MAT3500/4500 Topology
Autumn 2010
Mandatory Assignment (1 of 1)

Turn in your paper before 14:30 on November 4th, in the box in the 7th floor corridor in the Niels Henrik Abel building. Remember to use the official cover page, available next to the box. You are advised to turn in the paper early; experience shows that there are long queues around the deadline. If you will need to submit after the deadline, due to illness or other circumstances, apply to Robin Bjørnetun Jacobsen: office: B718 in the Niels Henrik Abel building, e-mail: studieinfo@math.uio.no, phone: 2285 5882. Remember that illness has to be documented by a medical doctor. See http://www.math.uio.no/academics/obligregler-eng.shtml for more information about the rules concerning mandatory assignments at the Department of Mathematics.

Students who do not get their paper accepted will not get access to the final exam. To get the paper accepted, you must achieve at least 50% correct answers. You may get partial credit for a problem if you have used a correct method, even without having reached the correct answer, so always show your work. In solving the problems you may collaborate with others and use all tools available. However, the paper you turn in must be written by you personally (by hand or typeset), and it should reflect your understanding of the material. If we are not certain that you understand your own paper, we may ask you to give an oral presentation of its contents.

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Let

$$C = \mathbb{R}^2 - \{(0,0)\}$$

be the complement of the origin $(0,0)$ in the Euclidean plane $\mathbb{R}^2$, with the subspace topology. Let

$$D = \{\mathbf{e}, \mathbf{n}, \mathbf{w}, \mathbf{s}\}$$

be a 4-element set. (Think of the compass directions: east, north, west and south.) Let $p\colon C \to D$ be the function given by

$$p(x,y) = \begin{cases} \mathbf{e} & \text{if } x > 0 \text{ and } y = 0, \\ \mathbf{n} & \text{if } y > 0, \\ \mathbf{w} & \text{if } x < 0 \text{ and } y = 0, \\ \mathbf{s} & \text{if } y < 0 \end{cases}$$

for $(x, y) \in C$. Give D the quotient topology from C, so that p is a quotient map. The word "map" always means "continuous function". Give justified answers to the following questions, which all carry equal weight.

(1) Find all open subsets of D.

(2) Is D a Hausdorff space?

(3) Is D compact?

(4) Is D homeomorphic to a product $A \times B$ of two 2-point spaces A and B?

(5) Find all closed subsets of D.

(6) Find the closure of each singleton set in D.

(7) Show that any map $f\colon D \to \mathbb{R}$ is constant.

(8) Is there a map $r\colon D \to C$ such that $p \circ r\colon D \to D$ is the identity map?

(9) Is p an open map?

(10) Is p a closed map?

(11) Is D connected?

(12) Construct a continuous path in D from $\mathbf{e}$ to $\mathbf{w}$.

(13) Let the maps $\alpha, \beta\colon C \to C$ be given by $\alpha(x, y) = (x, -y)$ and $\beta(x, y) = (y, x)$.

$$\begin{tikzcd} C \arrow[r, "\alpha \text{ or } \beta"] \arrow[d, "p"'] & C \arrow[d, "p"] \\ D \arrow[r, dashed, "k"] & D \end{tikzcd}$$

Which of the two maps $p \circ \alpha, p \circ \beta\colon C \to D$ factor as $k \circ p$, with $k\colon D \to D$ a map?

(14) Find all homeomorphisms $h\colon D \to D$.

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John Rognes, October 14th 2010