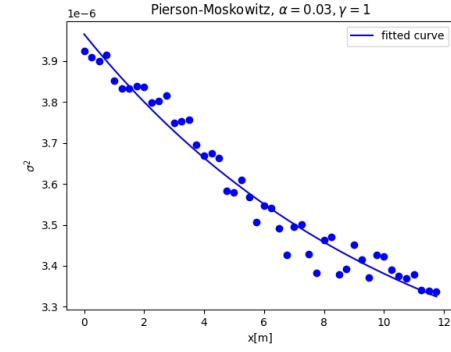
(a) $\alpha = 0.005$



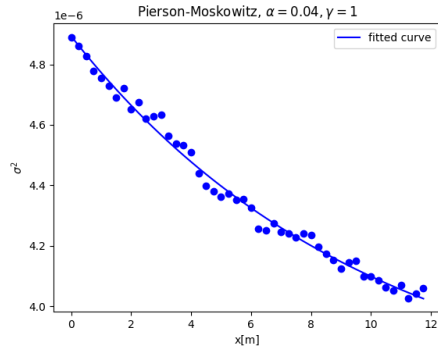
(b) $\alpha = 0.01$



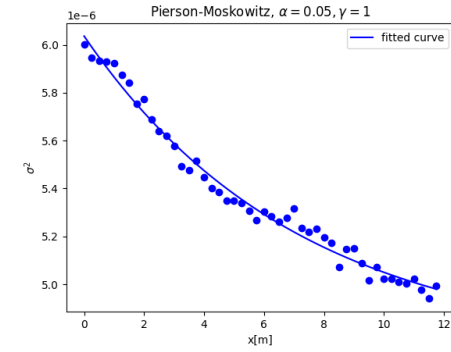
(c) $\alpha = 0.02$



(d) $\alpha = 0.03$



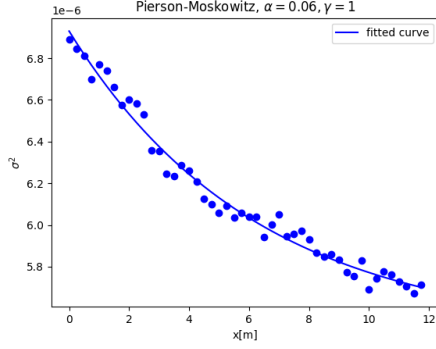
(e) $\alpha = 0.04$



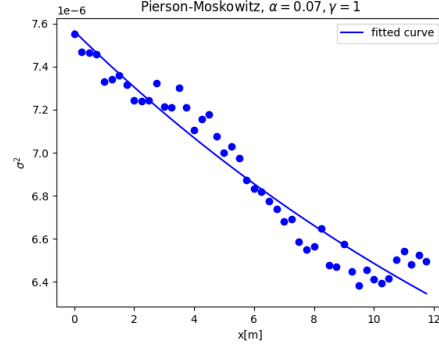
(f) $\alpha = 0.05$

Figure 4.5: The energy in the tank for different runs with the Pierson-Moskowitz spectrum for low steepness. $x = 0$ m is the position of the first probe. The dots represent the energy for each probe in the synthetic array.

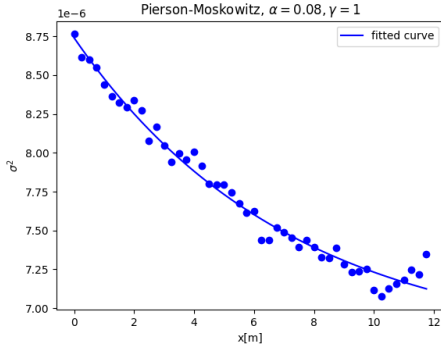
4.2. The exponent of the damping



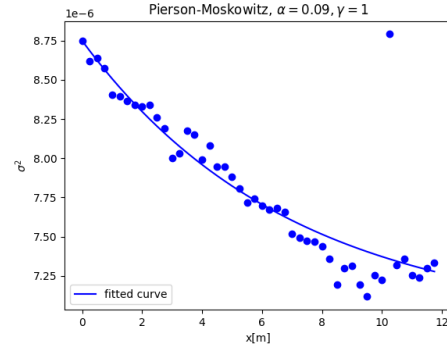
(a) $\alpha = 0.06$



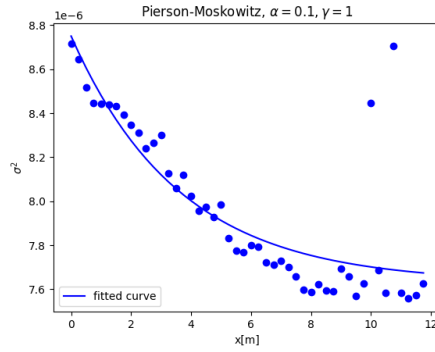
(b) $\alpha = 0.07$



(c) $\alpha = 0.08$



(d) $\alpha = 0.09$



(e) $\alpha = 0.1$

Figure 4.6: The energy in the tank for different runs with the Pierson-Moskowitz spectrum for high steepness. $x = 0$ m is the position of the first probe. The dots represent the energy for each probe in the synthetic array.

4.3. Slopes and steepness

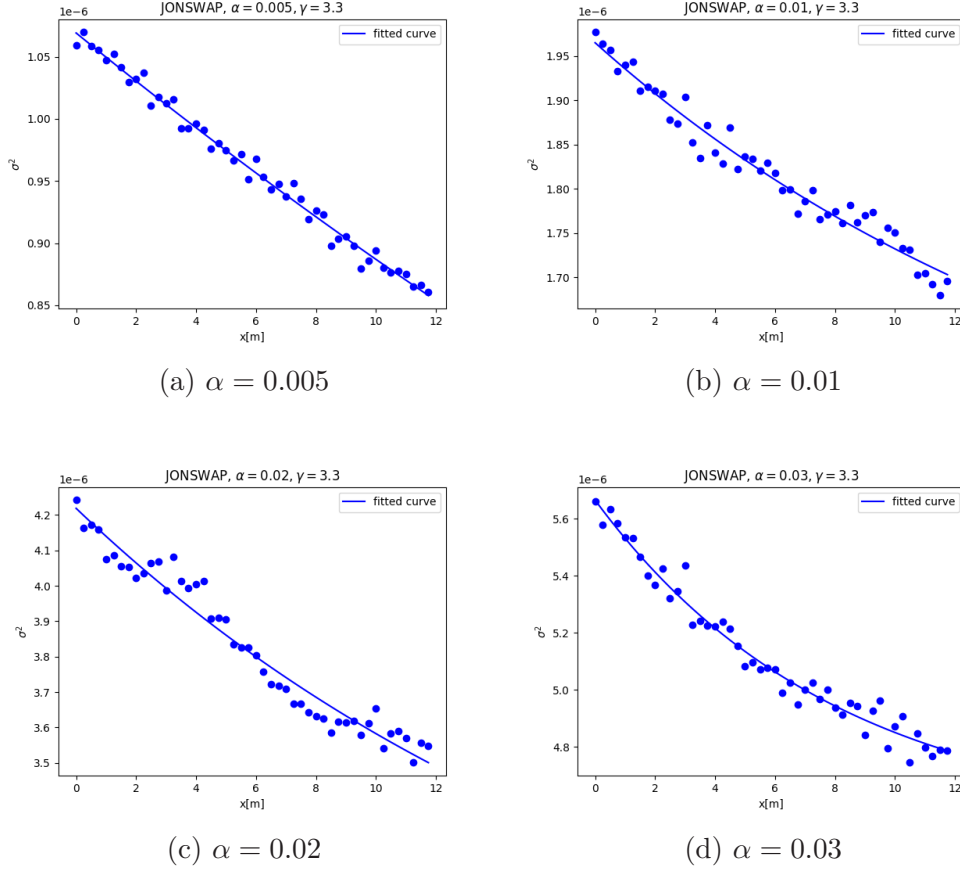


Figure 4.7: The energy in the tank for different runs with the JONSWAP spectrum for low steepness. $x = 0$ m is the position of the first probe. The dots represent the energy for each probe in the synthetic array.

We would think that the exponent gradually becomes bigger as the waves are steeper.

In Figure 4.11 we have plotted the exponent of equation (2.22) for Figure 4.5-4.8. Looking back at Figure 4.5-4.8, it seems that the energy for a lot of the JONSWAP runs is decaying more linearly than exponentially, compared to the Pierson-Moskowitz runs.

4.3 Slopes and steepness

After running the new waves using the Pierson-Moskowitz spectrum with $\gamma = 1$ and plotting the spectrum we find this new phenomenon. It seems that the group line is no longer a single line for some of the wave trains.

4.3. Slopes and steepness

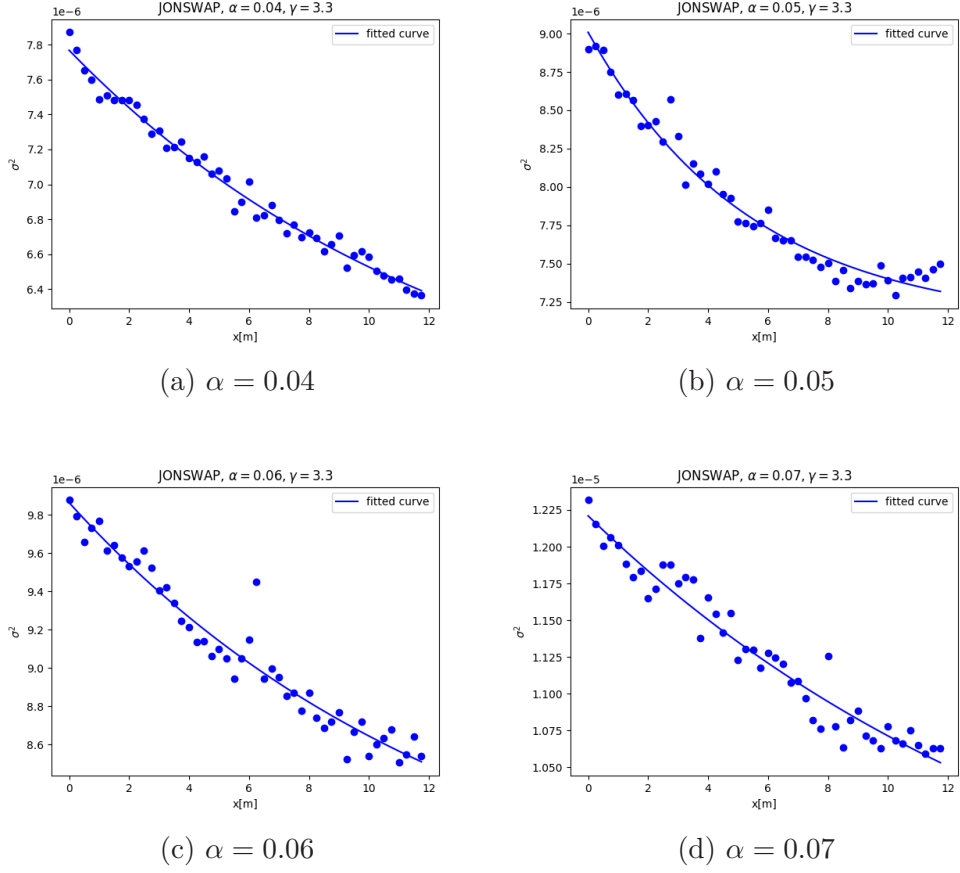


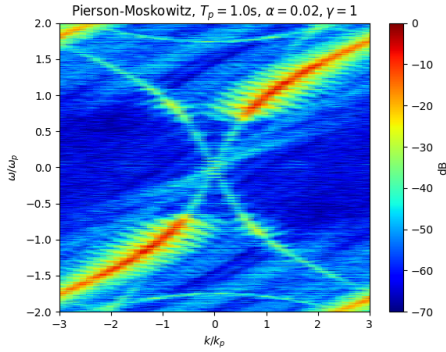
Figure 4.8: The energy in the tank for different runs with the JONSWAP spectrum for high steepness. $x = 0$ m is the position of the first probe. The dots represent the energy for each probe in the synthetic array.

In some cases, it looks like the group line can be separated into two group lines before they seem to merge as one for waves with higher steepness. The double group line appears when the waves seem to almost be breaking, so this could mean that breaking waves is a factor that contributes to the change in slope of the group line. In addition, for the cases with two group lines, it seems that the intensity slowly shifts from the lower group line to the upper group line when the steepness of the waves increases.

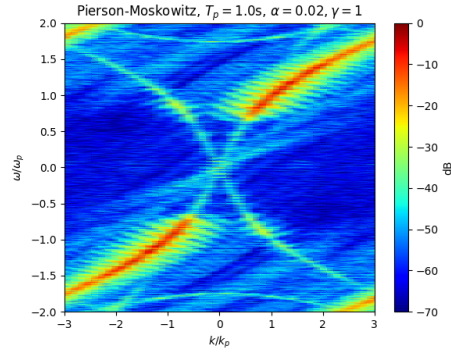
In Figure 4.1c and Figure 4.3c we can see this double group line.

From the runs, it seems like the slope of the group line is changing with the steepness. Therefore, we would like to plot the slope of the group line against the steepness to see if there is any interesting behaviour there. Considering that there seem to be two slopes for some of the steepnesses,

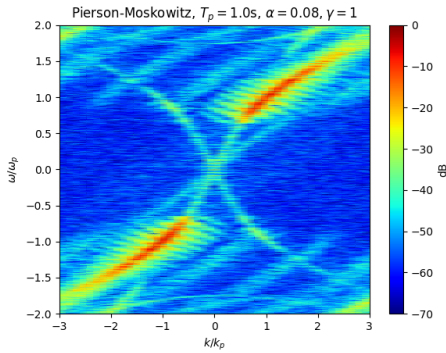
4.3. Slopes and steepness



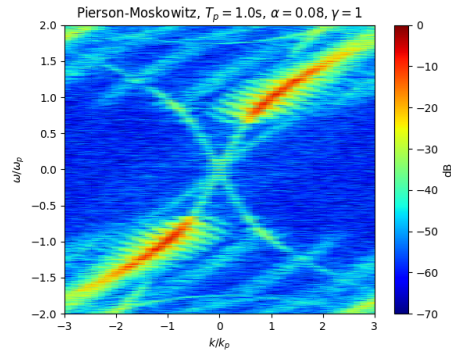
(a) No compensation, $\alpha = 0.02$.



(b) With compensation, $\alpha = 0.02$.



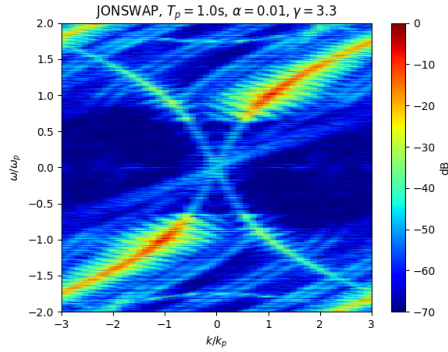
(c) No compensation, $\alpha = 0.08$.



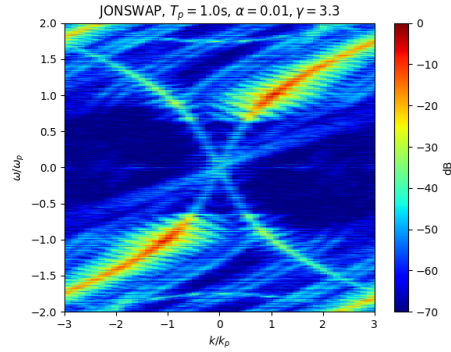
(d) With compensation, $\alpha = 0.08$.

Figure 4.9: Here we can see that the compensation of the damping does not affect the spectrum. The two upper figures (a) and (b) is for low steepness where (a) has no compensation and (b) with compensation. The two bottom figures (c) and (d) are for high steepness where (c) has no compensation and (d) with compensation. Here the Pierson-Moskowitz spectrum is used.

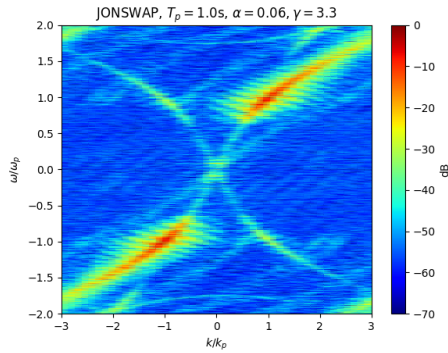
4.3. Slopes and steepness



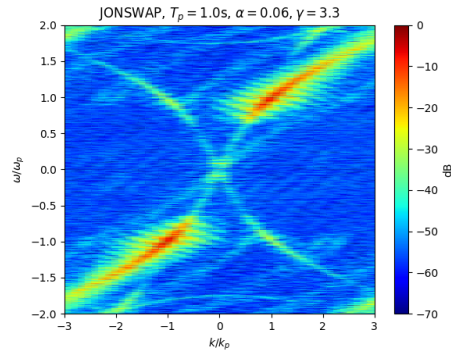
(a) No compensation, $\alpha = 0.02$.



(b) With compensation, $\alpha = 0.01$.



(c) No compensation, $\alpha = 0.06$.



(d) With compensation, $\alpha = 0.06$.

Figure 4.10: Here we can see that the compensation of the damping does not affect the spectrum. The two upper figures (a) and (b) is for low steepness where (a) has no compensation and (b) with compensation. The two bottom figures (c) and (d) are for high steepness where (c) has no compensation and (d) with compensation. Here the JONSWAP spectrum is used.

4.3. Slopes and steepness

γ	α	Steepness	Exponent
1.0	0.005	0.0317	3.039e-05
	0.01	0.04237	0.0464
	0.02	0.0593	0.0332
	0.03	0.0707	0.0959
	0.04	0.07816	0.1012
	0.05	0.0865	0.1401
	0.06	0.0924	0.1605
	0.07	0.0981	0.0487
	0.08	0.1037	0.1400
	0.09	0.1042	0.1442
	0.1	0.1051	0.2756
3.3	0.005	0.0366	0.0154
	0.01	0.0504	0.0553
	0.02	0.0730	0.0504
	0.03	0.0845	0.1247
	0.04	0.0986	0.0765
	0.05	0.1048	0.1850
	0.06	0.1125	0.0749
	0.07	0.1252	0.0594

Table 4.1: Here we can see the data from Figure 4.5-4.8. The exponent column refers to the exponent in equation (2.22).

these have been plotted in the same plot.

The way the slope is found is purely empirical. Noting the value on the y -axis at $k/k_p = \pm 1$, the slope was found by $\Delta y/\Delta x$. This, of course, brings some error into the results.

In Figure 4.12 we see the comparison of the steepness of the waves and the slope of the group line. We see that for the smaller steepness waves, the slope of the group line is relatively constant. Then, at around $\epsilon = 0.08$, the slope of the group line seems to do a jump and then slowly increase as the steepness increases. In addition, in the cases where there are two group lines, the gentler slope seems to align with the slope for the less steep waves. Similarly, the steeper slope aligns with the slope of the group lines for the steeper waves. All the numeric data of the steepness and the slopes can be seen in Table 4.2.

4.3. Slopes and steepness

γ	α	Steepness	Slope	Second slope
1.0	0.005	0.0317	0.29	-
	0.01	0.0424	0.295	-
	0.02	0.0593	0.31	0.49
	0.03	0.0707	0.32	0.51
	0.04	0.0782	0.34	0.53
	0.05	0.0865	0.49	-
	0.06	0.0924	0.495	-
	0.07	0.0981	0.52	-
	0.08	0.1037	0.54	-
	0.09	0.1042	0.535	-
	0.1	0.1051	0.545	-
3.3	0.005	0.0366	0.28	-
	0.01	0.0504	0.295	0.48
	0.02	0.0730	0.305	0.505
	0.03	0.0845	0.49	-
	0.04	0.0986	0.515	-
	0.05	0.1048	0.525	-
	0.06	0.1125	0.545	-
	0.07	0.1252	0.56	-

Table 4.2: Table of the data in Figure 4.1-4.4. By slope, the slope of the group line is meant, and by the second slope, the steeper group line is meant.

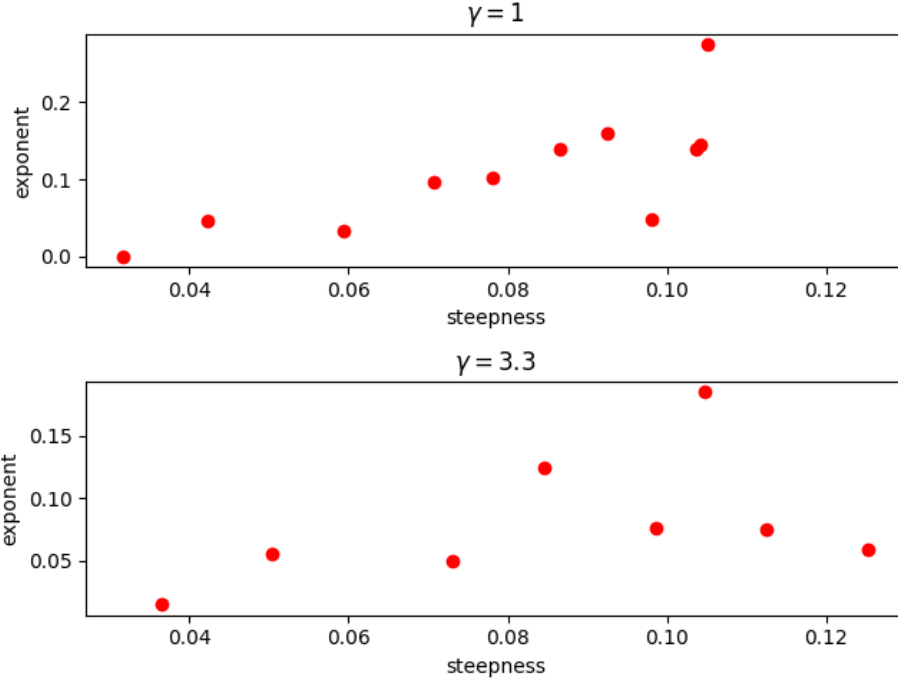


Figure 4.11: A comparison of the steepness and the exponent for the decay in energy. A visual representation of Table 4.1. The values of the exponent are positive because of how we defined it in equation (2.22). Here $\gamma = 1$ is referring to the Pierson-Moskowitz runs while $\gamma = 3.3$ is referring to the JONSWAP runs.

4.4 Comparing to simulations

After finding this two-group line feature, we want to compare this to simulations and earlier work that others have done. The equations used to simulate the waves are given by equations (2.26) and (2.27).

These equations are from Toffoli et al. (2006), where the first one should give us the linear properties while the second one is the reason for the non-linear properties. In the simulations, we are using both the JONSWAP spectrum and the Pierson-Moskowitz spectrum as input. The parameters are given to be as close to the real experiments as possible. The only notable change is the number of points in space, N , which is increased from 48 points to 100 points, but still over the same interval $[0, 11.75]$. In addition, the equations depend on θ , which gives waves going in different directions. As our experiments are long-crested, we have set all $\theta = 0$, making the equations somewhat simpler. This also makes the double sum in equation (2.24) a single sum, and the quadruple sum in equation (2.25) a double sum.

4.4. Comparing to simulations

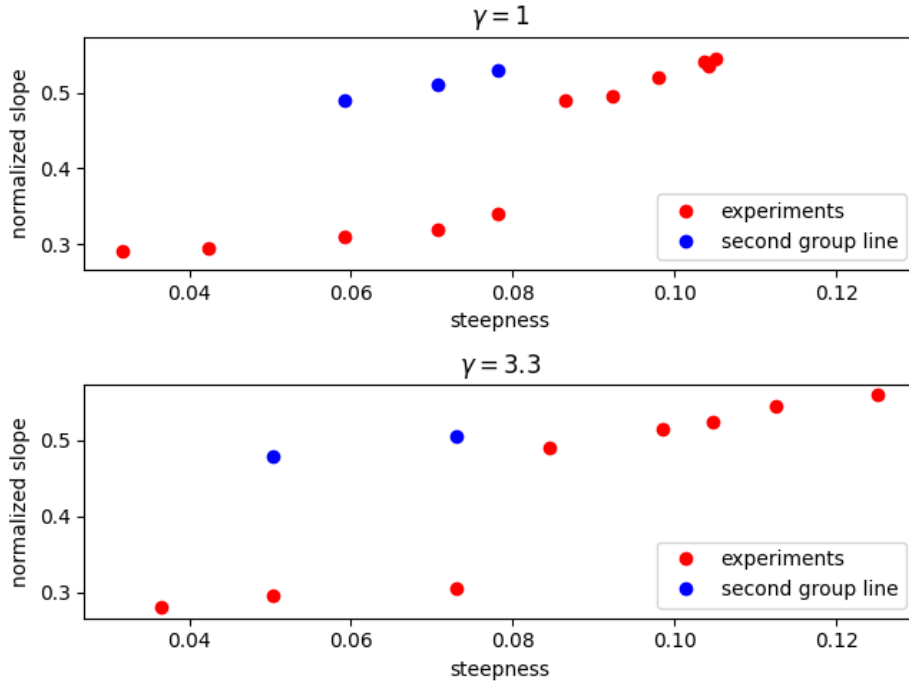


Figure 4.12: Here we show the steepness of the waves vs the slope of the group line. The blue dots represent the second group line in the cases where it was found. Here $\gamma = 1$ represents the Pierson-Moskowitz runs and $\gamma = 3.3$ represents the JONSWAP runs. The slope is normalised with the phase speed of the peak wave.

The results of the simulations can be seen in Figure 4.13 and Figure 4.14.

In Table 4.3 we can see the numerical results of the simulations.

The slope is found the same way as before. As we can see from Table 4.3, there are some irregularities, which comes from the fact that they are measured empirically. What seems to be the case for both γ is that the slope is constant and independent of the waves' steepness. In Figure 4.15 we have plotted Table 4.3 along with the results of the experiments.

We mentioned in Chapter 1 that there can be multiple factors that might be responsible for the group line. In the experiments we have both breaking and non-breaking waves. If breaking waves are partly responsible for the group line and second order, non-linear interactions are partly responsible for the group line, then the simulations will only show the part that the non-linear interactions are responsible for. This could be why we are only seeing one group line in the simulations, but have cases of two group lines in the experiments.

We can get from this that perhaps the equations we are using are not

4.4. Comparing to simulations

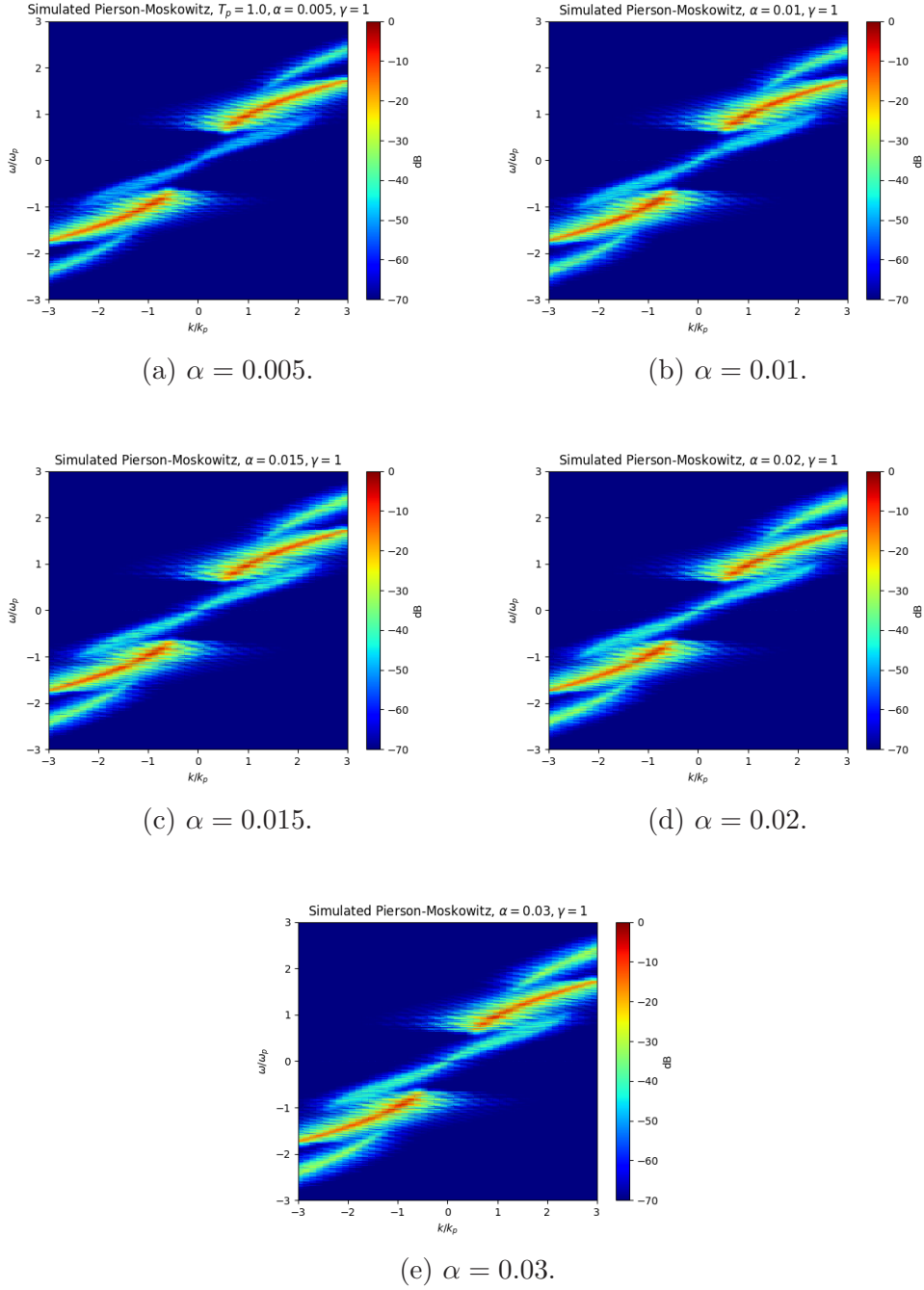


Figure 4.13: The spectra of the Pierson-Moskowitz simulations for different values of α .

4.4. Comparing to simulations

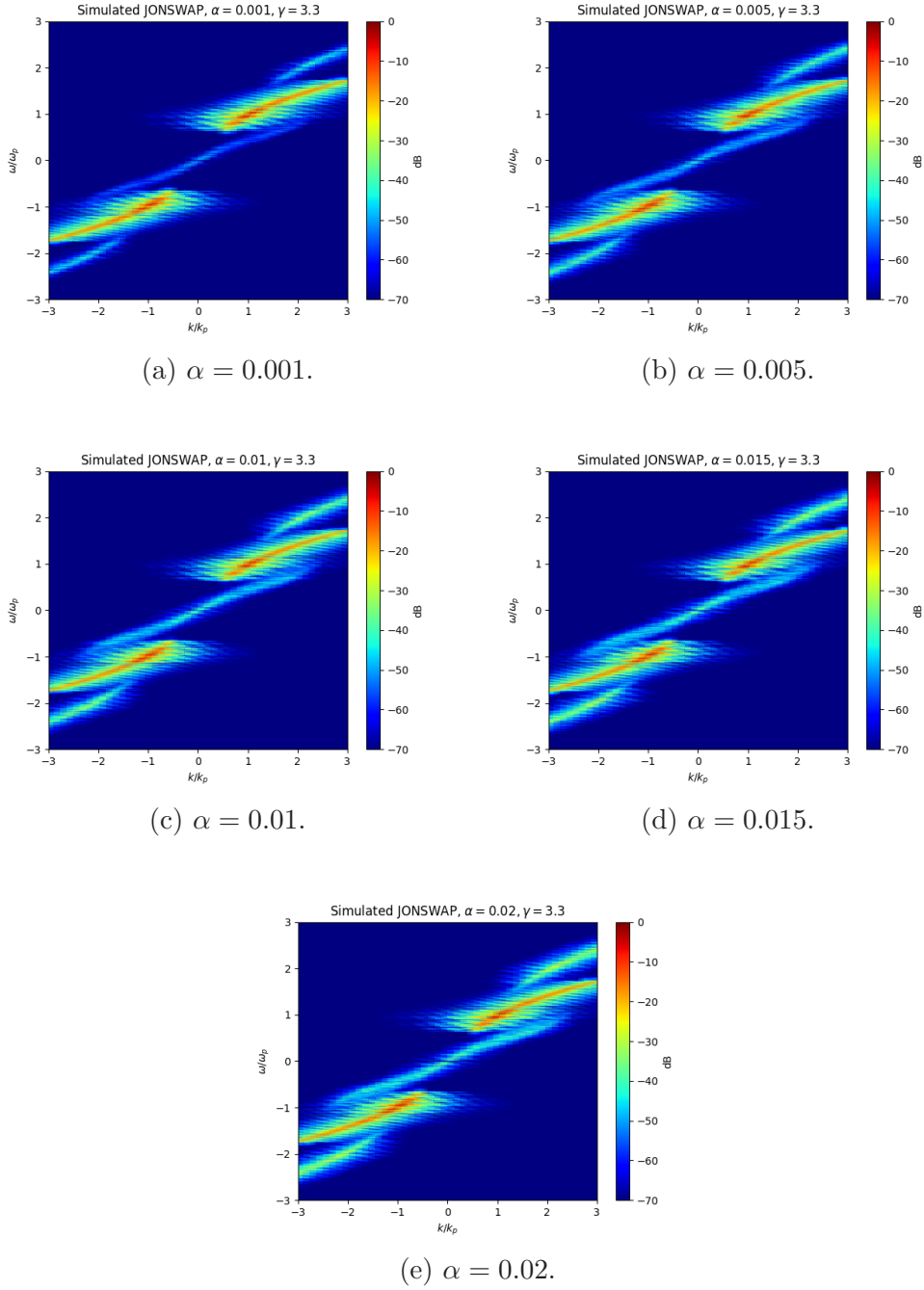


Figure 4.14: The spectra of the JONSWAP simulations for different values of α .

4.5. Comparing to other work

γ	α	Steepness	Slope
1.0	0.005	0.0428	0.3875
	0.01	0.0610	0.39
	0.015	0.0773	0.3925
	0.02	0.0918	0.4
	0.03	0.1191	0.3975
3.3	0.001	0.0240	0.3875
	0.005	0.0540	0.385
	0.01	0.0791	0.39
	0.015	0.0989	0.39
	0.02	0.1165	0.3875

Table 4.3: Steepness and slope of the group line for different α for the simulations. $\gamma = 1$ is for the Pierson-Moskowitz runs and $\gamma = 3.3$ is for the JONSWAP runs.

thorough enough and that there may be something about waves that the equations we currently have cannot describe.

4.5 Comparing to other work

To compare the results from the experiments and the simulations, we would like to compare this to radar images. In this case, we will compare with Kammerer and Hackett (2019), Plant and Farquharson (2012) and Platzer et al. (2019b). Kammerer and Hackett (2019) and Plant and Farquharson (2012) are recent articles, and Plant and Farquharson (2012) and Platzer et al. (2019b) are both focusing on the group line.

Platzer et al. (2019b) have two figures that we will be comparing with, Figure 3 and 4 which is here given as Figure 4.16a and 4.16b. Here they have plotted the (k, ω) wave spectrum of data acquired by two different radars, the MARSIG radar and the MEMPHIS radar, respectively. The grazing incidence angles were 1.378° and 0.794° , and the peak periods of the waves were $T_p = 8.21$ s and $T_p = 7.83$ s respectively. The slopes of the group line are both found empirically. Because they have units on their axis, we scale with ω_p and k_p found by T_p and the dispersion relation, see equation (2.4).

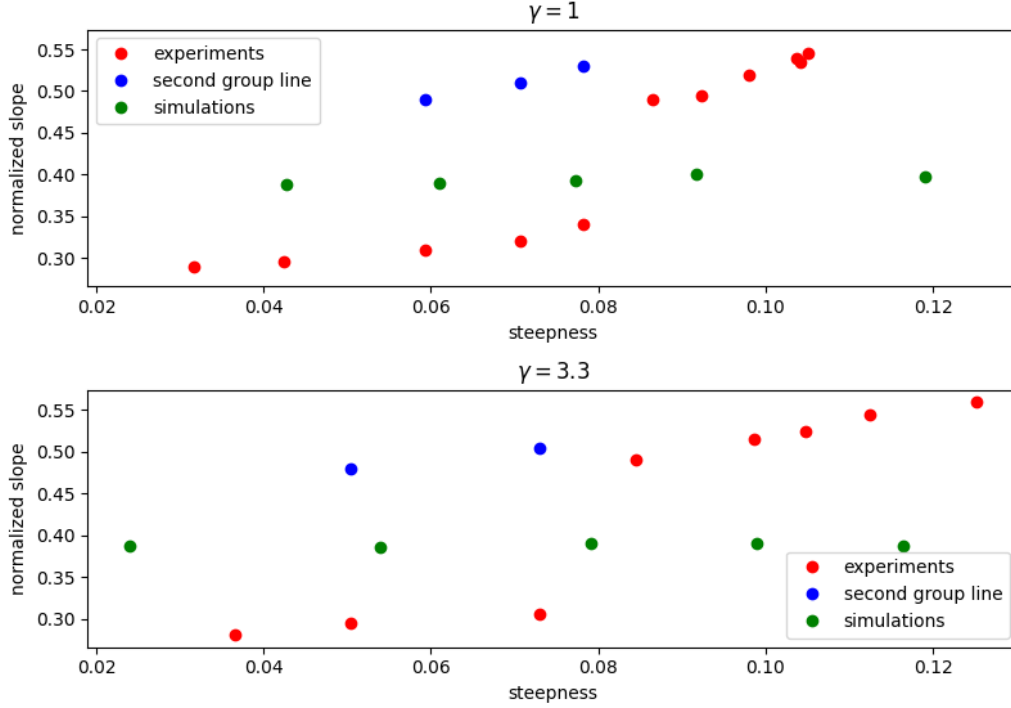


Figure 4.15: The exact figure as Figure 4.12, but here we have added data from the simulations. As we can see, the simulations do not change dependent on the steepness. $\gamma = 1$ is referring to the Pierson-Moskowitz runs while $\gamma = 3.3$ is referring to the JONSWAP runs. The slope is normalised with the phase speed of the peak wave.

In Kammerer and Hackett (2019), we are looking at Figure 7a which is here given as Figure 4.16c. They have used logarithmic axes with little information along the axes, making it hard to find the slope of the group line. However, the dispersion relation is included in the figure, so it was possible to find the slope of the group line by figuring out how the dispersion relation should behave for logarithmic axes. That does not help us in the end as they also have units on their axes, and there is no information given to find ω_p and k_p . Therefore, we cannot say anything more about how their results compare to ours.

Plant and Farquharson (2012) have used data from a CORAR radar on a cruise ship. The data used is collected from when the radar was not rotating during the end of the cruise. The figure we are looking at is Figure 1b which is here given as Figure 4.16d. The grazing incidence angle is near 1° , and the peak frequency is 0.162 Hz. The slope of the group line is also here found empirically and similar to when comparing to Platzer et al. (2019b), the

4.5. Comparing to other work

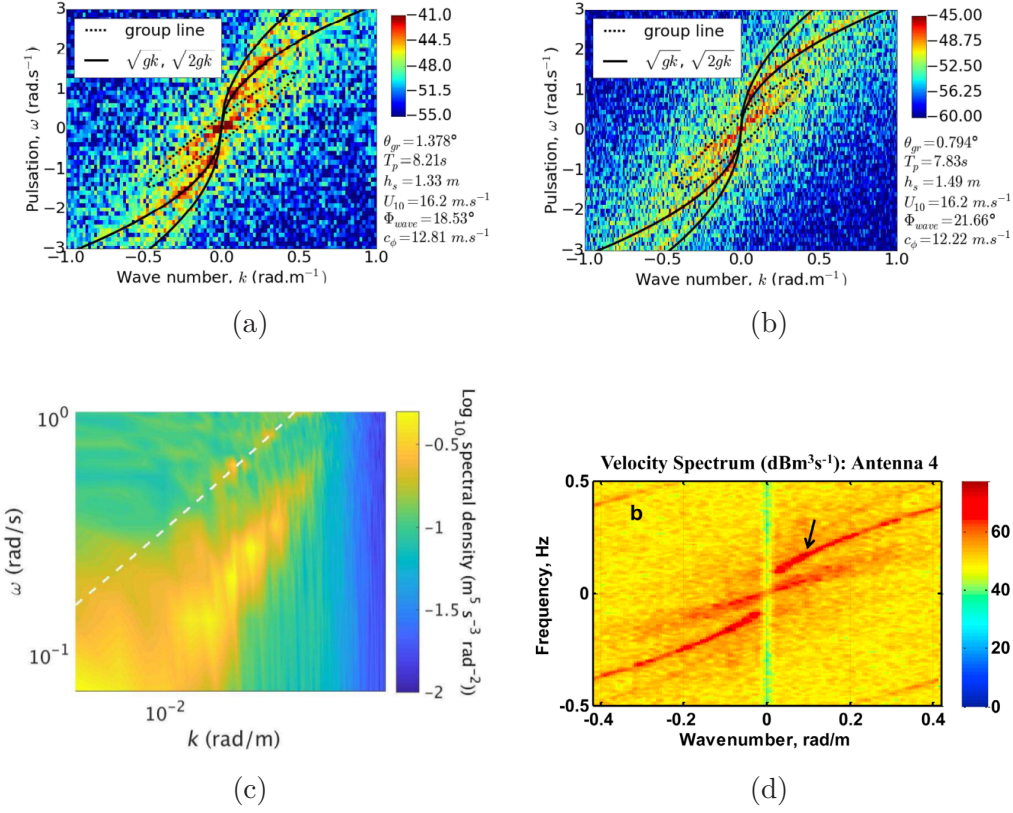


Figure 4.16: Here we have the four figures we are comparing with. (a) and (b) are Figure 3 and Figure 4 in Platzter et al. (2019b). (c) is Figure 7a in Kammerer and Hackett (2019). (d) is Figure 1b in Plant and Farquharson (2012). The dotted line in (c) is the dispersion relation and the arrow in (d) indicates the dominant wave peak.

axes were scaled before finding the slope.

In Figure 4.17 we have the same as in Figure 4.15 but we have also included the results of Plant and Farquharson (2012) and Platzter et al. (2019b). Because the steepness of the waves is not given in the articles, only the steepness of the slope has been considered and plotted. What we see is that the slope of Figure 4.16b aligns with the slope of Figure 4.16d, and they both align with the experiments where the steepness is low. This is unexpected as the data acquired by Platzter et al. (2019b) is data from close to shore, where we would expect more breaking compared to further out on the ocean.

A reason for this might be because of the way the radar is measuring the waves. In the introduction, we mention that shadowing might be a cause

4.5. Comparing to other work

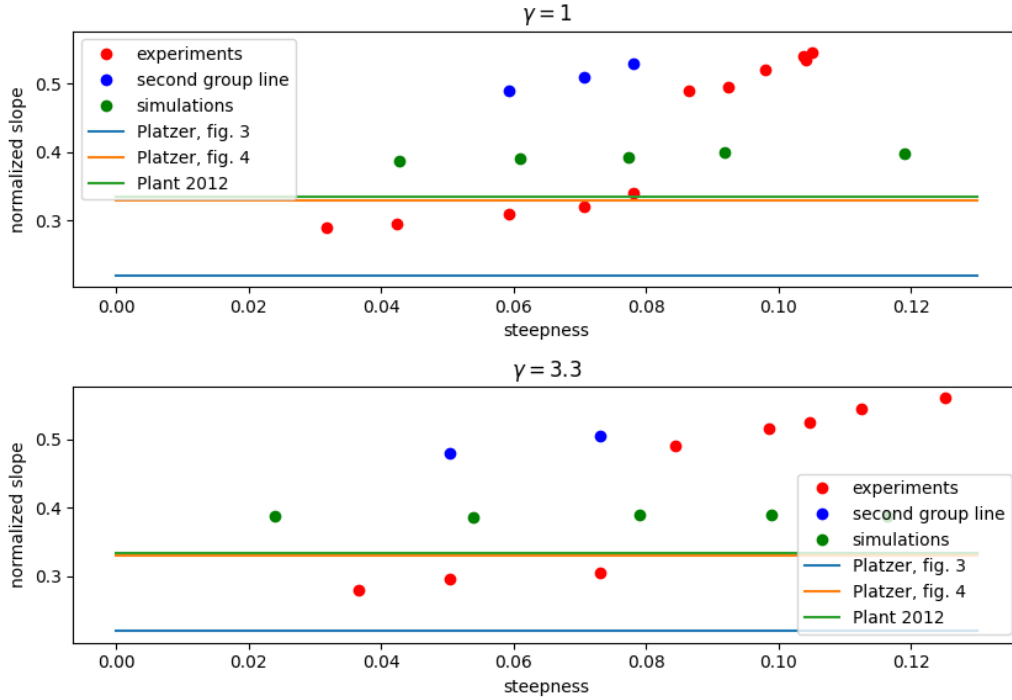


Figure 4.17: The same as Figure 4.15, but here we have added the slopes of Figure 4.16a, 4.16b and Figure 4.16d. The steepness is not mentioned in the article, and as such, we only give the slope of the group line. Here $\gamma = 1$ is referring to the Pierson-Moskowitz runs while $\gamma = 3.3$ is referring to the JONSWAP runs. The slope is normalised with the phase speed of the peak wave.

for the group line. However, it might also be the cause for why the slope is so gentle compared to our experiments where there are no shadowing happening. In Platzter et al. (2019b), it is mentioned in the article that the grazing incidence angle between the radar and the ocean of the two figures that we have been considering is both below 2° . In Plant and Farquharson (2012) the grazing incidence angle is about 1° .

Another reason could be because we have computed our spectrum based on the surface elevation, while Figure 4.16d of Plant and Farquharson (2012) and Figure 4.16b of Platzter et al. (2019b) are Doppler spectra. We would like to see how these will compare.

Using the boundary equations for the free surface, we have the dynamic boundary condition;

$$\frac{\partial \phi}{\partial t} + g\eta = 0 \quad (4.1)$$

and the kinematic boundary condition;

$$\frac{\partial \eta}{\partial t} = \frac{\partial \phi}{\partial z} \quad (4.2)$$

where η is the surface elevation, ϕ is the velocity potential and g is the acceleration due to gravity. Using equation (2.2) in equation (4.2) yields us

$$\begin{aligned} -i\omega\hat{\eta} &= k\hat{\phi} \\ \implies \hat{\phi} &= -\frac{i\omega}{k}\hat{\eta} \end{aligned}$$

where $\hat{\eta}$ and $\hat{\phi}$ is the Fourier transform coefficient. Using that $U = \frac{\partial \phi}{\partial x}$ gives us in the end

$$\hat{u} = ik\hat{\phi} = \omega\hat{\eta}$$

This tells us that the difference between the velocity and the surface elevation is a factor of ω . For low angular frequencies, the intensity should be more prominent in our case. In contrast, for high angular frequencies, the intensity should be less apparent than Plant and Farquharson (2012) and Platzer et al. (2019b). Other than the intensity of the different parts of the spectrum, the shape and features should still be the same.

However, Plant and Farquharson (2012) uses frequencies along the y -axis, while Platzer et al. (2019b) have the angular frequency along their y -axis, the same as us. This means that the y -axis of Plant and Farquharson (2012) is more compact and should be stretched with a factor of 2π , giving a gentler slope than what we have in Figure 4.17.

In Figure 4.17 the slope of Figure 4.16d from Plant and Farquharson (2012) is very close to the slope of the group line in Figure 4.16b from Platzer et al. (2019b). When considering that the y -axis is different, the slope in Plant and Farquharson (2012) should be gentler and, as a result, move downwards towards the slope of Figure 4.16a from Platzer et al. (2019b). When we then consider the angle of the three figures, Figure 4.16b has the largest angle of 1.378° , Figure 4.16d have an angle of $\sim 1^\circ$ and the smallest angle is Figure 4.16a with an angle of 0.794° . This could indicate that the

inclination angle of the radar could have an impact on the slope of the group line. There has not been found any research about this, meaning it could also just be a coincidence.

A third reason could be that in both the experiments and the simulations, the waves have been long-crested, while the waves in the radar images are short-crested. So it might be that for short-crested waves, the slope, in general, is gentler than for long-crested waves. We have knowingly used only one dimension in space to compare to the experiments in the simulations we have done. So to compare with the radar images, we should also simulate short-crested waves and see if the slope of the group line changes.

In order to make the short crested waves, equations (2.24) and (2.25) will be used instead of equations (2.26) and (2.27). This means that $E(\omega_i, \theta_j)$ also consist of a directional part, $E(\omega_i, \theta_j) = S(\omega_i)D(\omega_i, \theta_j)$. In this case, $S(\omega_i)$ is the JONSWAP spectrum while $D(\omega_i, \theta_j)$ is the directional spreading function (Toffoli et al., 2006) given by

$$D(\omega, \theta) = \frac{\Gamma(s(\omega) + 1)}{2\sqrt{\pi}\Gamma(s(\omega) + 1/2)} \cos^{2s(\omega)}\left(\frac{\theta}{2}\right) \quad (4.3)$$

where Γ is the Gamma function and $s(\omega)$ is a function, describing the magnitude of the directional spreading (Toffoli et al., 2006). In this case

$$s(\omega) = \begin{cases} \left(\frac{\omega}{\omega_p}\right)^5 s_{max} & \text{if } \omega \leq \omega_p \\ \left(\frac{\omega}{\omega_p}\right)^{-2.5} s_{max} & \text{if } \omega > \omega_p \end{cases} \quad (4.4)$$

where $s_{max} = 10$ corresponds to wind waves (Toffoli et al., 2006). We can see how the directional spreading function looks in Figure 4.18. Here we show $D(\omega, \theta)$ against θ to illustrate how the spreading of the waves are distributed. As we can see from the peak, we have $\theta = 0$ as the main direction. The different graphs correspond to different values of ω where the ω are in the same range as the simulations.

However, due to lack of time and the length of the simulations, we have not been able to explore this fully.

4.6 NFFT

In Section 3.1 we are mentioning that Taklo et al. (2015) had problems with parallel lines in their spectrum and that we got the same feature in the beginning. One of the early hypotheses for this feature was inaccuracies

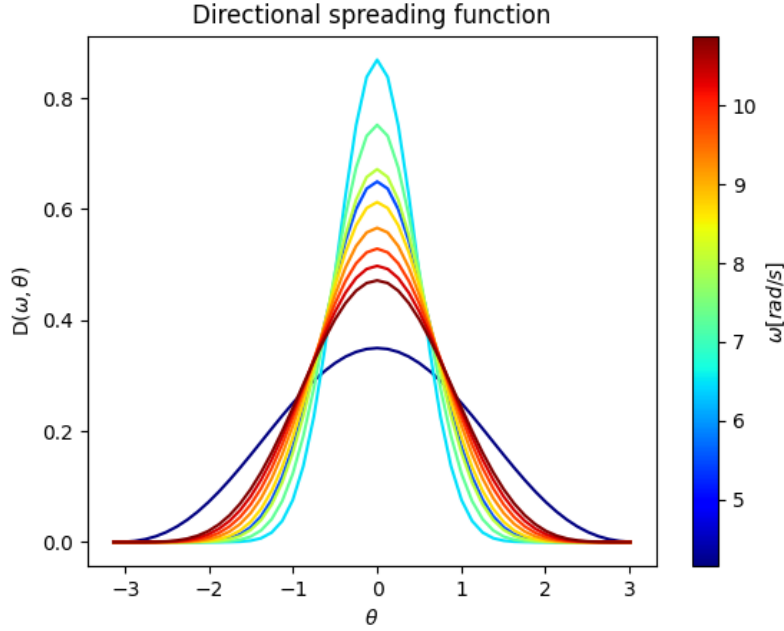


Figure 4.18: The directional spreading function. The different graphs are for different values of ω . The value of ω for each graph is related through the colour bar.

in the measurements. When calculating the spectrum, we assume that the probes are uniformly spaced down the tank, and if they are not, this might affect the results. The placement of the probes on the rail and moving the railing along the tank can give small displacements in the position that accumulate to a significant error. We, therefore, will try to simulate how much this possible error will affect the spectrum.

We simulated unidirectional linear waves using equation (2.2) and then computed the 2D spatio-temporal Fourier transform of the simulation. The result we then got can be seen in Figure 4.19. Here we have used the same number of points in time as in the experiments, $M = 33750$, the same $\Delta t = 0.008\text{s}$ and $T_p = 1.0\text{s}$. The number of points in space is $N = 60$ with $\Delta x = 20.34\text{cm}$, giving the total length $L = 12\text{m}$. The idea is first to see how much the error is causing, then try to accommodate the error by using NFFT(Nonuniformly spaced Fast Fourier Transform)(Potts et al., n.d.).

We now do the same, but this time we include an error in the placement of each probe. A random number of $[-0.02, -0.01, 0, 0.01, 0.02]$ is added to the position of the probe. This should be equivalent to an error of $\pm 2\text{ cm}$ in the experiments, 0.17% of the total length, or up to 10% of the distance

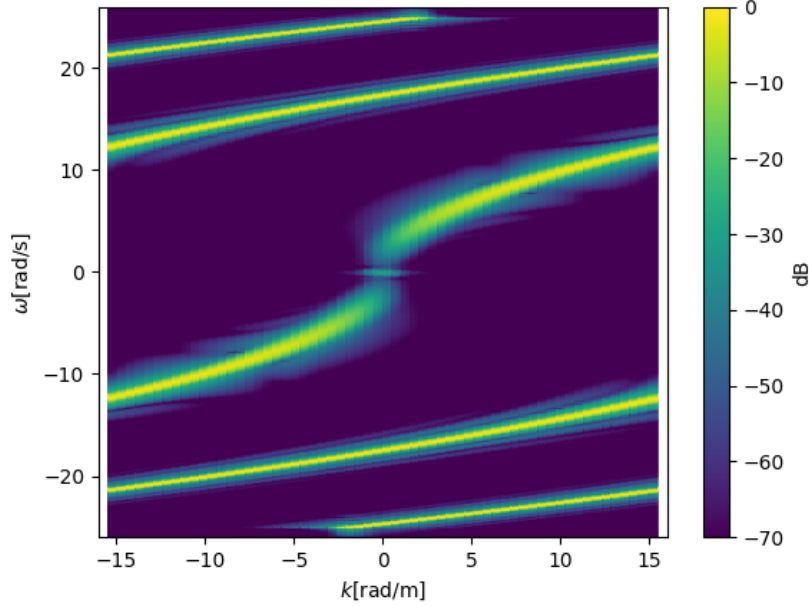


Figure 4.19: Here we have the spectrum of the simulated linear waves. We can see that the dispersion relation is folded due to aliasing.

between each probe. Instead of only adding a small error to each placement, an accumulation of errors could also be a viable implementation. However, since each position is already decided, a small displacement to the position seems accurate compared to the experiments. Now computing the spectrum with the added error gives us Figure 4.20. We can still clearly see where the dispersion relation is, but we also see that the displacement affects the spectrum by the energy being distributed to a lot of the frequencies. We now try to use NFFT to account for the displacement.

FFT assumes that the data points are uniformly distributed. Because this is not always the case, three universities (Chemnitz University of Technology, Osnabrück University and the University of Lübeck) have developed an algorithm to compute the DFT for the instances when the data is nonuniform. The NFFT algorithm can compute the DFT in multiple dimensions, of arbitrary data sizes and of complex values (Potts et al., n.d.).

It is possible to download the NFFT package for MATLAB or Octave from their web page (Potts et al., n.d.). There is also a version for Julia. In the case of Python, one can download the package from PyPI (Python Package Index) that relies on Numpy and SymPy. Both packages are written in C and need a C compiler to use.

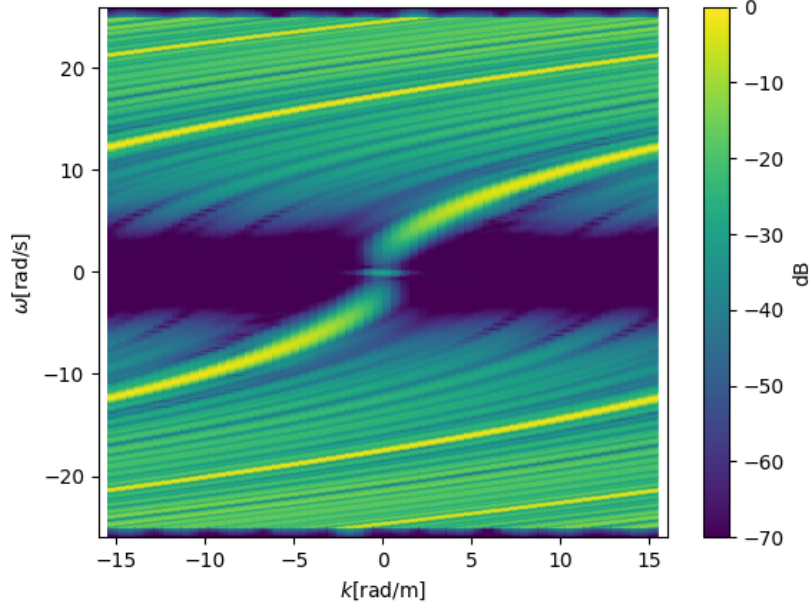


Figure 4.20: The computed spectrum when including error in the placement of the probes.

When using NFFT on the data with the error, we get Figure 4.21. From the figure, there does not seem to be an improvement to Figure 4.20. It also appears to be a lot less smooth compared to the spectrum of Figure 4.19 and 4.20. There is no obvious explanation for this, and the most straightforward reason might just be wrong usage of the algorithm.

4.7 Robustness

The steepness is a function of the characteristic amplitude, a_c , which again is a function of the standard deviation of the surface elevation, σ . We have used all probes to calculate the standard deviation for all simulations and experiments where we have used the steepness. However, it is also possible to not use all the probes, but only a couple of them. We want to know if the number of probes used for the standard deviation has a sizable impact or not.

The most interesting case is to see how much it will change Figure 4.12 as this is a phenomenon not yet seen. If the steepness changes, then the jump of the slope of the group line also changes, which means that the start of

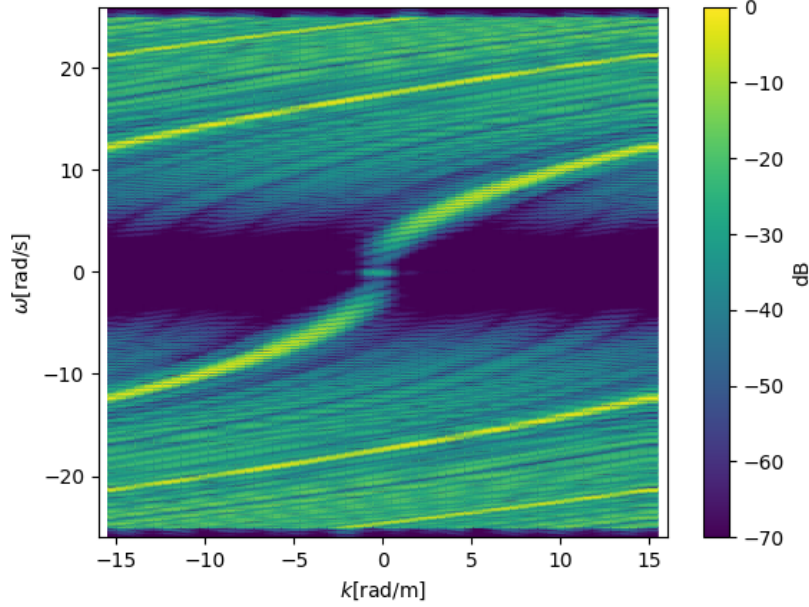


Figure 4.21: The spectrum where we have used NFFT to accommodate for the displacement of the probes.

the wave-breaking might not be the cause of the jump in the slope. Because all the probes are in the second half of the tank, we would expect more breaking by the first probe that is closer to the wavemaker and very little by the very last probe that is furthest from the wavemaker. This should give a higher steepness if we just consider the first probe compared to when we use all of them. This means that we would expect all the data points in Figure 4.12 to shift towards the right horizontally to higher steepness.

In Figure 4.22 we have plotted Figure 4.12 but here we have also plotted the data point for different steepness. The smaller, lighter dots represent the steepness when looking at the different number of probes when calculating the standard deviations of the surface elevation. The big dot has all 48 probes considered. The smaller dot represents 40, 30, 20, 10 and 1 probes in sequence. Furthermore, it is the probes closest to the wavemaker that is considered. As we earlier suspected, the dots are shifted right when only using the probes closest to the wavemaker.

Similarly, if we were to consider only the probes furthest from the wavemaker, the data points would shift to the left of the data in Figure 4.12, to lower steepness. There is also less change when the steepness is relatively low, this makes sense as there is no breaking for very low steepness, and the

4.8. What can produce the group line?

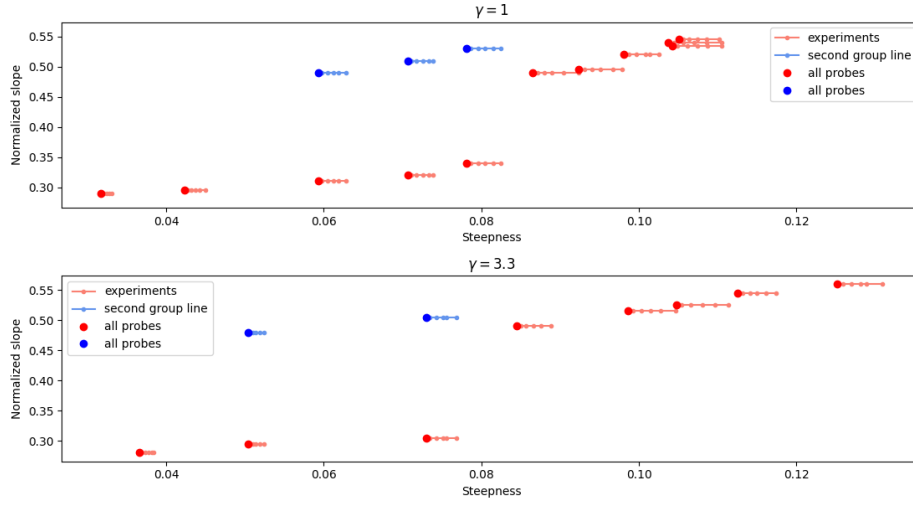


Figure 4.22: The dark dots are in the exact location as in Figure 4.12 and represent the steepness when we consider all the probes. The consecutive smaller points for the same slope is how the steepness changes with the number of probes considered when calculating the standard deviation of the surface elevation. The big dot has all 48 probes considered. The smaller dots represent 40, 30, 20, 10 and 1 probes in sequence. All the data can be seen in Table 4.4. $\gamma = 1$ is for the Pierson-Moskowitz runs and $\gamma = 3.3$ is for the JONSWAP runs. The slope is normalised with the phase speed of the peak wave.

waves would be pretty similar throughout the tank. However, the difference is not too big, even for very high steepness.

4.8 What can produce the group line?

Now that we have looked at different methods of measuring the surface elevation and different kinds of waves, can we say something about what is producing the group line?

If we are considering the experiments first, there have been both breaking and non-breaking waves, and they have both produced the group line. Also, the waves have been long-crested, and they have been measured top-down, meaning all the waves are considered with no shadowing. The simulations done for this thesis have also been long-crested, non-breaking waves, and they did also produce the group line. This means that breaking waves cannot be the sole reason for the group line.

We have also been considering radar images from different articles.

4.8. What can produce the group line?

γ	α	ϵ_{48}	ϵ_{40}	ϵ_{30}	ϵ_{20}	ϵ_{10}	ϵ_1
1.0	0.005	0.0317	0.0319	0.0322	0.0325	0.0328	0.0331
	0.01	0.0424	0.0428	0.0432	0.0437	0.0443	0.045
	0.02	0.0593	0.0598	0.0605	0.0612	0.0619	0.0628
	0.03	0.0707	0.0711	0.0718	0.0726	0.0733	0.0739
	0.04	0.0782	0.0787	0.0796	0.0805	0.0815	0.0825
	0.05	0.0865	0.0871	0.0880	0.0890	0.0904	0.0924
	0.06	0.0924	0.0931	0.0940	0.0951	0.0967	0.0979
	0.07	0.0980	0.0987	0.0997	0.1008	0.1014	0.1025
	0.08	0.1037	0.1045	0.1056	0.1070	0.1086	0.1105
	0.09	0.1041	0.1049	0.1061	0.1074	0.1087	0.11045
	0.1	0.1051	0.1056	0.1064	0.1074	0.1086	0.1102
3.3	0.005	0.0366	0.0369	0.0374	0.0378	0.0382	0.0384
	0.01	0.0504	0.0507	0.0510	0.0514	0.0519	0.0524
	0.02	0.0730	0.0735	0.0743	0.0751	0.0756	0.0768
	0.03	0.0845	0.0850	0.0857	0.0866	0.0876	0.0888
	0.04	0.0986	0.0993	0.1003	0.1015	0.1027	0.1047
	0.05	0.1048	0.1054	0.1066	0.1081	0.1097	0.1114
	0.06	0.1125	0.1132	0.1141	0.1150	0.1161	0.1174
	0.07	0.1252	0.1259	0.1270	0.1281	0.1289	0.1309

Table 4.4: Here we see the data of Figure 4.22. ϵ_n is the steepness where we have used the n probes closest to the wave paddle to calculate the standard deviation of the surface elevation. As we can see, the steepness decreases when the number of probes increases.

Compared to the experiments and simulations, these are short-crested and breaking. In addition, the waves in the articles that we have looked at are detected by radar with an inclination angle, and shadowing would be a reasonable explanation for the group line. However, Plant and Farquharson (2012) did consider shadowing in his article and did an analysis with data from an airship where the range of the grazing angles was between 25.6° and 53.1° . According to the article, these angles are too steep for any shadowing to happen and they are therefore disregarding shadowing in these experiments. They, however, still observe the group line and are thus concluding that shadowing cannot be a source of said feature. This would comply with the experiments done in this thesis as there is no shadowing in the experiments. As such, hydrodynamic interactions should be one of the leading causes for the group line.

In this thesis, we have considered long-crested, breaking and non-breaking waves. We have looked at long-crested, non-breaking waves in simulations, and we have been looking at the radar images of short-crested, breaking waves from Kammerer and Hackett (2019), Plant and Farquharson (2012) and Platzter et al. (2019b). If we are not entirely disregarding the shadowing hypothesis, then there are also different ways of acquiring the data. In the experiments, we used ultrasound probes that looked at the surface elevation from a top-down perspective. The articles we have discussed look at radar images taken with a grazing incidence angle. In total, there are 8, or 2^3 , different combinations that could be considered. We have only been looking at three of them in this thesis. This gives opportunities for a lot more research.

4.9 Bandwidth

During this thesis, we have been experimenting with the bandwidth of the spectrum. So far, four spectra, including the Gaussian, have been looked at. These can be seen all together in Figure 4.23.

Because the tail of the JONSWAP and the Pierson-Moskowitz spectrum is thin and narrow, we have opted in this thesis to narrow the bandwidth to only those ω that are above the 10 % threshold. While it takes away some frequencies, most of the frequencies will lie around the peak, and thus, there should be little significance in the cut frequencies. In Table 4.5 we can see the bandwidth for the different spectra.

From the wavenumber-frequency spectra, there seems to be no noticeable difference when changing the bandwidth of the spectrum. Neither in the shape, form or intensity of the group line nor in the full spectrum itself.

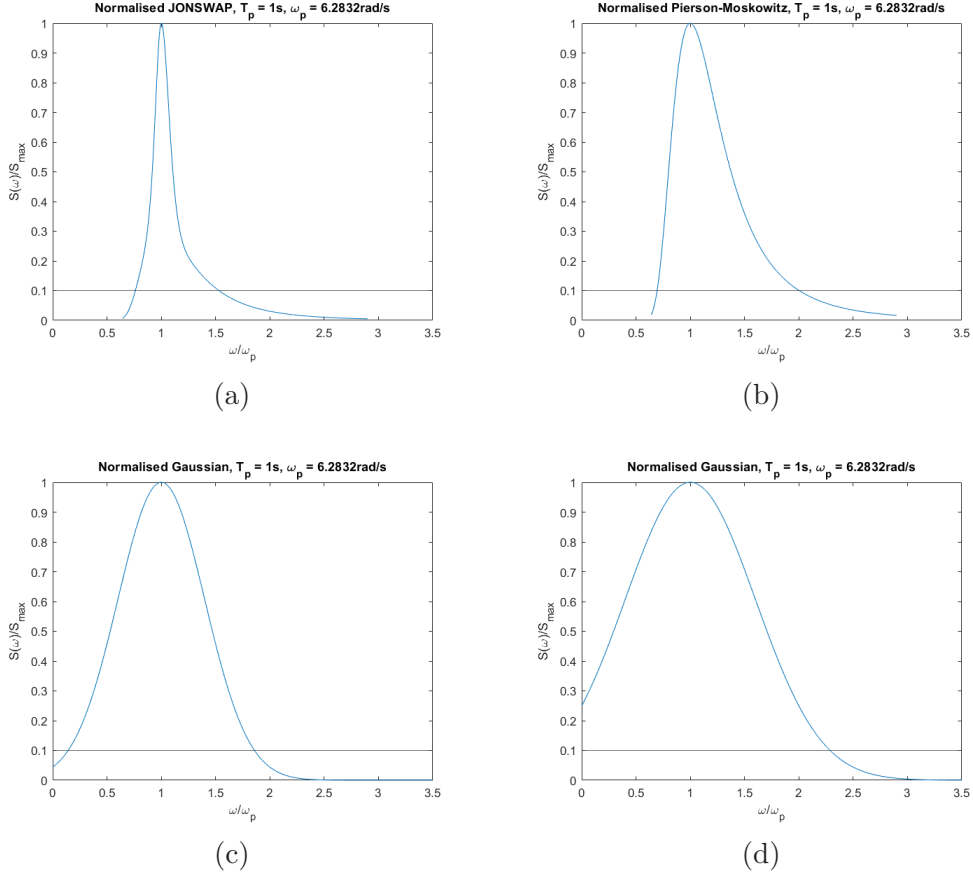


Figure 4.23: Here we have normalized all the different spectra used in this thesis. (a) is JONSWAP, (b) is Pierson-Moskowitz and (c) and (d) is Gaussian with $\sigma_g = 0.4$ and $\sigma_g = 0.6$ respectively.

Spectrum	$\omega_{min}[\text{rad/s}]$	$\omega_{max}[\text{rad/s}]$	Bandwidth $\left(\frac{\Delta\omega}{\omega_p}\right)$
JONSWAP	4.7812	9.6232	0.7706
Pierson-Moskowitz	4.3630	12.5898	1.3093
Gaussian, $\sigma_g = 0.4$	0.88975	11.6766	1.7168
Gaussian, $\sigma_g = 0.6$	0	14.3733	2.2876

Table 4.5: The table shows the bandwidth for the different spectra used. $\omega_p = 3.2832 \text{ rad s}^{-1}$ is the same for all the spectra.

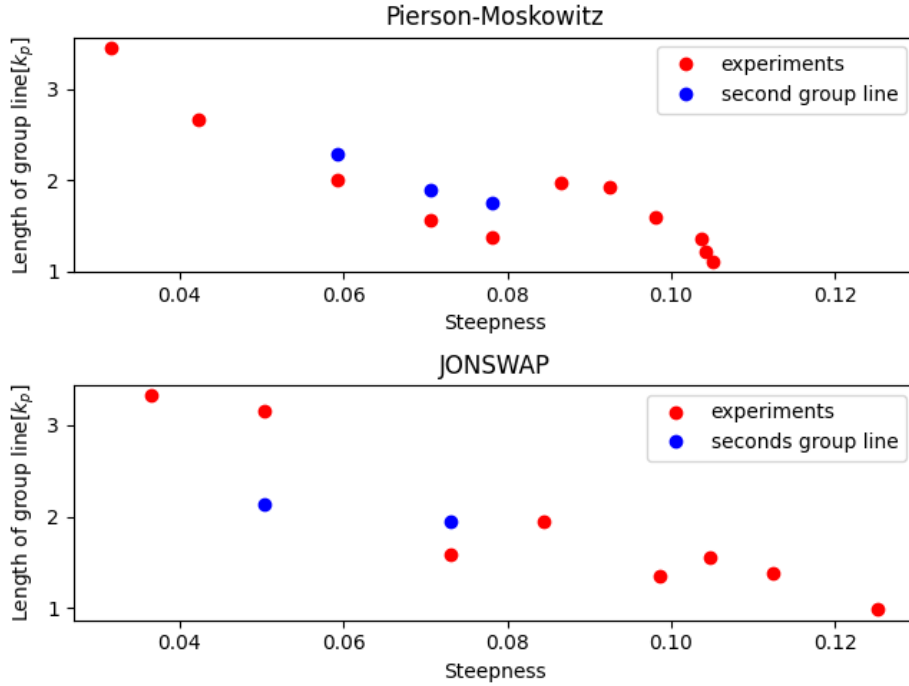


Figure 4.24: Here we see how the length of the group line changes with the steepness of the waves. The length is normalised as the axes are scaled, and we are looking at how far along the k -axis the group line goes. This means that the units on the y-axis are given by k_p .

From this, we can conclude that the bandwidth does not seem to affect the group line at all.

4.10 Length of the group line

When we look at the resulting spectra of the Pierson-Moskowitz, and the JONSWAP runs, we can see that the length of the group line varies for the different values of α . Because of this, we thought it interesting to see if the length is changing because of the steepness of the waves. Thus, in Figure 4.24 we have plotted the length of the group line against the steepness of the waves. Here, the length is not defined as the length seen in the spectrum, but rather, how far along the k -axis it goes.

It should be noted that the length has been empirically measured and that it is ambiguous how long it actually is. While some might see the group line as longer or shorter than what is measured, we have tried to be as consistent in the measuring as possible.

As we can see from Figure 4.24 there is a clear decrease in the length when the steepness is increasing. This is surprising as the second order contributions should be stronger with increasing steepness. However, it should be noted that there is also more noise in the spectra from the higher steepness waves and that the group line might be concealed in the noise. The numerical data of Figure 4.24 can be seen in Table 4.6.

When looking back at Figure 4.1-4.4 it seems that the intensity of the group lines are slowly shifting from the gentler slope to the steeper slope. In Figure 4.3a and 4.3b one can barely see something that could be a second group line. Looking at Figure 4.3c-4.3e, it seems that the gentler group line is stronger for the lower steepness and then the intensity slowly decreases. For the steeper slope, the opposite seem to happen. This could mean that second order, non-linear interactions and wave breaking are both responsible for the group line, but their contributions changes with the steepness of the waves.

In Figure 4.4 the steepness of the waves is so high that the breaking might overshadow the second order, non-linear interactions and this might be the reason why we only see one group line. This might also explain the results from Figure 4.24 where we see that the group line with steeper slope of always longer than the group line with gentler slope, except for the one case. It could be that the waves are not steep enough, and that the second order, non-linear interactions therefore contribute more than the wave breaking does, which makes the gentler group line longer than the steeper group line.

4.10. Length of the group line

γ	α	Steepness	Length of the group line [k_p]	Length of the second group line [k_p]
1.0	0.005	0.0317	3.45	-
	0.01	0.04237	2.67	-
	0.02	0.0593	2.00	2.29
	0.03	0.0707	1.56	1.89
	0.04	0.07816	1.37	1.75
	0.05	0.0865	1.97	-
	0.06	0.0924	1.93	-
	0.07	0.0981	1.59	-
	0.08	0.1037	1.35	-
	0.09	0.1042	1.21	-
	0.1	0.1051	1.11	-
3.3	0.005	0.0366	3.32	-
	0.01	0.0504	3.16	2.14
	0.02	0.0730	1.59	1.95
	0.03	0.0845	1.95	-
	0.04	0.0986	1.35	-
	0.05	0.1048	1.55	-
	0.06	0.1125	1.39	-
	0.07	0.1252	0.99	-

Table 4.6: Here we have the numerical data of Figure 4.24. The second group line is meant as the group line with the steeper slope. The length is given in k_p along the k -axis.

CHAPTER 5

Conclusion

The group line is a feature that appears in the frequency area close to zero in the (k, ω) spectrum. It is shaped like a cloud or a line and can be seen beneath the dispersion relation close to the origin.

We have been running experiments with long-crested, breaking and non-breaking waves in order to check if there is a noticeable difference in the group line between the breaking and non-breaking wave fields.

What we found is that the slope of the group line seems to change when the steepness of the waves increases and that there seems to be a jump in the steepness when the waves are breaking compared to when they are not. In addition, in some cases we have observed two group lines with slopes that fit into both of the breaking and the non-breaking cases. The second group line appears when the waves are almost breaking and it seems that the intensity of the two group lines shifts from the lower group line to the upper group line when the steepness of the waves and the intensity of the breaking increases.

To compare the experiments, we have been simulating long-crested, non-breaking waves and compared the group line with articles from Plant and Farquharson (2012) and Platzter et al. (2019b). We find that for the simulations, the slope of the group line does not change with the steepness of the waves. The slopes of Plant and Farquharson (2012) and Platzter et al. (2019b) seem to align with slope of the experiments with lower steepness. The difference with the experiments done and data from the articles are that we have used ultrasound probes and looked at long-crested waves while Plant and Farquharson (2012) and Platzter et al. (2019b) have data from radars that are looking at short-crested waves.

We have used the Pierson-Moskowitz spectrum and the JONSWAP spectrum for our waves in order to see if the bandwidth of the waves have an impact on the group line. From what we can see, there seems to be no

noticeable change in the the shape, form or intensity of the group line.

Lastly, we have been looking at the length of the group line. When plotting the length along the k -axis, we can see that the length of the group line has a consistent decline when the steepness of the waves increases.

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